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Use of IAs based approach for the Cooperative Periodic Market Problems

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Abstract: - This study investigates the two traders' periodic marketing problem in which both the spatial (where) and temporal (when) dimensions are to be decided simultaneously so as to maximize the total payoff during trading based on the itinerary. In literature, the periodic marketing problems are studied very few for the past decade, usually using the sequential strategy approaches. However, the solution by using sequential strategy tends to find the local optimal solution but not the global optimal solution. The difficulties encountered in the problem include: the spatial behavior of the individual consumers is uncertain, customers may from other neighboring sites based on the distance, the demand of a product is needed based on a specific time period (cycle), not all the market places need to be stayed and the same market place can be visited more than once. In this paper, an immune algorithm is used for solving such nonlinear periodic marketing problem. Numerical examples indicate that the proposed approach can efficiently and effectively search over promising solution regions to exhibit the superior performance of the proposed methodology for finding the optimal itinerant strategy during the specified planning horizon.

Key-Words: - Periodic marketing, Immune, Genetic algorithm, Sequential strategy

1 Introduction

In the real world, a special market system, where people get together to trade, with lowest level of trading function still exists only periodically. Such periodic marketplaces play an important role in the integration of the regional marketing system and the spatial organization of economic activities (Ghosh, 1982). Unlike the classical traveling salesman problem (Lawler et al., 1985), in the periodic marketing problem a traveling trader doesn't need to visit all cities (sites or market places) and the trader can visit the same city more than once with the condition of periodical product demand. Moreover, some customers in a city may travel to another city to purchase the product while they have the demand of the goods and the travel distance is not too far away.

The marketplace with the periodicity brings the advantage to the trades since their diverse economic role preclude them from full-time trading (Bromley et al., 1975) (Ghosh, 1982). However, it makes the traders face a hard decision problem because he/she has to decide how many and which cities to visit and when it is more profitable to be itinerant or fixed. Obviously, the location pattern of the problem can be characterized by both a spatial (where) and a temporal (when) dimension (Ghosh, 1982). The trader's objective is to maximize the total payoff within the specified planning horizon *T*.

While making the decisions of route planning for the periodic marketing problems, some factors are usually concerned by researchers (Eiselt et al., 1993). Generally, the factors include: (A) the number of traders: 1. single trader or 2. multiple traders; (B) customer behavior: 1. deterministic behavior manner, 2. probabilistic behavior manner; (C) Spatial structures: 1. linear market (one dimension), or 2. multidimensional space ; (D) pricing policy: 1. uniform delivered pricing, 2. discriminatory pricing; (E) rule of the game (for multiple traders): 1. cooperative 2. non-cooperative (competitive) (F) the number of commodities: 1. single, 2. multiple. Considering the different combination of the above six items, many works have been proposed. In traversing from the A1-B1-C1-D1-E1-F1 type to

A2-B2-C2-D2-E2-F2 type, the problem associated with the planning route decisions will become much more difficult and complex. In this paper, the problem in the form of A2-B2-C2-D1-E1-F1 is to be investigated and solved.

Although the issues about periodic marketing problem have been discussed by some researchers (Hay, 1971) (Symanski and Bromley, 1974), the spatial and temporal dimensions were seldom considered simultaneously. It is supposed that the conditions under which a trader becomes mobile and also establish the optimal location strategy for the trader should be identified by an ideal model of itinerant trading. Moreover, it is found that the single trader was considered in most of the literature in past decade (Bromley et al., 1975) (Ghosh, 1982) (Chen and Hsieh, 2003). In this study, the cooperative multiple traders' problem are investigated. The model has been illustrated in the next section. The periodic marketing problem, an *NP-hard* optimization, includes both temporal and spatial decisions at the same time. The exhausted search approach can provide the optimal solutions but it is not attractive in practice for solving the larger size problem, science it is very time consuming.

This paper is arranged as follows: in the next section the multiple traders' periodic marketing problem is briefly described; in Section 3 the general concept of an immune algorithms based approach is described and numerical examples of periodic marketing problems are solved and discussed in Section 4. Finally, the conclusion of the paper is summarized.

