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A Second-Order Combined Compact Upwind Difference Scheme for the Navier-Stokes Equations

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Abstract: - Based on the developed third- and fourth-order upwind compact finite difference(FD) schemes, a new high-order weighted upwind FD approach, which is called combined compact upwind FD method, is proposed for decreasing dispersive(phase) and dissipative errors of the finite difference approximations. The newly proposed combined compact upwind FD schemes have the characters of group velocity control scheme which can obtain a large group velocity control range. In this paper the optimum combined scheme is proposed by applying the dispersion-relation- preserving (DRP) idea. Furthermore a second-order projection algorithm which has at least third-order accuracy in spatial direction is developed for solving the incompressible, two-dimensional Navier-Stokes equations. Numerical examples are given to validate the performance and efficiency of the new projection algorithm proposed. The results show that the present method has desired accuracy and resolution. The proposed method can be extended to the solution of the complex fluid flow problems.

Key-Words: - high-order accuracy, combined compact upwind scheme, projection method, directly numerical simulation

1 Introduction

The physical phenominon changes in multi scale range in the real natrue, so the direct numerical simulations(DNS) of multi-scale flow prolem is asked to discribe all the scale component in some degree respectively so as to simulate the real physical phenominon as exactly as possible. For example, the DNS for turbulent and computational aeroacoustics(CAA) etc. How to compute the small scale (or high wave number) component is the main difficults in such problems, which request the finint difference(FD) scheme for spatial discretization has as low as possible dispersive errors and dissipative errors. The low order accurate upwind schemes have larger dispersive and dissipative errors, which are easy to cause computational instability, and such errors become more serious especially in short wave range. So People attach more and more importance to the high order accurate upwind compact schemes[1-5,12], which can decrease the computer efforts and memory storages, need a less stencil points relatively. It is obviously that the upwind schemes are prefer to approximate hyperbolic equations than symmetric ones, so the research of such schemes has obtained rapid progress; lots of high accurate upwind schemes are developed [1-5]. Lele proposed the compact FD schemes[1], which have increase the efficiency to capture the small scales, can reach the accuracy as high as spectral method. Adopting the thought of group velocity control, Ma and Fu [2] developed a kind of schemes called group velocity control(GVC) schemes and presented the high order accuracy compact difference method which can reflect the real physical flow more exactly and computational results of compressible mixing layers. Lele [1] proposed the resolving efficiency (RE) to weight the advantage or the disadvantage of a scheme. If a scheme can decrease dispersive and dissipative errors effectively, it can be though as a high RE one. Based on this opinion, we developed a combined compact upwind(CCU) scheme. The present scheme combinate a fourth-order compact upwind FD scheme (UCD4) we have proposed in Ref. [5] and the third-order ones(UCD3) in Ref. [4] with certain weight has at least third-order accuracy. The RE of CCUscheme has been improved in a certain degree.

Projection method introduced by Chorin [6] is suitable to solving the incompressible NS equations and further developed by Bell, et al [7,8] and gived the detail anlysis of perfomance of the Godunov projection method. Auterri and Lopez[9,10] presented some spectral projection algorithim and performed the DNS for some complex fluids flow. Fernandez[14] proposed the second-order accurate projection method in time . In this paper, a new kind

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of high order compact FD algorithm basec on projection method, which has at least third-order accuracy in spatial and second order in time, is designed to solve the incompressible NS equations in staggered grid system. Particularly, the new fourth-order accuracy FD scheme for approximating the pressure Poisson equation and the new explicit scheme for solving the pressure gradient are designed. At the end of this paper, the computational results about Taylor vortex array and driven cavity problem are listed. The results show that the present methods have desired accuracy and robustness, which are fit for directly numerical simulation of complex fluids flow problem.

