

Flank Milling Surface Design with the Least Square Approach

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Abstract: - Typically flank milling is used to machine ruled surfaces. In this paper, a method of designing doubly curved NURBS surfaces for flank milling using the least squares method is given. In this method, grazing points generated by the Bedi et al. [4] tool positioning method are used as sample points. The user defines the degree and the number of control points along two parametric directions of the surface. A NURBS surface is then fit to pass among the sample points using the least square technique [10, 11]. Accuracy of the surface is evaluated by the parametric error measurement method [12] and is controlled by increasing the number of control points. Surfaces that can be machined with great accuracy are generated. Examples are given to demonstrate this surface design method.

Key-words: - Curved surface design; Surface numerical analysis; Flank milling;

1. Introduction

Five-axis machining has become more attractive in today's manufacturing as it reduces machining time and improves surface finish. Flank milling, as one of the five-axis machining methods, has attracted attention of both industry and academic community. Different tool positioning methods for flank milling have been developed since the late 90's. All these methods focus on ruled surfaces and attempt to reduce the deviation between the machined surface and the given ruled surface. Some of these methods result in a good match between the given ruled surface and the final machined surface. Today, flank milling is used to machine impellers, turbine blades and other engineering parts.

In spite of being widely recognized that flank milling produces curved machined surfaces, no one has attempted to design free form surfaces that can be flank milled. However, some researchers in academics [1,2] and industry [3] have tried to use existing flank milling techniques, developed for ruled surfaces, to machine free form surfaces. In their methods, they first divide the given surface into multiple small surfaces and each small surface is treated as a ruled surface and the flank milling methods mentioned earlier are used to machine each small surface. Since the surface is machined in patches, the machined surface is discontinuous at the path boundaries. Also the geometry of the surface should be such that the

tool does not gouge the relevant path of the surface. As the machined surface is a curved surface, it only approximates the desired ruled surface. Most ruled surfaces are created to deliver a certain performance such as a flow pattern in an impeller application and any mismatch has a diverse effect on the efficiency. Similar losses can be witnessed in other engineering applications. The magnitudes of these losses in today's industry are attributed to manufacturing processes. If, on the other hand, a doubly curved surface (that can be flank milled) can be designed, then impellers and other engineering parts can be designed using surfaces that can be machined easily and exactly with little or no loss due to the manufacturing process. For impeller surfaces or blade surfaces, a designer can use the flank milled surface instead of the traditional ruled surface to optimize the performance of the part and not worry about compromises during machining. Thus, to define the machined surface generated by flank milling techniques, mathematically, is significant for manufacturing engineering.

In fact, the exact definition of the machined surface generated by flank milling is not known. However, studies by different researchers show that the machined surface produced by flank milling techniques can be closely represented by a grazing surface (or swept surface) composed of discrete grazing points. Bedi et al.[4] suggested that each grazing point on the grazing surface can be calculated

with a cross product method. Combining these grazing points results in a grazing surface. Mann et al.[5] generalized this method and applied it to surfaces of revolutions. Lartigue et al.[6] presented a similar method to determine an envelop surface generated by flank milling. Senatore et al.[7] used an analogous way to compute the grazing points and the envelop surface. Other researchers, like Li et al.[8], Menzel et al.[9], used the grazing points and grazing surface to optimize the cutting tool position. All these research results show that the machined surface can closely be approximated by the grazing surface.

Even though a grazing surface can be used to represent a machined surface, it is still hard for a design engineer or a manufacturing engineer to apply it in their design or manufacturing because it is difficult to obtain any information from a surface composed of discrete points and it is also difficult to connect this surface with other surfaces around it as its definition is not known in conventional NURBS forms. To use the grazing surface in engineering application, an effective way is to give the grazing surface a clear mathematical definition and make it easy be understood and apply. As Non-Uniform Rational B-Splines (NURBS) are commonly used in engineering applications, in this paper, the known grazing surface is approximated with a NURBS surface making it easy to apply in engineering applications.

To make a NURBS surface fit a given set of grazing points, a known effective method is the least square surface fitting method. By using this method, a NURBS surface which can be directly flank milled can be defined. Therefore, in this paper, a way to design a surface for flank milling with the least square method is developed. The paper is organised as follows. The flank milling surface design is introduced first; then, the mathematical background of the least square method is reviewed in the following section. The method to use least squares to design a surface for flank milling is discussed in section 3. Examples to demonstrate this surface fitting method are given in section 4 and discussion is presented in section 5. The final conclusion is addressed in section 6 to close this paper.

