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A New Approach to Stabilize a Class of Nonlinear Systems by ILC Method

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Abstract: - Iterative learning control methods are represented as powerful tools to control dynamics nowadays. Our new controller based on particular case of iterative learning control is radically different from the presented conventional method, which attempts to stabilize a class of nonlinear systems by satisfying the conditions of Lyapunov Stability Theorem.
Since our algorithm is model based, its robustness should be considered respect to perturbation in the system structure and Lipschitz condition number.
Depending on our designed method one shouldn't worry in this paper about finding the Lyapunov function on the considered systems any longer.

Key-Words: - Lipschitz Condition, Iterative Learning Control, Nonlinear Systems, Lyapunov Stability Theorem.

1 Introduction

Since the recent decade, the researchers have been focusing their efforts on learning control systems, as this kind of control technique is able to improve system performance efficiently.
Many scientists working on Iterative Learning Control(ILC), have presented different learning control schems. Among these for tracking control ,is the iterative learning control which was originally introduced by "Arimoto" in1984.
The main purpose of ILC is to find a control input iterativly.resulting in the plant ability to track the given reference signal on output trajectory over a finite time intreval.
Common ILC methods use the repetitive nature of the process to improve the tracking performance progressively[1][4][5][6].but from a new view point of ILC which is represented in this paper stabilizing a class of nonlinear systems would be obtained.
In section 2 problem formulation is represnted. Section 3 presents our controller designed method. Sections 4 discusses our rusults by showing application of our algorithm on some dynamics. And finally conclusion is included in section 5.

2 Problem Formulation

Consider the system

$$\dot{x} = f(x,t) + g(x,t) \times u \quad x \in R^n \ , u \in R^m \quad (1)$$

Where $f : D \times [0,\infty) \to R^n$ and $g : D \times [0,\infty) \to R^m$ are piecewise continuous in *t*, and *f* is locally Liptschitz in x on $D \times [0,\infty)$, $D \subset R^n$ is domain that contains the origin x=0.
Supposing the system (1) is perturbed as below:

$$\dot{x} = (f(x,t) + \Delta f) + (g(x,t) + \Delta g) \times U \qquad (2)$$

The perturbation could be resulted from modeling, aging, or uncertainties and disturbances which exist in any realistic problem. In a typical situation, though perturbation is not known, but some information like knowing an upper bound is available. Here the perturbation is represented as an additive term on right-hand side of the state equation. Uncertainties which don't change the system order can always be represented in this form.
In general if a perturbation is considered as $\Delta h(x,t)$, it can be classified in two types as below :
$h(x,t) = 0$ is vanishing perturbation , and $h(x,t) \neq 0$ called nonvanishing perturbation.
In this paper, vanishing perturbation has been investigated for *g* function whereas *f* perturbation is considered to be Lipschitz.
It is necessary to fulfill these four assumptions:
1. g perturbation is vanishing; $\Delta g(0,t) = 0$, and its upper bound is known as $\|\Delta g(x,t)\| < \delta$
2. f is piecewise continuous in time, and locally Liptschitz in $D \times [0,\infty), D \subset R^n$, in state $\| f(x,t) - f(0,t) \| \leq M \|x\|$
3. *f(0,t)=0* ; x=0 is an equilibrium point of the

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unperturbed system.
4. f purturbation, $\Delta f(x,t)$, satisfies the lipschitz condition.

3 Controller Design Method

For a closed loop system, state space equation is given by (1), Usually stabilization of the closed loop system can be prepared by a suitable controller u(x,t) as a function of state x and time t.
For such a system in relation (1) by the theorem discussed below, we claim that a stabilizer as form of feedback control and a Lyapunof Function for stabilizing the system are found.

THEOREM:

Consider the closed loop system (3) with the following controller:

$$u = k\,x \qquad (3)$$

Where K is the matrix which governed by the following law:

$$M \times I + g(x,t) \times k = Q^d \qquad (4)$$

where M is bound of lipschitz condition, I is unit matrix with the proper dimension, and Q^d is a desired negative difinite matrix which is selected by the designer based on the rate of descending the Lyapunov criterion.
Then (3) will be asymptotically stable around the origin.