2 Problem Formulation

Nomenclature

- N the number of the cities
- M the number of traders
- T planning horizon
- R_{mit} the m^{th} trader's payoff from city i at time period t
- C_{mij} traveling cost to next city j from city i for the m^{th} trader
- C_g the unit sales price of a product
- X_{mit} 1 if city i is located by the m^{th} trader at time t, 0 otherwise
- Y_{mijt} 1 if city j is to be located by the m^{th} trader from city i, 0 otherwise
- I demand interval for the product
- u_m exponent on distance to the m^{th} trader

The objective of the periodic marketing problem is to maximize the sum of all traders' payoff by selecting the best marketing site from all cities for each trader based on each time period within the planning horizon. In other words, find the best strategy for each trader's trading to obtain the best sequence of itinerary that yields the maximum total payoff of all traders. The model is modified the periodic marketing problem by Ghosh, 1982 and described as follows:

$$Max. \sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{i=1}^{N} R_{mit} X_{mit} - \sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{i=1}^{N}\sum_{j=1}^{N} C_{mij} Y_{mijt} \quad (1)$$

Subject to

$$\sum_{i=1}^{N} X_{mit} = 1 \text{ for all } t \text{ and } m \quad (2)$$

$$X_{mit} \le \sum_{j} Y_{mijt} \text{ for all } i, t \text{ and } m \quad (3)$$

$$X_{mit} \le \sum_{k} Y_{m,k,i,t-1} \text{ for all } i, t \text{ and } m \quad (4)$$

Several assumptions are made as follows:

(A) only a single product is to be sold and each consumer buys only a single unit of the product during a single visit to the marketplace.

(B) the spatial behavior of individual customer is uncertain, i.e., the number of people expected to visit a specified site i of market from any neighboring city j can be expressed as $P_{jt} d_{ij}^{-u_m}$, where d_{ij} is the distance from i to j and u_m is an exponent on distance to the m^{th} trader which influence the spatial behavior of customers. Based on the assumption, R_{mit}, the m^{th} trader's expected payoff at location I at time t, can be expressed as follows:

$$R_{mit} = C_g \sum_{j=1}^{N} P_{jt} d_{ij}^{-u_m} \quad (5)$$

(C) the demand of a product at every marketplace satisfies each customer for I time periods, so that the number of people in city j who have the demand at time t can be formulated follows:

$$P_{jt} = \begin{cases} P_{j,t-1} - w_{j,t-1} & if \ t \le I \\ P_{j,t-1} - w_{j,t-1} + w_{j,s} & if \ t > I \end{cases} \quad (6)$$

where, $s = t - I$ and $w_{jt} = P_{jt} d_{ij}^{-u_m}$.

(D) the price of production does not vary over space and time (i.e., uniform delivered pricing).

The geometric programming and approximate techniques used for solving the global optimum planning are generally time consuming due to the complex transformation. Furthermore, the exact solutions for the optimization problems are not necessarily desirable because it's arduous to achieve the exact solutions, and even when they are available, their merit may turn into marginal. Due to the difficulties of applying the approximate and exact techniques, a major part of the work on solving this optimization problem is devoted to developing heuristic/metaheuristic algorithms. Above all, the genetic algorithms become very popular tools for solving the problem successfully. Although genetic algorithms can be easily designed and implemented without the requirement of sophisticated mathematical treatment, the difficulties are in the determining appropriate values for the parameters. If the parameters are not assigned properly, the genetic algorithms will more likely converge to a local optimum and hard to reach the global optimum. One of the characteristics of immune algorithms based approach mentioned in previous section, the global optimum could be more easily achieved than genetic algorithms since the diversities of the feasible spaces can be better ensured. For the above reason, the immune algorithms based approach is applied for solving the periodic marketing problems in this research.

3 Immune Algorithms

The natural immune system of all animals is a very complex system for defense against pathogenic organisms. A two-tier line of defense is in the system including the innate immune system and the adaptive immune system. The basic components are lymphocytes and antibodies (Farmer et al., 1986). The cells of the innate immune system are immediately available to combat against a wide variety of antigen without previous exposure to them. The antibody production in response to a determined infectious agent (antigen) is the adaptive immune response mediated by lymphocytes which are responsible for recognition and elimination of the pathogenic agents (De Castro and Timmis, 2002). The cells in the adaptive system are able to develop an immune memory so that they can recognize the same antigenic stimulus when it is presented to the organism again. Also, all the antibodies are produced only in response to specific infections. There are two main types of lymphocytes: B-lymphocytes (B-cell) and T-lymphocytes (C-cell). B-cell and T-cell carry surface receptor molecules capable of recognizing antigens. The B-cells produced by the bone marrow show a distinct chemical structure and can be programmed to make only one antibody that is placed on the outer surface of the lymphocyte to act as a receptor. The antigens will only bind to these receptors with which it makes a good fit.