2 Construction of CCU and Accuracy Analysis

The definitions of some difference operators that used in this paper are listed below:

$$\Delta_{xx} f_{i,j} = \frac{f_{i+1,j} - 2f_{i,j} + f_{i-1,j}}{\Delta x^2} + O(\Delta x^2)$$

$$\Delta_{yy} f_{i,j} = \frac{f_{i,j+1} - 2f_{i,j} + f_{i,j-1}}{\Delta y^2} + O(\Delta y^2)$$

$$\delta_x^0 f_{i,j} = \frac{f_{i+\frac{1}{2},j} - f_{i-\frac{1}{2},j}}{\Delta x} + O(\Delta x^2)$$

$$\delta_y^0 f_{i,j} = \frac{f_{i,j+\frac{1}{2}} - f_{i,j-\frac{1}{2}}}{\Delta y} + O(\Delta y^2)$$

$$\Delta_x^0 f_{i,j} = \frac{f_{i+1,j} - f_{i-1,j}}{2\Delta x} + O(\Delta x^2)$$

$$\Delta_y^0 f_{i,j} = \frac{f_{i,j+1} - f_{i,j-1}}{2\Delta y} + O(\Delta y^2)$$

2.1 Construction of CCU scheme

Consider the following convection diffusion equation

$$\frac{\partial u}{\partial t} + \frac{\partial f}{\partial x} = 0, \qquad f = au \qquad (1)$$

Let f^+ be the f for the case $a > 0$ and f^- be the f case for $a < 0$, then the third-order CCU34 scheme we proposed is

$$F_i^+ = \sigma \overline{F}_i^+ + (1-\sigma)\overline{\overline{F}}_i^+ \qquad (2)$$

$$F_i^- = \sigma \overline{F}_i^- + (1-\sigma)\overline{\overline{F}}_i^- \qquad (3)$$

where $\overline{F}_i^+$, $\overline{\overline{F}}_i^+$, $\overline{F}_i^-$ and $\overline{\overline{F}}_i^-$ can be calculated by the following compact upwind schemes

$$\overline{F}_{i-1}^+ + 2\overline{F}_i^+ = \frac{1}{2\Delta x}(-5f_{i-1} + 4f_i + f_{i+1}) \qquad (4)$$

$$\overline{F}_{i+1}^- + 2\overline{F}_i^- = \frac{1}{2\Delta x}(-f_{i-1} - 4f_i + 5f_{i+1}) \qquad (5)$$

$$\overline{\overline{F}}_{i-1}^+ + 2\overline{\overline{F}}_i^+ = \frac{1}{2\Delta x}(-7f_{i-1} + 8f_i - f_{i+1}) + \Delta x S_i \qquad (6)$$

$$\overline{\overline{F}}_{i+1}^- + 2\overline{\overline{F}}_i^- = \frac{1}{2\Delta x}(f_{i-1} - 8f_i + 7f_{i+1}) - \Delta x S_i \qquad (7)$$

where $F^\pm$, $\overline{F}^\pm$ and $\overline{\overline{F}}^\pm$ are the difference approximation of the $\partial f^\pm / \partial x$, Δx the mesh size, $\sigma \in [0,1]$, called combination-optimization-controlling parameter, we should choose the suitable value of σ in order to improve the RE of CCU scheme as high as possible. The formula (4) and (5) are third-order compact upwind schemes (UCD3) presented by Fu and Ma (1989) [4] , (6)and (7) fourth-order compact upwind schemes (UCD4) presented by us [5]. S_i which note the difference approximation of the $\partial^2 f / \partial x^2$, is calculated by the following fourth-order Padé scheme

$$\frac{1}{12}S_{i-1} + \frac{5}{6}S_i + \frac{1}{12}S_{i+1} = \Delta_{xx} f_i \qquad (8)$$

2.2 The optimum CCU scheme

We choose $\sigma = 0.71$ by employing dispersion-relation-preserving (DRP) method[3,12] to optimize in this paper.The Fourier accuracy analysis [1,2,4] is employed to test the performance of CCU scheme. The real part and imaginary part of modified wavenumber are response the dispersive error and disspastive error of FD scheme, respectively. The modified wavenumbers of UCD3, UCD4 and CCU scheme are listed below:

(1) UCD3

$$\bar{k}_i = \frac{(8+\cos\omega)\sin\omega}{5+4\cos\omega}, \quad \bar{k}_r = \frac{4(\sin\frac{\omega}{2})^4}{5+4\cos\omega} \qquad (9)$$

(2) UCD4

$$\bar{\bar{k}}_i = \frac{(38+17\cos\omega-(\cos 2\omega)^2)\sin\omega}{25+25\cos\omega+4(\cos\omega)^2}, \quad \bar{\bar{k}}_r = \frac{8(\sin\frac{\omega}{2})^6}{25+25\cos\omega+4(\cos\omega)^2} \qquad (10)$$