PROOF:

By defining the Lyapunov function, as below and following the proof procedure, a quoted controller will be derived as follow:

$$V = \frac{1}{2} x^T x \Rightarrow \dot{V} = x^T \dot{x}$$

$$\dot{V} = x^T (f(x,t) + g(x,t) \times u(x,t))$$

$$\dot{V} = x^T f(x,t) + x^T g(x,t) \times u(x,t)$$

By using the *Schwartz Inequality* we have:

$$x^T f = \; < x, f > \; \le \; \|x\| \, \|f\|$$

and by implementing the lipschitz condition it changes to:

$$x^T f = \; < x, f > \; \le \; \|x\| \, \|f\| \; \le \; \|x\| M \|x\| = x^T M I x$$

Now by selecting the controller as u=Kx, the below relation is achieved:

$$\dot{V} \le x^T MI \; x + x^T gK \; x = x^T (MI + gK) x$$

Therfore the desired negative definite matrix Q^d is considered as:

$$M \times I + g(x,t) \times k = Q^d$$

Since the K matrix can be obtained from the following cases as below:

Case1. *g is invertible matrix*. In this case K is obtained simply by the following relation (test 6):

$$K = g^{-1}(Q^d - M \times I)$$

Case 2. *g is pseudo-invertible matrix*. Here K can be expressed by (test 7):

$$K = (g^T g)^{-1} g^T (Q^d - M \times I)$$

Our algorithm is also reliable for the system (1) in which inputs and outputs numbers are different.
If g(x,t) is not invertible or pseudo-invertible, it means that the inputs are interaction together and the system has contorolability problem, in this case the mentioned ILC method is more effective.

COROLLARY: in general case, g has been $n \times m$ dimention (whre n is the number of states and m is the number of inputs): in this case Iterative Learnig Control can be used to find the desired $K_{m \times n}$ matrix which satisfy the relation (5) by the following method (test 1) :

$$K^{i+1} = K^i + \eta \Delta e^i$$

where i represents the iteration index and $\eta_{m \times n}$ is the learning factor matrix .

$$\Delta e^i = Q^d - Q^i \quad \text{where} \quad Q^i = M \times I + gK^i$$

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$$\begin{aligned}\Delta e^{i+1} &= Q^d - Q^{i+1} = Q^d - M \times I - gK^{i+1} \\ &= Q^d - M \times I - g(K^i + \eta \Delta e^i) \\ &= Q^d - (M \times I + gK^i) - \eta g \Delta e^i \\ &= (Q^d - Q^i) - \eta g \Delta e^i = (I_{n\times n} - \eta g)\Delta e^i\end{aligned}$$

$\left\|\Delta e^{i+1}\right\| \le \left\|I_{n\times n} - \eta g\right\| \left\|\Delta e^i\right\|$ the convergence condition now is obtained :

$$\max\left\|I_{n\times n} - \eta g\right\| \le 1$$

It should be considered that this type of control is used when the system and the controller have different rate of processing time, meaning that the desired controller works much faster than the process possibly by using new high-rate software nowadays.

<u>Remark 1:</u> Robustness Respect to Perturbation in f and g Functions

Considere f and g are perturbed by Δf and Δg terms. Suppose the perturbation of Δf is vanishing which satisfies the linear growth bound: $\left\|\Delta f(x,t)\right\| < \gamma\left\|x\right\| \ \forall t \ge 0, \forall x \in D$ where γ is nonnegative constant. Also for g perturbation, it is supposed that its upper bound is known, therefore it can be written as:
$\left\|\Delta g(x,t)\right\| < \delta \ \forall t \ge 0, \forall x \in D$ where δ is a positive number.
By these assumptions, $f' = f + \Delta f$ would remain Lipschitz and according to mentioned theorem K matrix can be obtained from the new relation below:

$$\begin{aligned}&(M+\gamma)\times I_{n\times n} + (g(x,t)+\Delta g)\times k = Q^d \\ &(M+\gamma)\times I_{n\times n} + g(x,t)\times K = Q^d - \Delta g \times K = \overline{Q}^d\end{aligned}$$