2. Mathematical background

The least square technique is a mathematical method to make a curve or a surface fit a given sample of points so that the square of the deviation between the fitted and sample points is minimized. The fitted curve or surface does not necessarily pass through each sample point. The knot vectors and the order of the surface are usually given by the user.

2.1 Fitting a curve to point data

Assume that a set of discrete points $\{Q_l\}$ are given, $l = 0, 1, 2, \dots, k$. A p^{th} degree non-rational B-spline curve is used to approximate these points. The parameter of each sample point uu ($uu \in [0, 1]$) is chosen either using the equal spacing method or the chord length method or the centripetal method [10]. The knot vector as selected by the user is given by [10]

$$u_0 = u_1 = \dots = u_p = 0,$$

$$u_{m-p} = u_{m-p+1} = \dots = u_m = 1,$$

$$d = \frac{m+1}{n-p+1}, \quad i = \text{int}(jd), \quad \alpha = jd - i,$$

$$u_{j+p} = (1 - \alpha) \cdot uu_{i-1} + \alpha \cdot uu_i,$$

$$j = 1, 2, \dots, n - p, \quad (1)$$

where m is the number of knots counted from 0 in the knot vector and n (also counted from 0) is the number of control points defined by the user. Therefore, the knot vector of the interpolation curve is defined as

$$U = \{u_0, u_1, \dots, u_{m-1}, u_m\}.$$

A NURBS curve [10] is defined as

$$Q_l = \sum_{i=0}^n N_{i,p} P_i, \quad (2)$$

where p_i ($i = 0, 1, \dots, n$) are the control points that need to be determined and $N_{i,p}$ are the basis function.

Equation (2) can be written in matrix form as

$$Q = NP, \quad (3)$$

where $Q = \begin{bmatrix} Q_0 \\ Q_1 \\ M \\ Q_l \end{bmatrix}, \quad P = \begin{bmatrix} P_0 \\ P_1 \\ M \\ P_n \end{bmatrix},$

$$N = \begin{bmatrix} N_{0,p}(uu_0) & N_{1,p}(uu_0) & \Lambda & N_{n,p}(uu_0) \\ N_{0,p}(uu_1) & N_{1,p}(uu_1) & \Lambda & N_{n,p}(uu_1) \\ M & M & O & M \\ N_{0,p}(uu_l) & N_{1,p}(uu_l) & \Lambda & N_{n,p}(uu_l) \end{bmatrix}.$$

Solving equation (3), the control points for the approximating curve can be determined as follows:

$$P = (N^T N)^{-1} N^T Q. \quad (4)$$

Using the results from equation (4), p^{th} degree NURBS curve that approximates the sample points can be produced.

2.2 Fitting a surface to point data

Suppose a set of discrete points $\{Q_{k,l}\}$ are given, $k = 0,1,\Lambda, r$ and $l = 0,1,\Lambda, s$. A $(p, q)^{th}$ degree non-rational B-spline surface with $(m + 1) \times (n + 1)$ control points can be used to approximate these discrete points. Two parameters, u and v , are used to describe the approximating surface. In Fig. 1, u varies along the column direction (parameter l) from 0 to s . Similarly, v varies along the row direction (parameter k) from 0 to r .

The parameter corresponding to each sample point $uu_{k,l}$ ($uu_{k,l} \in [0,1]$) along u direction is calculated using either the equal spacing method or the chord length method or the centripetal method [10]. The parameter uu_l for each column is

$$\overline{uu}_l = \frac{1}{r+1} \sum_{k=0}^r uu_{k,l}, \quad (5)$$

where $l = 0,1,\Lambda, s$.

