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Study of prediction of linear motion guide rigidity through grey modeling of linear differential and linear difference equations

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Abstract: The purpose of this paper is to compare the prediction models GM (1,1) constructed through grey linear differential equation and grey linear difference equation, and to apply the prediction models to study of linear motion guide rigidity prediction. Based on measurement data study of prediction of linear motion guide rigidity could be conducted to serve as the basis for quality improvement and effective selection. Grey modeling is employed for rigidity prediction to provide references for linear motion guides manufacturers and users. The prediction model established through grey linear differential equation is based on the continuous system, which is superior to that established by grey linear differential equation. The prediction results also show the error of grey linear differential modeling is less significant than that of grey linear difference modeling.

Key-words: grey linear differential equation grey linear difference equation linear motion guide

1. Introduction

Linear motion guides can be seen as a special bearing. It plays an important role in LCD, semiconductor automatic processing equipment, medical apparatus, CNC machine tools and nano-micro-processing device manufacture process. Any automatic equipment related to linear motion needs to use this important part. So, linear motion guide is a critical part. In addition, rigidity is one of the quality factors of linear motion guides. This paper based on the measurement data and less amount of time employs grey prediction modeling, which gives linear motion guide rigidity prediction. Results of linear motion guide rigidity prediction are available to manufacturers and users. They can also serve as reference for testing of other types of linear motion guides, and it reduce testing cost and profitability of the industry could be enhanced.

Developed by Deng [1], the grey theory is about systematic analysis of limited information. Under the constraint of incomplete information, it engages in prediction, analysis and decision-making pertinent to the system. The grey theory has helped many social, economic and environmental studies attain significant results [2-9].

In essence, the information that grey theory deals with is discrete signal. Yet conventional grey modeling employs linear differential equation first for modeling, and then causes discreteness. As a result, how to define the interval of signal sampling becomes a problem. Conversion of a continuous modeling system into a discrete modeling system is also prone to modeling system errors. This study engages in modeling directly through grey linear difference equation and avoids the problems mentioned above.

This paper comprehensively explains the prediction models constructed through grey linear differential equation and grey linear difference equation. It also applies the prediction models to study of linear motion guide rigidity prediction and compares the differences of the outcomes.

2. Grey Linear Differential Modeling

Traditional prediction model grey modeling GM(1,1) employs accumulated generation operation (AGO) and first order constant coefficient linear differential equations to process primitive sequence, determines the coefficient of linear differential equations through least squares algorithm, and after the solution is obtained engages in system recovery, prediction and evaluation through inverse accumulated generation operation (1-IAGO).

The white differential Shadow Equation of GM (1,1) is $\frac{dx^{(1)}}{dt} + ax^{(1)} = b$, in which a is the developed coefficient and b is the Grey controlled variable of the system. The method of modeling is as follows:

1) Input primitive sequence

Input initial sequence of the model

$$x^{(0)} = \{x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \cdots, x^{(0)}(n)\} \quad (1)$$

2) AGO

Form new sequence through AGO:

$$x^{(1)}(i) = \sum_{j=1}^{i} x^{(0)}(j)$$

$$x^{(1)} = \{x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), \cdots, x^{(1)}(n)\} \quad (2)$$

3) Determine the whitening value $z^{(1)}(k)$

Through AGO the grey background values of the grey derivatives $D(k)$ of sequence $x^{(1)}$ include the hazy set. The whitening value $z^{(1)}(k)$ that contains hazy set is

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1)$$

4) Determine B and Yn through least square method

$$Yn = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, \quad B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \\ -z^{(1)}(n) & 1 \end{bmatrix},$$

$$\hat{a} = \begin{bmatrix} a \\ b \end{bmatrix} \quad (3)$$

5) Solve the equation for a, b

Throu $Y_n = B\hat{a}$ and matrix computation rules we can determine

$$\hat{a} = (B^T B)^{-1} B^T Y_n = \begin{bmatrix} a \\ b \end{bmatrix} \quad (4)$$

6) List response equation $x^{(1)}(k+1)$

Through the shadow equation of white differential

$\frac{dx^{(1)}}{dt} + ax^{(1)} = b$ we obtain

$$x^{(1)} = ce^{-at} + \frac{b}{a} \quad (5)$$

7) Determine the response equation

$$\hat{x}^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a} \right] e^{-ak} + \frac{b}{a} \quad (6)$$

8) Recover the prediction value $\hat{x}^{(0)}(k+1)$

Through 1-IAGO we obtain

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k)$$

$$= (1 - e^{a}) \left[x^{(0)}(1) - \frac{b}{a} \right] e^{-ak} \quad (7)$$

9) Examine the errors of the prediction model

$$e(k) = \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \times 100\% \quad (8)$$

2. Grey Linear Difference Modeling

The grey linear difference prediction model GM (1,1) of this paper employs AGO and first order constant coefficient linear differential equation to process primitive sequence, determines the coefficient of linear differential equations through least squares algorithm, and after the answer is obtained engages in system recovery, prediction and evaluation through 1-IAGO.