In this new case,(besides the perturbation in the system) it seams that the desired selected Q^d matrix has been changed to $\overline{Q}^d$ matrix, where $\overline{Q}^d = Q^d - \Delta g \times K$, in designing Q^d it should be noticed that it must be negative enough so that besides additive $-\Delta g \times K$ term, $\overline{Q}^d$ would still remain negative (test 5). And the convergence condition of the mentioned ILC method in the new case can be achieved as:

$$\begin{aligned}\left\|I_{n\times n} - \eta g'\right\| &= \left\|I_{n\times n} - \eta(g+\Delta g)\right\| \\ &\le \left\|I_{n\times n} - \eta g\right\| + |\eta|\left\|\Delta g\right\| \\ &\le \left\|I_{n\times n} - \eta g\right\| + \delta|\eta| \le 1\end{aligned}$$

$$\max(\left\|I_{n\times n} - \eta g\right\|) \le 1 - \delta|\eta|$$

<u>Remark 2:</u> Robustness Respect to Lipschitz Condition Number

Although our designed controller is derived according to relations (3) , (4) and the Lipschitz condition number is important in this process, if the designer select this number incorrectly such as M' in the below relation:
$M' = M + \Delta M$
This method is still reliable if M' is laid in the below range (test 4):

$$M' \in [\sup\{\frac{\left\|f(x,t)\right\|}{\left\|x\right\|}\}, \infty)$$

<u>Remark 3:</u> Selecting the Desired Matrix Q^d

Choosing the Q^d matrix affects the performance of the response. In other word, Q^d is appeared in the derivative of Lyapunov function and if it is selected larger (in the sence of negative definite matrices) it means that the derivative of Lyapunov function would be more negative and it forces the above mentioned function to reach the origin more faster and vice versa (test 2).

<u>Remark 4:</u> Forgetting Factor

Forgetting factor enables the designer to control the speed of the response during the process of the system.
According to previuos remark, by selecting Q^d matrix at the beginning of the process, the designer can perform the mentiond task only once,but by using forgetting factor as term of $1 - \exp(-t/T)$ (where t represents the time and T is the total time of process) the rate of descending the Lyapunov function can be adjusted, so the speed of states to reach the origin is controllable during the process (test 3).

<u>Remark 5:</u> Sliding Surface

In view point of sliding mode control, the sliding surface is known as a special case of lyapunov surface. By using our algorithm one can claim that has found a sliding surface. Consider the following relations:

$$Y = CX, \quad C = [c_1, \ldots, c_n] \quad c_i \neq 0, \quad X = [x_1, \ldots x_n]^T$$

where Ci's are selected in such a way that CX would be hurwitz.
By defining the Lyapunov function and following the procedure below we will have:

$$V = \frac{1}{2} Y^T Y \Rightarrow \dot{V} = Y^T \dot{Y} = (CX)^T (CX)' = X^T C^T C\dot{X}$$
$$\dot{V} = X^T C^T C(f + gu) = X^T C^T Cf + X^T C^T Cgu$$
$$\dot{V} \leq X^T C^T CMX + X^T C^T CgKX = X^T (C^T CMI + C^T CgK)X;$$
$$Q^d = C^T CMI + C^T CgK$$

by adjusting K matrix we make $\dot{V}$ to be negative then based on the Lyapunov theorem Y moves to the origin and $CX \longrightarrow 0$ so,

$$c_1 x_1 + \ldots + c_n x_n = 0 \quad or \quad c_1 x + c_2 x^{(1)} \ldots + c_n x^{(n-1)} = 0$$

because of Y is Hurwitz, then all of the states moves to origin. In this case S=CX is called sliding surface which has been claimed to find by our method.

4 Experimental Results

Now the results of our designed method are illustrated in nonlinear systems with our problem formulation for each section separately.