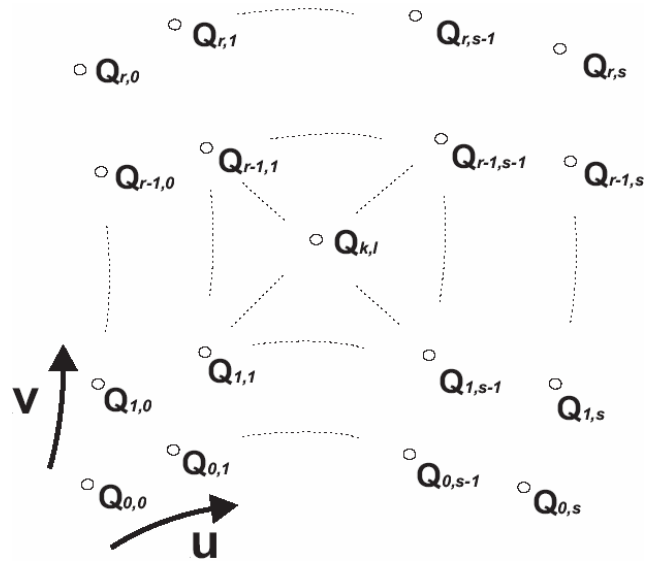


Fig. 1 Discrete sample points and their position.

Similarly, the parameter for each sample point $vv_{k,l}$ ($vv_{k,l} \in [0,1]$) along the v direction is calculated and the parameter vv_k for each row is

$$\overline{vv}_k = \frac{1}{s+1} \sum_{l=0}^s vv_{k,l}, \quad (6)$$

where $k = 0,1,\Lambda, r$.

After the parameter of each sample point is defined, the knot vector along two parameter directions (u and v directions) need be determined. As the parameter value of each sample point must be unique, the knot vector in the two directions are decided by using the sample points along any row or column (simply treats sample points of the row or the column as the sample points of a curve). The technique to decide the knot vector of a curve described in 2.1 can be used to define the knot vectors of the surface. For more details on selecting the knot vector, see [10,11].

Using equation (1), the knot vector of the approximating surface along the u direction is decided and is given by

$$U = \{u_0, u_1, \Lambda, u_{n+p}, u_{n+p+1}\}.$$

Similarly, the knot vector of the approximating surface along the v direction can also be obtained and is given by

$$V = \{v_0, v_1, \Lambda, v_{m+q}, v_{m+q+1}\}.$$

With the least square approach [11], a non-rational B-spline equation can be obtained by manipulating the following,

$$Q = N_v P N_u^T, \quad (7)$$

where N_v and N_u are matrixes of the B-spline basis function. P is the matrix of control points and Q is the matrix of given sample points.

$$Q = \begin{bmatrix} Q_{r,0} & Q_{r,1} & \Lambda & Q_{r,s} \\ M & M & N & M \\ Q_{1,0} & Q_{1,1} & \Lambda & Q_{1,s} \\ Q_{0,0} & Q_{0,1} & \Lambda & Q_{0,s} \end{bmatrix},$$

$$N_v = \begin{bmatrix} N_{0,q}(\overline{vv_r}) & N_{1,q}(\overline{vv_r}) & M & N_{m,q}(\overline{vv_r}) \\ M & M & N & M \\ N_{0,q}(\overline{vv_1}) & N_{1,q}(\overline{vv_1}) & \Lambda & N_{m,q}(\overline{vv_1}) \\ N_{0,q}(\overline{vv_0}) & N_{1,q}(\overline{vv_0}) & \Lambda & N_{m,q}(\overline{vv_0}) \end{bmatrix},$$

$$P = \begin{bmatrix} P_{m,0} & P_{m,1} & \Lambda & P_{m,n} \\ M & M & N & M \\ P_{1,0} & P_{1,1} & \Lambda & P_{1,n} \\ P_{0,0} & P_{0,1} & \Lambda & P_{0,n} \end{bmatrix},$$

$$N_u = \begin{bmatrix} N_{0,p}(\overline{uu_s}) & N_{1,p}(\overline{uu_s}) & \Lambda & N_{n,p}(\overline{uu_s}) \\ M & M & N & M \\ N_{0,p}(\overline{uu_1}) & N_{1,p}(\overline{uu_1}) & \Lambda & N_{n,p}(\overline{uu_1}) \\ N_{0,p}(\overline{uu_0}) & N_{1,p}(\overline{uu_0}) & \Lambda & N_{n,p}(\overline{uu_0}) \end{bmatrix}.$$

Solving equation (7), the control points of the approximating NURBS surface can be obtained using

$$P = (N_v^T N_v)^{-1} N_v^T Q (N_u (N_u^T N_u)^{-1}), \quad (8)$$

Using the results from equation (8), $(p, q)^{th}$ degree NURBS approximating surface can be generated to approximate the sample points. Changing the parameterization, degree, the number of control points and the knot vector will result in a changed surface. These can be used to find a surface suited for the application.