The one-order constant-coefficient linear differential equation is

$$x^{(1)}(k) + ax^{(1)}(k-1) = b$$

1) Input primitive sequence

Input initial sequence of the model

$$x^{(0)} = \left\{ x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \cdots, x^{(0)}(n) \right\} \quad (9)$$

2) AGO

Form new sequence out of the primitive sequence through AGO:

$$x^{(1)}(i) = \sum_{j=1}^{i} x^{(0)}(j)$$

$$x^{(1)} = \left\{ x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), \cdots, x^{(1)}(n) \right\} \quad (10)$$

3) Determine B and Yn through least square method

$$Yn = \begin{bmatrix} x^{(1)}(2) \\ x^{(1)}(3) \\ \vdots \\ x^{(1)}(n) \end{bmatrix}, \quad B = \begin{bmatrix} -x^{(1)}(1) & 1 \\ -x^{(1)}(2) & 1 \\ \vdots & \\ -x^{(1)}(n-1) & 1 \end{bmatrix},$$

$$\hat{a} = \begin{bmatrix} a \\ b \end{bmatrix} \quad (11)$$

4) Solve the equation for a, b

Through $Y_n = B\hat{a}$ and matrix computation rules we can determine

$$\hat{a} = (B^T B)^{-1} B^T Y_n = \begin{bmatrix} a \\ b \end{bmatrix} \quad (12)$$

5) Solve $\hat{x}^{(1)}(k)$

Through one-level constant-coefficient linear differential equation we obtain

$$\hat{x}^{(1)}(2) = b - ax^{(1)}(1)$$

$$\hat{x}^{(1)}(3) = b - ax^{(1)}(2)$$

$$M$$

$$\hat{x}^{(1)}(n) = b - ax^{(1)}(n-1) \qquad (13)$$

6) <u>Recover the prediction value</u> $\hat{x}^{(0)}(k+1)$

Through 1-IAGO we obtain

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \qquad (14)$$

7) <u>Examine the errors of the prediction model</u>：

$$e(k) = \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \times 100\% \qquad (15)$$

3. Case Study

For linear motion guide rigidity test, we design a jig (see Fig. 1) in conjunction with SHIMADZU UH100A universal testing machine as shown in Fig. 2. Due to the design of the jig, we can obtain the tensile and strain of the linear motion guide under tension. Through the tensile and strain and we can then obtain the rigidity. This experiment employs BRH25A linear motion guide manufactured by ABBA Linear Technology Company (Fig. 3). Rigidity of 3,000kg-6,000kg is taken for modeling data. Rigidity is predicted to be between 6,500kg and 8,000kg. See (Table. 1).

a) Steps of grey linear differential modeling and analysis are shown below:

1) Model's primitive sequence

$$x^{(0)} = \{x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \cdots, x^{(0)}(7)\}$$
$$= \{4213.5, 4252.7, 4228.3, \Lambda, 4115.2\}$$

2) AGO

$$x^{(1)} = \{x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), \cdots, x^{(1)}(7)\}$$
$$= \{4213.5, 8466.2, 12694.5, \Lambda, 29329.9\}$$

3) Determine B and Yn through least square method

$$Yn = \begin{bmatrix} 4252.7 \\ 4228.3 \\ M \\ 4115.2 \end{bmatrix}, B = \begin{bmatrix} -6339.9 & 1 \\ -10580.4 & 1 \\ M & \\ -27272.3 & 1 \end{bmatrix}$$

$$\hat{a} = (B^T B)^{-1} B^T Y_n = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0.0065 \\ 4295.3 \end{bmatrix}$$

4) List the response equation

$$\hat{x}^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a} \right] e^{-ak} + \frac{b}{a}$$

5) Solve $\hat{x}^{(1)}(k)$ for 1-IAGO

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k)$$
$$= (1 - e^{a}) \left[x^{(0)}(1) - \frac{b}{a} \right] e^{-ak}$$
$$= \{4254.4, 4226.9, 4199.6, \Lambda, 4013.3\}$$

6) Error Examination

$$e(k) = \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \times 100\%$$
$$= \{-0.04, 0.03, -0.04, \Lambda, -1.54\}$$

b) Steps of grey linear difference modeling and analysis are shown below:

1) Model's primitive sequence

$$x^{(0)} = \{x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \cdots, x^{(0)}(7)\}$$
$$= \{4213.5, 4252.7, 4228.3, \Lambda, 4115.2\}$$

2) AGO

$$x^{(1)} = \{x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), \cdots, x^{(1)}(7)\}$$
$$= \{4213.5, 8466.2, 12694.5, \Lambda, 29329.9\}$$

2) Determine B and Yn through least square method

$$Yn = \begin{bmatrix} 8466.2 \\ 12694.5 \\ M \\ 29329.9 \end{bmatrix}, B = \begin{bmatrix} -4213.5 & 1 \\ -8466.2 & 1 \\ M & \\ -25214.7 & 1 \end{bmatrix}$$

$$\hat{a} = (B^T B)^{-1} B^T Y_n = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -0.994 \\ 4281.6 \end{bmatrix}$$