Test 1- *g is rectangular (ILC method*)

Consider the system with the following state equations:

$$\dot{X} = f(X) + g(X)u$$
$$X = [x_1\ x_2\ x_3]^T;\ f(X) = \begin{bmatrix} \sin x_1 \\ \sin x_3 \cos x_2 \\ \sin(x_1\ x_3) \end{bmatrix}$$
$$g(X) = \begin{bmatrix} 2 + \cos x3 & 0 & 0 & 0 \\ 3 & 3 + \sin x_2 & 0 & 0.1 \\ 0 & 0.2 & 0.3 & 0.1 \end{bmatrix}$$
$$Q^d = -2 \times \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}, MI = 4 \times I_{3\times3}, \eta = -\begin{bmatrix} 0.003 & 0.001 & 0.001 \\ 0.001 & 0.001 & 0.001 \\ 0.001 & 0.001 & 0.01 \\ 0.001 & 0.001 & 0.001 \end{bmatrix}.$$

It should be noticed that the g matrix is rectangular (it means that the number of input and output are different). Now the results for controlling this system by our designed method are plotted as below:

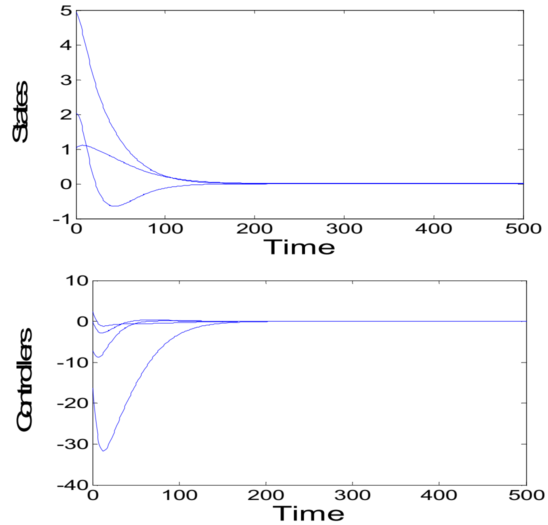


Fig 1- ILC method- the states and controllers

Test 2- *selecting the desired Q^d matrix*

In the above mentioned system the Q matrix is changed and the results are shown as follow:

$$Q^d = -150 \times \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix},$$

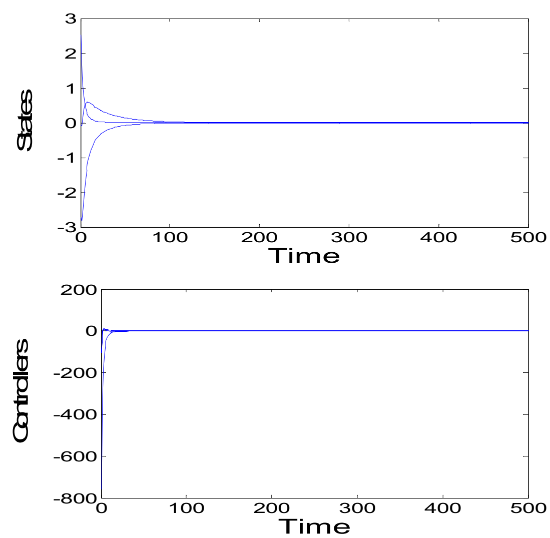


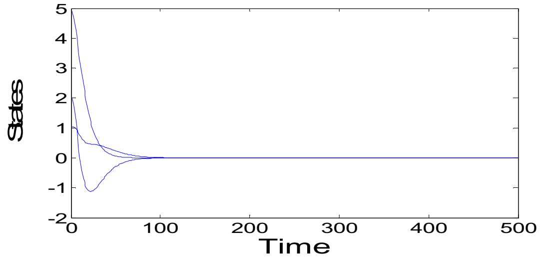
Fig 2- Effect of the desired Q^d matix- the states and controllers

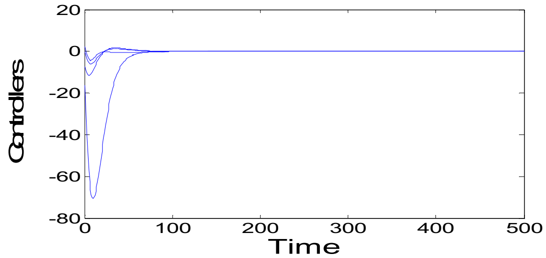
Now compare the amplitude of the controller and the speed of convergence for states and the controller in test 1 and test 2