To distinguish and eliminate the intruders of the organism is the main task of the immune system so that it must has the capability of self/non-self discrimination. As mentioned previously, various antibodies can be produced and then can recognize the specific antigens. The portion of antigen recognized by antibody is called epitope which acts as an antigen determinant. Every type of antibody has its own specific antigen determinant which is called idiotope. Moreover, in order to produce enough specific effector cells to against an infection, and activated lymphocyte has to proliferate and then differentiate into these effector cells. This process is called clonal selection and followed by the genetic operations such that a large clone of plasma cell is formed. Therefore, the antibodies can be secreted and ready to bind antigens. According to above facts, Jerne (1973) proposed an idiotype network hypothesis which is based on the clonal selection theory. In his hypothesis, some types of recognizing sets are activated by some antigens and produce an antibody which will then activate other types of recognizing sets. By this way, the activation is propagated through entire network of recognizing sets via antigen-antibody reactions. It is noted that the antigen identification is not done by a single or multiple recognizing sets but by antigen-antibody interactions. From this point of view, for solving the combinatory optimization problems, the antibody and antigen can be looked as the solution and objective function respectively. Based on the nature immune system, the clone process is included in genetic algorithm for improving the performance for solving the periodic marketing problems.

3.1 Computation Procedures

The computation procedures of the proposed immune based genetic algorithm illustrated in Figure 1 work as follows and the discussion comes in sequence:

Step 1. Randomly generate an initial population of antibodies.

Step 2. Evaluate the corresponding affinity value for each individual.

Step 3. Choose the best k individual with highest affinity values and then clone these k antibodies.

Step 4. The set of the clones in previous step will suffer the genetic operation process, i.e., crossover and mutation (Michalewicz, 1996).

Step 5. Update the memory set. The process includes replacement and elimination. Firstly, calculate the new affinity values of these new individuals (antibodies). Select those individuals who are superior to the individuals in the memory set, and then the superior individuals replace the inferior individuals in the memory set. Secondly, antibodies will be eliminated while their structures are too similar. So the individuals in the memory set can keep the diversity.

Step 6. Check the stopping criterion, if not stop then go to Step 2. Otherwise go to next step.

Step 7. Stop. The best solution(s) can be provided from the memory set.

Note the clone size for each selected individual is an increasing function of the affinity with the antigen. In other words, the number of posterity of each antibody is proportional to their fitness values, i.e., the higher the fitness, the larger the clone size. (De Castro and Von Zuben, 2000)

In our implementation, the integer solutions are represented by strings of binary digits. Each string consisting of substring includes the length of trading cycle for each traders and the marketplace selected for each trader. The details have been described in next section. For further details about the clonal selection and affinity maturation processes in Step 5, please refer to De Castro and Von Zuben (2000). The stopping criterion is the maximum iterations in this article.

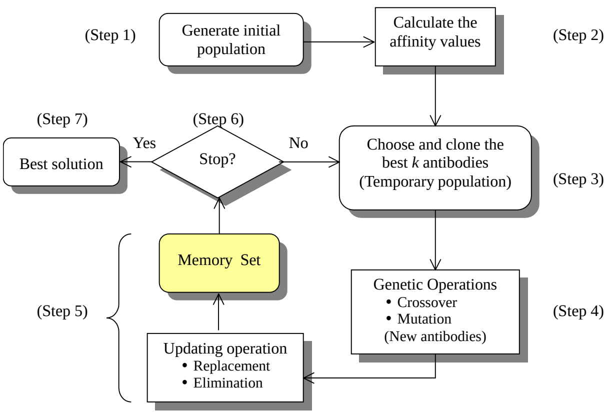

Fig.1 The immune based approach

3.2 The Representation Mechanism

The structure of the solution representation mechanism is proposed. In our implementation, solutions can be represented by strings of binary digits with a special structure as illustrated in Figure 2. The string is divided into $M+1$ substrings and to be decoded by two stages. The first substring represents the length of the cycle and the other subsrings represent the potential marketplaces in the cycle for each of M traders respectively. The cycle length L is to be determined in the first decoding stage. Consequently, each of the M substrings is then divided by L substrings representing the potential marketplaces in the cycle for each trader. While the marketplaces in a cycle are obtained for each trader, all traders' tours can be determined by fulfill their cycles into the planning horizon individually (see Figure 3). Moreover, due to the proposed solution representation in immune based approach, the complexity of the precedent constraints (2), (3) and (4) can be minimized tremendously. It makes the violation of all constraints be avoided.