2.3 Surface error measurement

After the approximating NURBS surface is built, the error between the given discrete sample points and the approximating surface needs be measured to evaluate the quality of the approximating surface.

Different surface error measurement methods have been developed since the late 90's. These methods were compared and analyzed in a previous paper [12] and will be directly used in this research. As the given surface is built from discrete sample points and the parametric value of each sample points ($\overline{uu_l}, \overline{vv_k}$) is known, the parametric surface error measurement method is selected for use in this research. Although, this method over estimates the surface error, if the two surfaces are very close, the measurement result will still closely reflect the surface error distribution. On the other hand, the parametric error measurement method provides a simple surface error computation process compared to other surface error measurement methods.

Using the parametric surface error measurement method, the approximating surface error can be estimated. If the error exceeds user defined tolerance, more control points need be used to generate the approximating surface. The computation steps are the same as before. The degrees of the surface can be kept the same or changed depending on the simulation result. This computation process is repeated until the surface error is controlled and brought in the range the user desires.

3. Surface design with the least square method

The machined surface generated by the flank milling technique can be described using the method given in the section 2. To generate this machined surface, a grazing surface comprising of grazing points is

required. The method to generate the grazing points is given below.

3.1 Tool positioning method and the grazing points

Different tool positioning methods generate different machined surfaces. Consequently, different grazing surfaces can be obtained with different tool positioning methods. In this research, Bedi et al.'s tool positioning method is used to produce the machined surface.

In Bedi et al.'s tool positioning method [4], the cutter is imagined to roll along two guiding curves. The two guiding curves are any two curves on the given design surface.

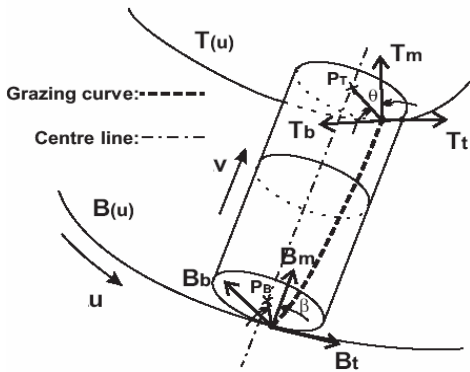


Fig. 2 Cylindrical cutter and guiding curves.

As Fig. 2 shows, $T(u)$ and $B(u)$ are the two guiding curves, a cylindrical cutter with radius R is rolling along them. The cutter contacts the guiding curves at two points, $T(u)$ and $B(u)$, and the two touching points have the same parametric value u . Frenet frames are set up at each contact points. T_t and B_t are the tangent vectors, T_m and B_m are the main normal vectors, T_b and B_b are the binomial vectors. Mathematical equations can be developed based on the geometrical relationship between the cutter and the guiding curves and are summarized as

$$P_T - T(u) = R \cos(\theta) T_m(u) + R \sin(\theta) T_b(u), \quad (9)$$

$$P_B - B(u) = R \cos(\beta) B_m(u) + R \sin(\beta) B_b(u), \quad (10)$$

$$(P_T - P_B)(\cos(\theta) T_m(u) + \sin(\theta) T_b(u)) = 0, \quad (11)$$

$$(P_T - P_B)(\cos(\beta) B_m(u) + \sin(\beta) B_b(u)) = 0. \quad (12)$$

The solution of these equations results in the cutter position at parameter u .

The grazing curve corresponding to this tool position can also be developed using the method developed in [4]. Assume that velocities at two contact points ($T(u)$ and $B(u)$) are V_T and V_B . These velocities can be linearly interpolated along the tool axis direction and the velocity at any point between the contact points is given by

$$V = V_B(1-v) + V_T v, \quad 0 \leq v \leq 1. \quad (13)$$

P_T and P_B are two points on the tool axis. These two points correspond to contact points $T(u)$ and $B(u)$. Any point between P_T and P_B can also be linearly interpolated and is given by

$$P = P_B(1-v) + P_T v, \quad 0 \leq v \leq 1. \quad (14)$$

The grazing point between contact points $T(u)$ and $B(u)$ is calculated as [4]

$$G = P + \frac{V * T_{axis}}{|V * T_{axis}|} R, \quad (15)$$

where T_{axis} is the vector of the cutter axis.