Test 3- *forgetting factor*

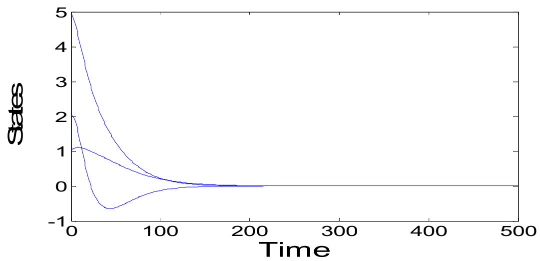
The forgetting factor and Its effect is illustrated in the following manner:

A-Without forgetting factor:





B-With forgetting factor:



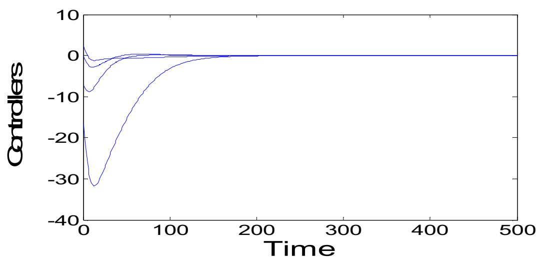
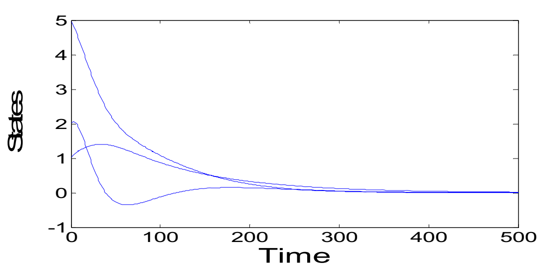


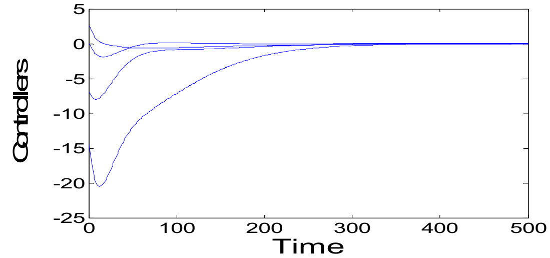
Fig 3- Effect of the forgetting factor- A without and B with forgetting factor-the states and controllers

It is obvious that the rate of convergence is different in both cases shown above.

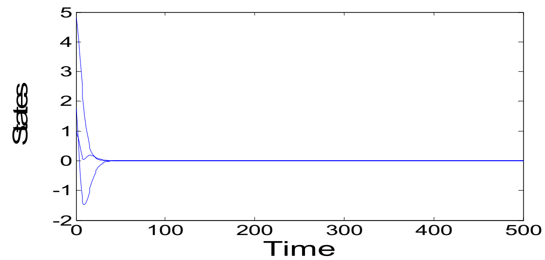
Test 4- *Robustness Respect to Lipschitz Conditon*

In this case the lipschitz condition number has reduced to half of its value comparing to test 1. the plots entirely show the difference in two considered cases.





Other experiment shows that this number has been increased to ten times larger than its value in test 1.



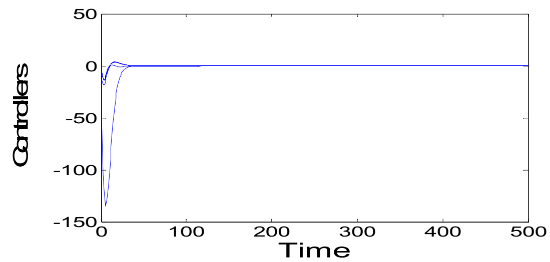
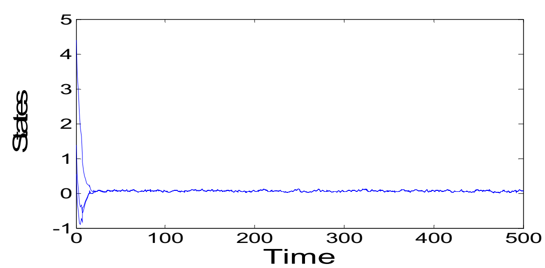


Fig 4- Lipschitz condition number robustness-the states and controllers

Test 5- Robustness Respect to perturbation in *f* and *g*

In this experiment f and g are changed by f+5*rand and g+2*rand respectively.The results are shown as below:



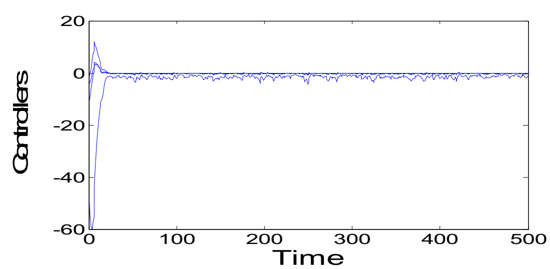


Fig 5- robustness respect to f and g perturbation-the states and controllers

Test 6- *g is invertible matrix*

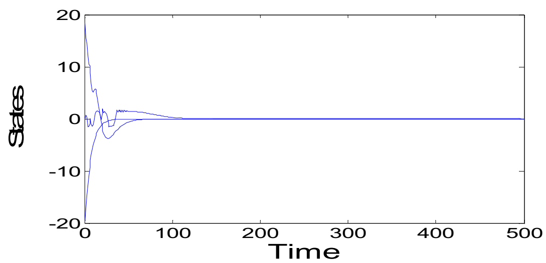
According to the mentioned theorem discussed previously (case 1) the results for the following system achived as below:

$$\dot{X} = f(X) + g(X)u$$

$$X = [x_1 \; x_2 \; x_3]^T ; \; f(X) = \begin{bmatrix} -200 \sin x_2 \cos x_1 \\ 100 \sin x_{3\,2} \\ \sin(x_1 \, x_3) \end{bmatrix}$$

$$g(X) = \begin{bmatrix} 2+\cos x3 & 0 & 0 \\ 3 & 3+\sin x_2 & 10 \\ 0 & 0 & 0.1 \end{bmatrix}$$

$$Q^d = -2 \times \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} \quad , \quad MI = 4 \times I_{3\times3}$$



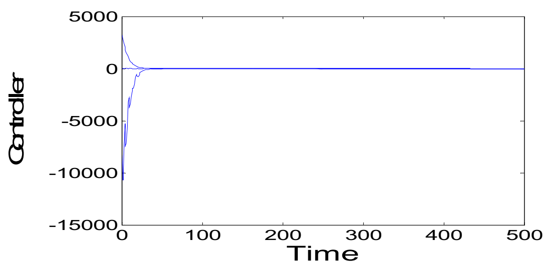
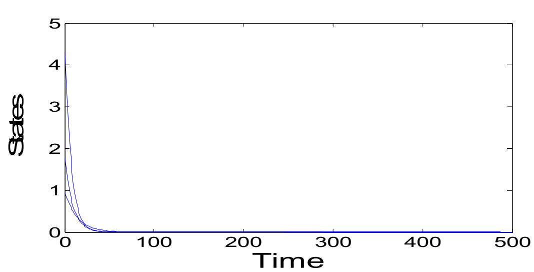


Fig 6- g matrix is invertible-the states and controllers

Test 7- *g is pseudo-invertible matrix*

In this test, the same system and similar initial condition as in test 1 are considered. The results are shown and the comparison between the controller's amplitutes are considerable.



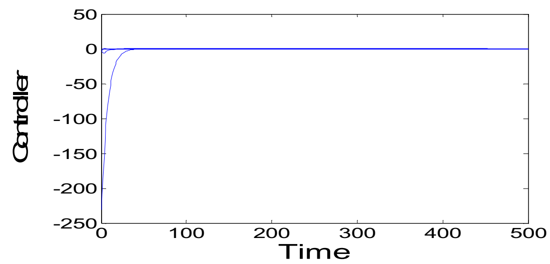


Fig 6- g matrix is pseudo-invertible-the states and controllers

5 Conclusion

One of the main problem in any nonlinear system is to satisfy the stability of these systems and the Lyapunov stability Theorem is the most powreful method to achieve this purpose but the chief worrying matter in this theorem is to find a proper Lyapunov function.

In this paper, from a new view point of ILC and using a well-known Lyaponov function and our represented method, this problem has been solved and the stability for such systems are obtained.

Acknowledgement

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