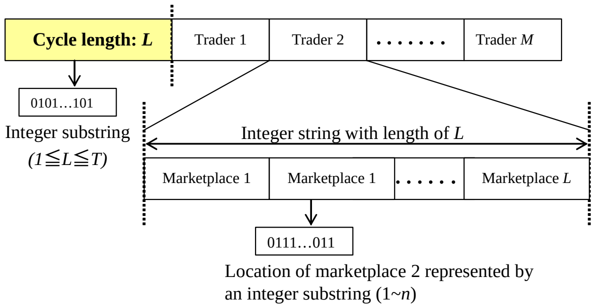

Fig.2. Solution representation

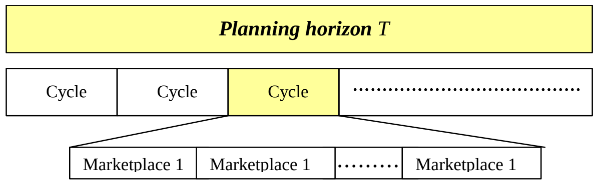

Fig.3 A number of cycles is composed in horizon T.

3.3 Affinity of Antibodies

All the antibodies are to be evaluated by the affinity to the antigen. The affinity of every antibody (a coded string) is taken to be the expected revenue during the planned horizon. In the combinatorial optimization model, the affinity function of immune algorithm is the objective function (1). It is to find the best sequence of cities so as to maximize all traders' payoff and minimize their transportation costs within the predefined length of time period.

3.3 Genetic Operators and Evaluation

The implementation of genetic operators including the crossover operator and mutation operator requires the selection of the crossover point(s) and mutation point(s) for each string under a predetermined crossover probability and mutation probability. The crossover operator provides a through search of the sample space to produce good solutions. The mutation operator performs random perturbations to selected solutions to avoid the local optimum.

4 Numerical Results and Discussion

To evaluate the performance of the proposed approach for the periodic marketing problems, an example problem with four cities are solved. For each city, the corresponding location and population were shown in Figure 4.

In the experiment, several assumptions are made: the price of unit product (C_g) is 20 and the traveling cost (C_{ij}) for the trader is 10 per unit of distance. The distance between any pair of cities is measured by using Euclidean distance in our example. Our method is implemented in MATLAB® on the Pentium-4 2.0 GHz personal computer and the suitable parameters' settings of the proposed immune algorithm were experimentally determined. The proposed approach was used to analyze the example periodic marketing problem and compared with the solution obtained by using exhausted search method.

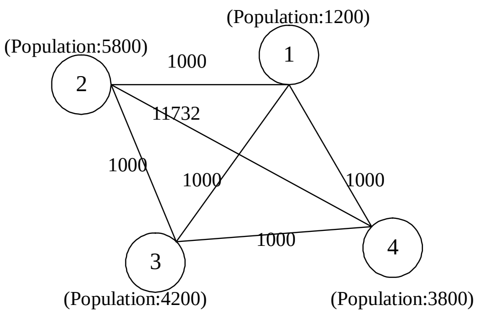

Fig. 4 The numerical instance.

The results obtained by using exhausted search method and proposed approach are compared and shown in Table 1. The numerical results of variant instances show both of the optimal solutions found by using exhausted search method and the by the proposed method. However, with the increase of problem size, the CPU time for solving each example by the proposed is slightly and linearly increased. Oppositely, the CPU time is increased exponentially by the exhausted search approach. For example, while the three cities are considered and T=3, the CPU time used for the exhausted search approach and proposed approach are 10 seconds and 39 seconds respectively. However, while four cities are considered and T=5, the CPU time used for the exhausted search approach and proposed approach are 143790 seconds and 68 seconds respectively. According to the observation, it shows the proposed is able to find the optimal solutions effectively and efficiently.