Different tool positions will generate different grazing points on the grazing surface. Enough tool positions need be used so that grazing points can be scattered along the whole machined surface. These grazing points can be used as sample points to find a NURBS surface that fits them closely.

3.2 Surface design with the least square

The method to fit a least squares NURBS surface to a given set of sample points was developed in section 2. In this method, the degrees and the number of control points of the approximating surface along two parameter directions (u and v) need be decided first. The degree and the number of control points along the guiding curves direction (u direction) are important factors that control the characteristic of the final

machined surface, thus the degree of the approximating surface along u direction is selected to be the same as the degree of the guiding curves. Alternatively, the degree can be selected to be degree 2 or 3, as lower degrees surface do not exhibit unwanted oscillations and can make it much easier to design good fitting surfaces [10,11]. The number of control points along the guiding curve direction can initially be the same as the number of control points of the guiding curves. Of course, more control points can be added in this direction if desired.

The degree and the number of control points along the parameter v direction (the tool axis direction) are initially set to 2 or 3 for the same reasons explained above. The numbers of control points along the v direction are initially set to 3 or 4.

After the degree and the number of control points have been selected, the position of each control point is calculated using the method described in the section 2. Consequently, the approximating NURBS surface can be generated. The error between the approximating surface and the given grazing points can be measured using the parametric error measurement method [12]. If the maximum error exceeds the user defined tolerance, more control points can be added along the two parameter directions. Degrees of the surface can also be changed depending on error analysis and simulation results. With an increased the number of control points, the error between the approximating NURBS surface and the grazing surface can be effectively controlled.

In the section 2, the parametric value of each sample point (uu, vv) is given by equations (5) and (6). An alternative method to get this parametric value is to use the parametric value of each grazing point directly. When the grazing surface is calculated with equations (13), (14) and (15), each grazing point corresponds to a certain parametric value (u, v). This parametric value more closely reflects the grazing point's location than the average value from equations (5) and (6). Thus, for simplicity, the parametric value of each grazing points is taken to be the value from equations (5) and (6). For this reason, $uu = u, vv = v$ at each grazing point.

In sequence, the knot vectors of the approximating NURBS surface can use unit knot vectors in both of

u and v directions in place of the knot vectors obtained from equation (1). The main concern in equation (1) is to ensure that each knot span at least contains one uu or vv so that the knot vector truly reflect sample points' distribution on the given surface. For the parametric value of each grazing point (u and v), equal space between two consecutive tool positions is normally used. Thus, a unit vector can guarantee that the number of grazing points at each knot span is equal and it also promises the knot vector itself reflects the grazing points' distribution exactly. Therefore, the unit knot vectors can be used instead of knot vectors from equation (1).

After these changes, the whole computation process is greatly simplified. The resulting surface may be different and the difference between the two surface parameter setting methods will be analyzed in the next section.

4. Examples

Examples that apply the proposed method are given in this section. Two guiding curves are given and the control points of these curves are listed in Table 1. A cylindrical cutter is used to machine the surface and the radius of the cylindrical cutter is $R = 5mm$. The knot vectors of the two curves are both $[0,0,0,1,1,1]$. The degree of the guiding curves is 2.

	T₀	T₁	T₂	B₀	B₁	B₂
x	75	30	0	60	30	15
y	15	30	60	0	30	75
z	-5	-5	-5	-45	-45	-45
w (weight)	1	1	1	1	1	1

Table. 1 Control points for guiding curves [mm]

A NURBS surface that can be exactly flank milled is designed. Bedi et al.'s tool positioning method is used in this design and the grazing points on the grazing surface are calculated by equation (15). Using these grazing points and the surface fitting method described above, the desired surface can be obtained.