(3) CCU

$$\tilde{k}_i = \sigma\bar{k}_i + (1-\sigma)\bar{\bar{k}}_i, \quad \tilde{k}_r = \sigma\bar{k}_r + (1-\sigma)\bar{\bar{k}}_r \qquad (11)$$

where $\omega = k\Delta x\ (0 \le \omega \le \pi,\ k \ge 1)$ is wavenumber. Variations of k_i and k_r for some difference approximations are given in Fig.1 and 2. The CCU scheme has largely improved the RE of

UCD3 and UCD4 in the far smaller error tolrance range which can be confirmed in Table1. The dissipative error of the CCU scheme is smaller than UCD3, but larger than UCD4, which can be found in Fig. 2. But the dispersive error is far smaller than the UCD3 and UCD4, which are the most important characteristics. The range of well-resolved wavenumber may be defined by the error tolerance ε [1]

$$\left|\frac{k_i(\omega)-\omega}{\omega}\right| < \varepsilon$$

where $k_i(\omega)$ is modified wavenumber. Suppose ω_f is the shorest well-resolved wavenumber. The fraction $e(\varepsilon)=\omega_f/\pi$ may be regarded as a measure of the RE of a scheme. We list the $e(\varepsilon)$ of different FD schemes under different error tolerance ε in Tabel 1, The results show that the RE of CCU scheme is far higher than that of UCD4, and better than UCD3 under the smaller tolerance error and wide range of wavenumber.

3 Projection Method

In this paper we consider the non-dimensional time dependent incompressible Navierf-Stokes (NS) equations of the form

$$U_t + (U\cdot\nabla)U + \nabla p = \frac{1}{\text{Re}}\Delta U \qquad (12)$$

$$\nabla\cdot U = 0 \qquad (13)$$

where $U=(u,v)$ and p represent the velocity vector and pressure, respectively, Re is the Reynolds number. According to projection method [6], the firstly, an intermediate velocity $U^*=(u^*,v^*)$ is introduced and calculated by neglecting the contribution of the pressure gradient terms:

$$\frac{U^*-U^n}{\Delta t} = \frac{1}{\text{Re}}\Delta U^n - (U^n\cdot\nabla)U^n \qquad (14)$$

The secondly, the intermediate velocity U^* can be decomposed into the sum of two vectors:

$$U^* = U^{n+1} + \nabla\phi \qquad (15)$$

where vector U^{n+1} with zero divergence denotes the velocity fields at the next time level, vector $\nabla\phi$ with zero curl. And then combination formula (12),(14) and (15) we have

$$\frac{U^{n+1}-U^*}{\Delta t} = -\nabla p^{n+1} \qquad (16)$$

$$\nabla\cdot U^{n+1} = 0 \qquad (17)$$

The pressure Poisson equation is obtained by combining equation (12) and (13) as follows

$$\nabla^2 p^{n+1} = \frac{1}{\Delta t}\nabla\cdot U^* \equiv \frac{1}{\Delta t}D(U^*) \qquad (18)$$

After solving the pressure and pressure gradient, we can obtain the velocity fields at time level $n+1$ form equation (12). As we known that this projection method is only the first-order accurate in time, we called it one step Euler projection method.

In order to improve the accuracy, we introduce the following second-order accuracy explicit predictor-corrector projection method in time[13].

Predictor stage:

$$\frac{U^*-U^n}{\Delta t/2} = \frac{1}{\text{Re}}\Delta U^n - (U^n\cdot\nabla)U^n \qquad (19)$$

$$\nabla^2 p^{n+1/2} = \frac{2}{\Delta t}\nabla\cdot U^* \equiv \frac{2}{\Delta t}D(U^*) \qquad (20)$$

$$\frac{U^{n+1/2}-U^*}{\Delta t/2} = -\nabla p^{n+1/2} \qquad (21)$$

Correction stage:

$$\frac{U^{**}-U^n}{\Delta t} = \frac{1}{\text{Re}}\Delta U^{n+1/2} - (U^{n+1/2}\cdot\nabla)U^{n+1/2} \qquad (22)$$

$$\nabla^2 p^{n+1} = \frac{1}{\Delta t}\nabla\cdot U^{**} \equiv \frac{1}{\Delta t}D(U^{**}) \qquad (23)$$

$$\frac{U^{n+1}-U^{**}}{\Delta t} = -\nabla p^{n+1} \qquad (24)$$

The computing procedure in each stage of predictor-correctior method is the same with one step Euler projection method, here we are not going to give the more details.