4) List response equation

$$\hat{x}^{(1)}(k) = b - a\hat{x}^{(1)}(k-1)$$

5) Solve $\hat{x}^{(1)}(k)$ for 1-IAGO

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k)$$

$$= \{4254.4, 4226.8, 4199.4, \Lambda, 4012.7\}$$

6) Error Examination

$$e(k) = \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \times 100\%$$

$$= \{-0.04, 0.04, -0.04, \Lambda, -1.52\}$$

5. Experiment Outcome

Table 1 shows actual rigidity of 3000kg -8000kg obtained from the test. This paper employs 3000-6000kg as data for grey linear differential modeling and grey linear difference modeling and uses it for prediction of rigidity of 6500-8000kg. The predicted results obtained by the grey linear differential modeling and grey linear difference modeling are shown in (Table. 1) and (Fig. 4). The mean absolute percentage error (MAPE) of the grey linear differential modeling and grey linear difference modeling from 3000kg to 6000kg are 0.051 and 0.052%, and the MAPE of the grey linear differential modeling and grey linear difference modeling from 6500kg to 8000kg are 1.082% and 1.069% respectively. The prediction results above indicate that as far as modeling sequence is concerned, the error of grey linear differential modeling is less significant than that of grey linear difference modeling.

6. Conclusions

This paper presents the prediction model of grey linear difference equation and compares it with that of the traditional grey linear differential equation, as well as applies it to prediction of rigidity of linear motion guides. The research indicates that the prediction errors of both the GM (1,1) model, established through grey linear differential equation, and the GM (1,1) model, established through grey linear difference equation, are within the reasonable range. But, the prediction outcome of GM (1,1) model established through grey linear differential equation is similar. And the GM (1,1) model, established through grey linear difference equation, is simpler than the GM (1,1) model, established through grey linear differential equation. Its computation is faster, and its mathematic foundation is more comprehensive.

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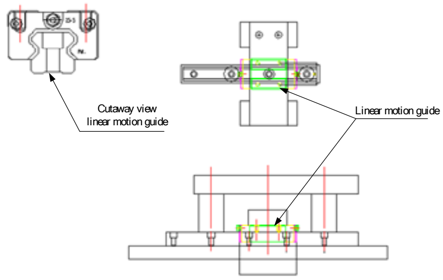

Fig.1. Jig for testing

Fig.2. BRH25A linear motion guide

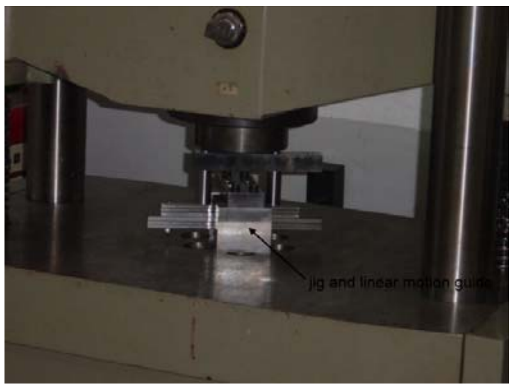

Fig.3. Linear motion guide and jig on universal testing machine

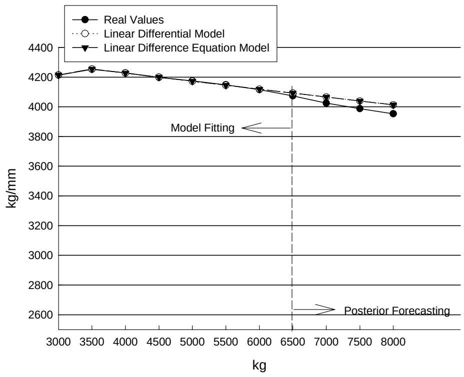

Fig.4.Real values and model values for linear motion guide rigidity from 3000- 8000kg

Table1

Model values and forcast errors(unit:K=kg/mm)

Load kg（F）	Real value elongation mm（δ）	Real value Rigidity （K=F/δ） kg/mm	Grey Linear Differential (kg/mm)	error （%）	Grey Linear Difference (kg/mm)	error （%）
3000	0.712	4213.5	-	－	－	－
3500	0.823	4252.7	4254.4	-0.04	4254.4	-0.04
4000	0.946	4228.3	4226.9	0.03	4226.8	0.04
4500	1.072	4197.8	4199.6	-0.04	4199.4	-0.04
5000	1.198	4174.6	4172.5	0.05	4172.2	0.06
5500	1.326	4147.8	4145.5	0.06	4145.2	0.06
6000	1.458	4115.2	4118.7	-0.09	4118.4	-0.08
MAPE 3000-6000				0.05		0.05
6500	1.596	4072.7	4092.1	-0.48	4091.7	-0.47
7000	1.739	4025.3	4065.7	-1	4065.2	-1
7500	1.881	3987.2	4039.4	-1.31	4038.9	-1.3
8000	2.024	3952.6	4013.3	-1.54	4012.7	-1.52
MAPE 6500-8000				1.08		1.07

$$MAPE = \frac{1}{n}\sum_{k=1}^{n}[\left|\hat{x}^{(0)}(k) - x^{(0)}(k)\right| / x^{(0)}(k)]$$