First, the method described in the section 2 is used to design this surface. The degree of the surface in both the u and v directions is selected to be 2 and a three by three polygon of control points is initially selected to generate this NURBS surface. The parametric value of each sample point (grazing points) is given by equations (5) and (6) and the knot vectors along the u and v directions are calculated using equation (1). Using equation (8), control points of the approximating NURBS surface can be decided. After selecting these, the NURBS surface is constructed. The error between the approximating surface and the grazing points is calculated using the parametrical error measurement method and the result is plotted in Fig.3 below. The deviation between the approximating surface and the grazing points on the machined surface is in the range [0, 0.076]. The maximum surface error is close to 0.076.

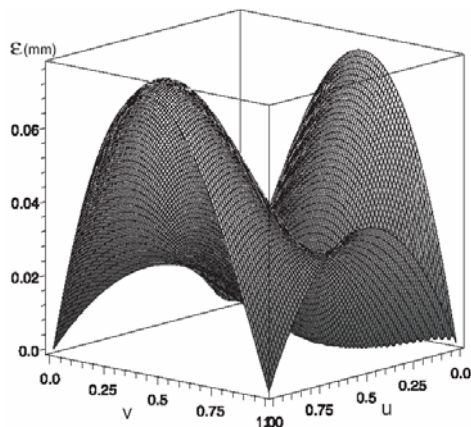


Fig. 3 Error distribution of the approximating NURBS surface.

The alternative way described in the section 3 is next used to design the same surface. The degree of the surface in both u and v directions is kept as 2 and the number of control points of the approximating surface are selected to be three by three. The parametric value of each sample points as used in the grazing point calculations of equations 13, 14 and 15 are used along with unit knot vectors in both u and v directions. The approximating NURBS surface is designed. The surface error is measured and plotted in Fig. 4 below.

The deviation between the approximating surface and the grazing points is in the range [0, 0.0172]. The maximum surface error is less then 0.0172. Compared

to Fig. 3, the maximum surface error reduces from 0.076 to 0.0172. The approximating NURBS surface with unit knot vectors and inheriting parametric value (for each sample points) generate small surface error. Thus, unit knot vectors and parametric values inherited from the grazing point calculations are used for each sample point for the remaining examples.

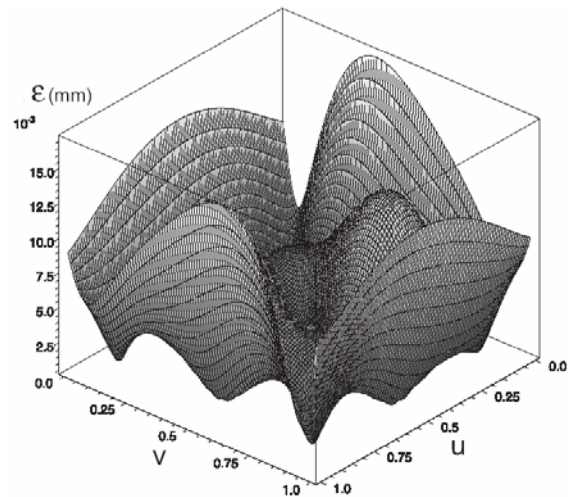


Fig. 4 Error distribution of the 3 by 3 NURBS surface for unit surface knot vector.

If the maximum surface error, 0.0172, still exceeds user defined tolerance, more control points can be added to reduce this maximum surface error. To demonstrate the process of surface error control, the number of control points is increased from three by three (the v direction) by four (the u direction) and then to four (the v direction) by five (the u direction). The degree of the approximating surface is kept as 2 along both parameter (u and v) directions. The approximating NURBS surfaces can be built using the least square method given before. The parametric error measurement method is used to calculate these surface errors and final results are plotted in Fig. 5 and Fig. 6.

The deviation for the three by four NURBS surface is in the range [0, 0.014]. The deviation for the four by five NURBS surface is in the range [0, 0.00255]. Compared to the three by three NURBS surface (Fig. 4), the maximum surface error reduces from 0.0172 to 0.014 and to 0.00255 as the number of control points are increased from three by three to three by four and four by five. With an increased number of control

points, the approximating surface error can be effectively controlled.