4 Discretization of Governing Equations

All the discretization based on staggered grid system. The first momentum equation is approximated at point $(i+\frac{1}{2}, j)$ in *x* direction, the second one at the point $(i, j+\frac{1}{2})$ in *y* direction. In discretization of the momentum equations, the values u at point $(i, j+\frac{1}{2})$ and v at $(i+\frac{1}{2}, j)$ are needed, which can be calculated by midpoint interpolation scheme [1]. The convection terms are discretized by using CCU scheme. For example, the term $u\frac{\partial u}{\partial x}$ is split as follows

$$u\frac{\partial u}{\partial x} = u^+\frac{\partial u}{\partial x} + u^-\frac{\partial u}{\partial x} \qquad (25)$$

Where $u^+ = \frac{u+|u|}{2}$ (u > 0) and $u^- = \frac{u-|u|}{2}$ (u < 0) · The term $u^+ \frac{\partial u}{\partial x}$ and $u^- \frac{\partial u}{\partial x}$ are approximated at point $(i+\frac{1}{2}, j)$ by $u^+_{i+\frac{1}{2},j} F^+_{i+\frac{1}{2},j}$ and $u^-_{i+\frac{1}{2},j} F^-_{i+\frac{1}{2},j}$, respectively. Where $F^+_{i+\frac{1}{2},j}$ and $F^-_{i+\frac{1}{2},j}$ can be approximated by CCU scheme (2) and (3), respectively. The viscous terms can be discretized using formula (8). After discretizing all the convection and viscous terms, the intermediate velocity u^* and v^* at point $(i+\frac{1}{2}, j)$ and $(i, j+\frac{1}{2})$ can be calculated by using formula (3), respectively.

The pressure Poisson equation is approximated at the point (i, j). According to paper [11], the pressure Poisson equation can be approximated with the following fourth-order compact scheme

$$\begin{aligned}
&\frac{1}{12\,\Delta x \Delta y}[(\lambda+\gamma)(p_{i+1,j+1}+p_{i+1,j-1} \\
&\quad + p_{i-1,j+1} + p_{i-1,j-1} - 20\,p_{i,j}) \\
&\quad + 2(5\lambda-\gamma)(p_{i+1,j}+p_{i-1,j}) \\
&\quad + 2(5\gamma-\lambda)(p_{i,j+1}+p_{i,j-1})] \\
&= \frac{1}{12\,\Delta x \Delta t}\{\frac{1}{2}[27\,\delta_x^0 u^*_{i,j} - (u^*_{i+\frac{3}{2},j} - u^*_{i-\frac{3}{2},j})] \\
&\quad + (\Delta x)^3 \Delta_{xx}(\delta_x^0 u^*_{i,j} + \delta_y^0 v^*_{i,j})\} \\
&\quad + \frac{1}{12\,\Delta y \Delta t}\{\frac{1}{2}[27\,\delta_y^0 v^*_{i,j} - (v^*_{i,j+\frac{3}{2}} - v^*_{i,j-\frac{3}{2}})] \\
&\quad + (\Delta y)^3 \Delta_{yy}(\delta_y^0 v^*_{i,j} + \delta_x^0 u^*_{i,j})\}
\end{aligned} \tag{26}$$

where $\lambda = \frac{\Delta x}{\Delta y}$, $\gamma = \frac{\Delta y}{\Delta x}$.