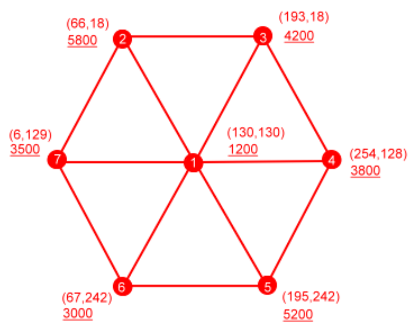

Fig. 5 An extended numerical instance.

Moreover, an extend example is illustrated in Figure 5. The numbers in the parentheses are the coordinates of the cities and the numbers with underline are the population size of the city. Another approach is also proposed which is greedy algorithm. The greedy approach is obeying the following process:

(1) *At the end of each time period, every trader determines the most profitable of the alternative locations and compares the expected payoff at the location to that expected at the present location.*

(2) *If the payoff at the alternative site exceeds that at the present site, the trader moves to that site.*

The greedy approach is used by the real world by making decision more conveniently. The solutions obtained by using proposed approach and greedy approach are compared in Table 2. As shown in the table, the solution found by using greedy is worse than those by immune algorithm. For a small size problem, the greedy can find solutions as well as the immune algorithm. While the problem is becoming larger; the performance of greedy is become worse. It shows the proposed immune algorithm is able to provide good quality of solutions.

Generally, from the numerical results, several observations are illustrated as following:

(1). Optimal itinerary strategy may not be unique and the proposed immune algorithm is able to provide the multiple optimal itinerating routs for the two traders. Given City = 5 and I=4 in Table 2, immune algorithm provided 9 different itinerary strategies while greedy algorithm provided only one itinerary strategy.

(2). Itinerary strategy found by immune algorithm in the planning horizon, a cycle can be found for each of trader. For instance in Table 2, while I=4 and City=7, the Trader_1 is with a cycle 2-3-2-2 and Trader_2 is with another 5-4-5-5.

According to the numerical examples, it is indicates that the proposed immune algorithm can provide the optimal or near optimal itinerary strategy for the traders.

4 Conclusion

This paper provides a metaheuristic approach, using the proposed immune algorithm based searching approach, to solve the periodic marketing problem with two cooperative traders which is represented by the itinerant trading model. In the problem, the traveling cost, consumer travel and product demand characteristics are all considered. In our problem, both the spatial (where) and temporal (when) dimensions are to be decided simultaneously so as to maximize the total payoff during trading based on the itinerary with specified length of the planning horizon. The advantage of the proposed methodology is the solution to the normative questions. For example, what is the most profitable strategy for the itinerant trading plan and how many of goods are meet the least demands within the planned horizon for all traders.

A more considerations of this research that we are going to study include multiple types of product with multiple traders in this model, etc. It is to investigate that the best strategy for the multiple traders with selling multi-products should be to compete or cooperate with each other

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Table 1. Solution comparison of exhausted search approach and immune algorithm.

		Exhausted sSearch				Immune Algorithm			
City	T	Optimal route		Payoff	CPU	Best route		Payoff	CPU
		Trader_1	Trader_2	($\times10^6$)		Trader_1	Trader_2	($\times10^6$)	
City 1~3	4	2-2-2-2	3-3-3-3	**0.43145**	10	2-2-2-2	3-3-3-3	**0.43145**	39
		3-3-3-3	2-2-2-2			3-3-3-3	2-2-2-2		
	5	2-2-2-2-2	3-3-3-3-3	**0.63890**	936	2-2-2-2-2	3-3-3-3-3	**0.63890**	49
		3-3-3-3-3	2-2-2-2-2			3-3-3-3-3	2-2-2-2-2		
	6	2-2-2-2-2-2	3-3-3-3-3-3	**0.64617**	87660	2-2-2-2-2-2	3-3-3-3-3-3	**0.64617**	61
		3-3-3-3-3-3	2-2-2-2-2-2			3-3-3-3-3-3	2-2-2-2-2-2		
City 1~4	4	3-2-2-2	4-4-4-3	**0.56345**	1404	3-2-2-2	4-4-4-3	**0.56345**	55
		3-4-4-4	2-2-2-3			3-4-4-4	2-2-2-3		
		4-4-4-3	3-2-2-2			4-4-4-3	3-2-2-2		
	5	2-2-2-2-2	3-3-3-3-3	**0.75797**	143790	2-2-2-2-2	3-3-3-3-3	**0.75797**	68

Note: I=2, u_1=0.2 u_2=0.2C_g=20, C_{ij} =10

Table 2. Solution comparison of immune algorithm and greedy algorithm for extended instance.