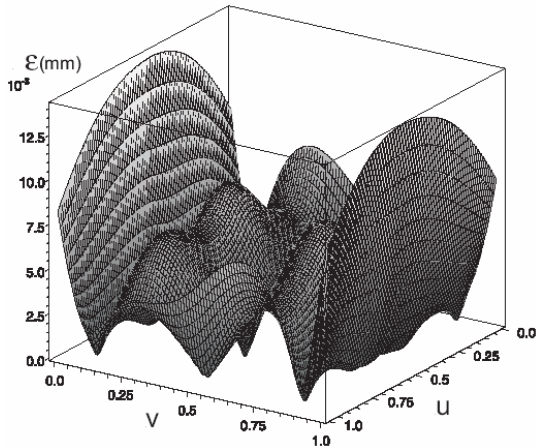


Fig. 5 Error distribution for 3 by 4 NURBS surface.

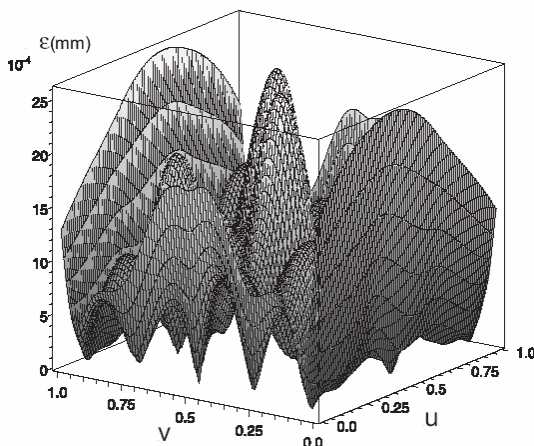


Fig. 6 Error distribution for 4 by 5 NURBS surface.

This method was also successful tried on a few other examples with good results.

5. Discussion

Using the grazing points on the grazing surface, the machined surface can be approximated by a NURBS surface through the least square approach. The accuracy of this surface is influenced by the distribution of the grazing points, the degree and the number of control points of the surface.

The distribution of the grazing points plays an important role in the least square surface fitting

method. High density of grazing point patches on the grazing surface will make the approximating surface match these patches well and impose the accuracy of the surface match. For flank milling surface design, the design surface needs to closely match the machined surface, thus, the grazing points should be evenly scattered along the grazing surface. With more grazing points, the computation time increases. A table of sample points vs. time for the example given in the section 4 is shown in Table 2. The computer used in this example was a Pentium 4 CPU 3.06GHz with 1.00 GB of RAM. The software used for calculation was Maple, a symbolic algebra package. Of course, other software like C++ can also be used. However, the trend of the variation will be the same.

The degrees and the number of control points of the approximating surface also significantly affect the surface design. Properly selecting the degrees and the number of control points are important in the least square approach. Normally, a good guess at these numbers is required initially, then, control points can be added or deleted as desired in different parameter directions or alternatively the degrees of the approximating surface can be changed to make the surface fit the grazing points closely. More control points and low degree values will result in good surface fits.

Clearly, a high accuracy surface can be produced if sufficient and evenly distributed grazing points are given, proper degree of the surface is selected, and adequate control polygons are used. To achieve this target requires a large mesh of sample points and a long computation time is needed. Furthermore, the process is not automatic and requires a lot of user input. If all user selections are blended into an algorithm, the mesh of sample points will be very large and computation time will be extensive. For engineering application, the designer requires a simple method with few parameters to select. One must remember that the designer is an expert in fluid flow or other engineering field and may know little about surface design. Selection of number of control points, knot vectors etc. may dissuade them from using the least square method. If, on the other hand, a brute force algorithm is generalized, it will also increase the computation time. Thus, another easy to handle flank milling surface design method that simplifies the computation process is being studied.

S.Ps C.Ps	30*30	50*50	70*70	100*100
3*3	28.9s	88.5s	202.2s	562.1s
3*4	32.1s	93.1s	214.1s	585.5s
4*5	45.2s	154.5s	374.2s	1054.9s

Table 2. the number of sample points vs. the computation time. **S.Ps**: Sample Points. **C.Ps**: Control Points.

6. Conclusion

This paper presents a method for designing free form surfaces that can be flank milled. The least square method is used to develop this type of surface. Bedi et al's tool position method is used to calculate the grazing points on the grazing surface and these grazing points are treated as discrete sample points. The error between the design surface and the grazing surface is evaluated with the parameter error measurement method and is controlled by the number of control points and degrees of the design surface. High accuracy surface can be obtained with this method. The method, however, is complicated by the amount of user inputs it requires and by the increases in computation time with increase in sample points.

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