In projection method algorithm, before computing velocity, we must know the value of the pressure gradient at the same point. In staggered grid system, as we know above, the pressure gradient component $\partial p / \partial x$ and $\partial p / \partial y$ are approximated at point $(i+\frac{1}{2}, j)$ and $(i, j+\frac{1}{2})$, respectively. Combined the projection techniques, we derivate the following explicit schemes to complete such task. According to Taylor's series, we have

$$\begin{aligned}
(\frac{\partial p}{\partial x})_{i+\frac{1}{2},j} &= \delta_x^0 p_{i+\frac{1}{2},j} + \frac{\Delta x^2}{24}\Delta_{yy}\delta_x^0 p_{i+\frac{1}{2},j} \\
&\quad - \frac{\Delta x^2}{24\Delta t}(\Delta_{xx} u^*_{i+\frac{1}{2},j} + \delta_y^0 \delta_x^0 v^*_{i+\frac{1}{2},j}) \\
&\quad + O(\Delta x^4 + \Delta x^2 \Delta y^2 + \Delta y^4)
\end{aligned} \tag{27}$$

$$\begin{aligned}
(\frac{\partial p}{\partial y})_{i,j+\frac{1}{2}} &= \delta_y^0 p_{i,j+\frac{1}{2}} + \frac{\Delta y^2}{24}\Delta_{xx}\delta_y^0 p_{i,j+\frac{1}{2}} \\
&\quad - \frac{\Delta y^2}{24\Delta t}(\Delta_{yy} v^*_{i,j+\frac{1}{2}} + \delta_x^0 \delta_y^0 u^*_{i,j+\frac{1}{2}}) \\
&\quad + O(\Delta x^4 + \Delta x^2 \Delta y^2 + \Delta y^4)
\end{aligned} \tag{28}$$

5 Numerical Examples

In this part, we take into consideration of the numerical solutions of NS equations (12) and (13) by using the new numerical method presented in this paper.

Example 1 (Taylor's array) [6]. The problem (12) and (13) with the initial conditions are taken as:

$$\begin{cases} u(x,y,0) = -\cos(Nx)\sin(Ny) \\ v(x,y,0) = \sin(Nx)\cos(Ny) \end{cases} \tag{29}$$

where $0 \le$ x, y $\le 2\pi$ and N is an integer. The exact solution for this case is known as:

$$\begin{cases} u(x,y,t) = -\cos(Nx)\sin(Ny)\exp(-2N^2 t/\mathrm{Re}) \\ v(x,y,t) = \sin(Nx)\cos(Ny)\exp(-2N^2 t/\mathrm{Re}) \end{cases}$$

Tables 3 and 4 show RMS (root-mean-square) errors and rate of convergence Rate (CR) for velocity components u , v and vorticity ψ with different parameters. The definitions of RMS error and CR can be found in paper [14]. It is observed that the whole convergence rate of our FD algorithm is at least three.

Example 2 (driven-cavity) . The problem (12) and (13) with the boundary conditions are taken as:

$$\begin{cases} u = 1, & y = 1 \\ u = v = 0, & x = 0,\ 1, \quad y = 0 \end{cases} \tag{30}$$

This is a classcial example to test the performance of the numerical method, the results given by Ghia[15] has been regarded as the benchmark solutions. In present paper, we give the results of $\mathrm{Re} = 400{,}1000$ using a 129×129 grid. In Fig.3 and 4, the streamline of steady flow has been given. In Fig.5 and 6 we present comparisons of the u -velocity along the vertical centerline and the v -velocity along the horizontal centerline of the square cavity, respectively. It is easy to find that the present results are coincide with the Ghia's very well.

6 Conclusion

There are some new characteristics of present method. Firstly, the new weighted combined compact upwind scheme which has high resolving efficiency and strong dissipation restrain capblity is proposed. Such combination methods improve the performance of the intrinsic shortcomings of the former schemes. Secondly, we proposed the high-order accuracy compact difference projection algorithm for incompressible NS equations based on the staggered grid system. Such algorithm has at least third-order accuracy on the whole. The numerical examples verify that the present method is fit of DNS of complex flows, include the flow problem with large gradient and high Reynolds number.

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Table 1. Resolving Efficiency of the Schemes Shown in Fig. 1

Scheme	$e(\varepsilon = 0.1)$	$e(\varepsilon = 0.05)$	$e(\varepsilon = 0.005)$
UCD4[5]	0.660	0.550	0.310
UCD3[4]	0.825	0.800	0.345
UCD5[2]	0.725	0.635	0.430
CCU	0.795	0.750	0.660

Table 3. RMS errors with the rate of convergence for u, v, ψ at Re=2000, N=2,*t*=2

Grid	*u*-error	Rate	*v*-error	Rate	ψ -error	Rate
11×11	6.28241(-3)	—	6.28241(-3)	—	3.01721(-2)	—
21×21	3.91373(-4)	4.29	3.91373(-4)	4.29	1.74202(-3)	4.41
31×31	8.35782(-5)	3.96	8.35782(-5)	3.96	3.58175(-4)	4.06
41×41	2.66855(-5)	4.08	2.66855(-5)	4.08	1.14364(-4)	4.08
51×51	1.09885(-5)	4.06	1.09885(-5)	4.06	4.71530(-5)	4.05

Note: 5.00791(-3)= 5.00791×10^{-3} etc.