		Immune Algorithm			Greedy Algorithm		
City	I	Trader_1	Trader_2	Payoff($\times10^7$)	Trader_1	Trader_2	Payoff($\times10^7$)
3	2	2-2-2-2-2-2-2-2-2-2	3-3-3-3-3-3-3-3-3-3	**0.1075495199000**	2-2-2-2-2-2-2-2-2-2	3-3-3-3-3-3-3-3-3-3	**0.1075485199000**
5		3-2-3-2-3-2-3-2-3-2 2-3-2-3-2-3-2-3-2-3 2-3-2-3-2-3-2-3-2-3 4-5-4-5-4-5-4-5-4-5	4-5-4-5-4-5-4-5-4-5 5-4-5-4-5-4-5-4-5-4 4-5-4-5-4-5-4-5-4-5 3-2-3-2-3-2-3-2-3-2	**0.1795485199391**	2-3-2-3-2-3-2-3-2-3	5-4-5-4-5-4-5-4-5-4	**0.1795485199391**
7		4-5-4-5-4-5-4-5-4-5	3-2-3-2-3-2-3-2-3-2	**0.2182644781197**	2-3-3-2-3-7-2-3-7-2	5-4-4-5-4-6-5-4-6-5	0.1604102156703
3	3	2-2-2-2-2-2-2-2-2-2 3-3-3-3-3-3-3-3-3-3	3-3-3-3-3-3-3-3-3-3 2-2-2-2-2-2-2-2-2-2	0.0859996580300	2-2-2-2-2-2-2-2-2-2	3-3-3-3-3-3-3-3-3-3	0.0859996580300
5		5-5-5-5-5-5-5-5-5-5 2-2-2-2-2-2-2-2-2-2	2-2-2-2-2-2-2-2-2-2 5-5-5-5-5-5-5-5-5-5	**0.1329748560102**	2-3-3-2-3-3-2-3-3-2	5-4-4-5-4-4-5-4-4-5	0.1304961655664
7		2-2-2-2-2-2-2-2-2-2	5-5-5-5-5-5-5-5-5-5	**0.1695141041139**	2-3-3-2-3-7-2-3-7-2	5-4-4-5-4-6-5-4-6-5	0.1604102156703
3	4	2-2-2-2-2-2-2-2-2-2 3-3-3-3-3-3-3-3-3-3	3-3-3-3-3-3-3-3-3-3 2-2-2-2-2-2-2-2-2-2	**0.0650671033000**	2-2-2-2-2-2-2-2-2-2	3-3-3-3-3-3-3-3-3-3	**0.0650671033000**
5		2-3-3-2-2-3-3-2-2-3 2-3-3-3-2-3-3-3-2-3 2-3-2-2-2-3-2-2-2-3 2-3-2-2-2-3-2-2-2-3 2-3-3-3-2-3-3-3-2-3 2-3-2-2-2-3-2-2-2-3 2-3-3-2-2-3-3-2-2-3 2-3-3-3-2-3-3-3-2-3 2-3-3-2-2-3-3-2-2-3	5-4-4-4-5-4-4-4-5-4 5-4-5-5-5-4-5-5-5-4 5-4-5-5-5-4-5-5-5-4 5-4-4-4-5-4-4-4-5-4 5-4-4-5-5-4-4-5-5-4 5-4-4-5-5-4-4-5-5-4 5-4-5-5-5-4-5-5-5-4 5-4-4-4-5-4-4-4-5-4 5-4-4-5-5-4-4-5-5-4	**0.1090671033006**	2-3-3-3-2-3-3-3-2-3	5-4-4-4-5-4-4-4-5-4	**0.1090671033006**
7		2-3-2-2-2-3-2-2-2-3	5-4-5-5-5-4-5-5-5-4	**0.1355705058268**	2-3-3-3-2-3-7-7-2-3	5-4-4-4-5-4-4-4-5-4	0.1342727635773

Note: a cycle is denoted by an underline.