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51×51	1.09885(-5)	4.06	1.09885(-5)	4.06	4.71530(-5)	4.05

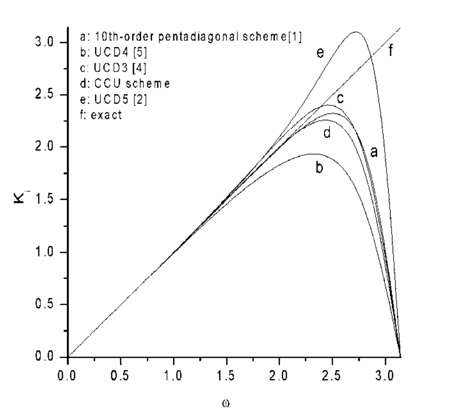

Fig.1 Real part of modified wavenumber

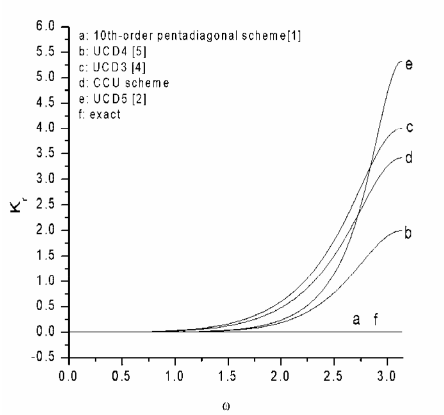

Fig. 2 Imaginary part of modifie wavenumber

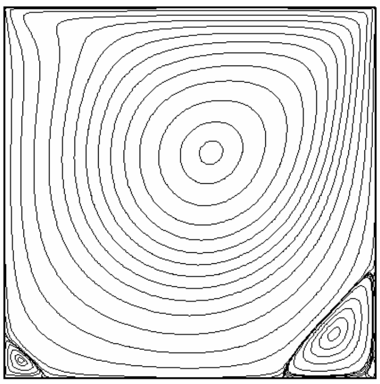
Fig.3 Streamline for Re=400

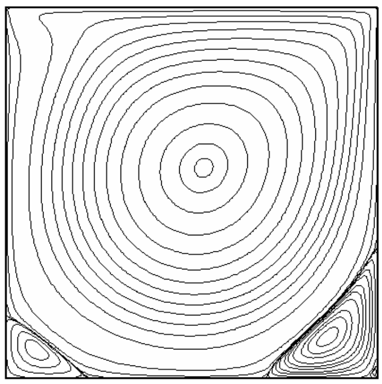
Fig. 4 Streamline for Re=1000

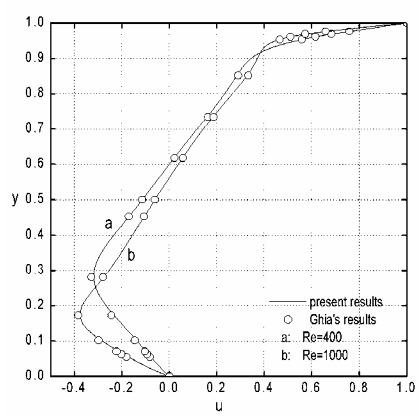

Fig.5 Comparison of u -velocity

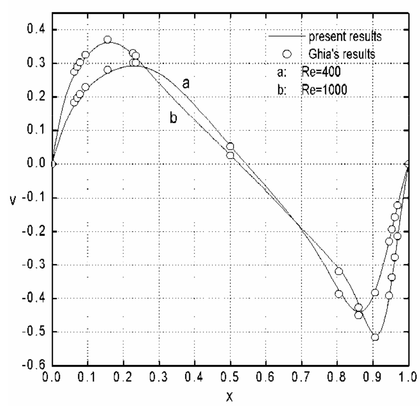

Fig.6 Comparison of v -velocity