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A New Iterative Learning Control Method for Stabilizing a Class of Nonlinear Systems

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Abstract: - Iterative learning control methods are applied ever-increasingly in control of systems. In recent years considerable improvements have been made in theory and application of ILC methods.
Nowadays ILC's methods in most academics are described as the methods based on repetitive process from beginning to the end, or a kind of repetitive control.
In this paper a new method based on ILC is represented which is differ from conventional methods of ILC, not having repeating property and the system is controlled only once from the beginning to the end of the process.
By freezing the time and moving in the new virtual axis, called index, our designed method tries to find the best value for control at this time step, so it is obvious that this control process is off-line method.
The mentioned ILC method based on Lyapunov Stability Theorem and by satisfying the convergence condition for our designed ILC method, the stability of the closed loop is obtained automatically.

Key-Words: - Iterative Learning Control, Nonlinear Systems, Lyapunov Stability Theorem.

1 Introduction

In the last two decades, iterative learning control has been extensively studied, achieving significant progress in both theory and application.
Conventional iterative learning control (ILC) is a relatively new control strategy that updates the control input to improve the system performance by means of system repetition over a finite time interval[3][4][5].
In this paper, a new type of ILC method is represented. Different from conventional methods which control a process in finite time repetitively, by fixing the time and moving in through the new virtual axis, called index, this new mentioned method tries to find the best value for control at this time step. In this process, using a Lyapunov Function and satisfying the condition of Lyapunov Theorem the stability of the investigated systems are obtained.
In section 2 problem formulations is represented. Section 3 presents our controller designed method. Sections 4 discuss our results by showing application of our algorithm on some dynamics. And finally conclusion is included in section 5.

2 Problem Formulation

In this paper, we focus on the design a controller for a class of SISO nonlinear system whose dynamical equation can be expressed in the following form:

$$x^{(n)} = f(X) + g(X)u \qquad (1)$$

Where $X = [x, \dot{x}, ..., x^{(n-1)}]$, it can also be expressed as below;

$$\begin{cases} \dot{x}_1 = x_2; \\ \dot{x}_2 = x_3; \\ . \\ . \\ . \\ \dot{x}_n = f(X) + g(X)u \end{cases} \qquad (2)$$

where t is the time ,x(t) is the output variable, $x^{(i)}(i = 1,...,n-1)$ is the ith derivative of *x(t)*, *u(t)* is the input and f and g are known which are uniformly continuous respect to x and *f(0,t)=0* .

3 Controller Design Method

Lemma:

Suppose real series $\{U_k\}_{k\geq 0}$, $\{V_k\}_{k\geq 0}$ and $\{W_k\}_{k\geq 0}$ satisfy

a) $U_{k+1} = W_k U_k + V_k$
b) $\lim_{k\to\infty} V_k \to 0$

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c) $\|W_k\| \le \rho < 1, \forall k > q$

Where q is a finite positive integer. Then $\lim_{k\to\infty} U_k \to 0$

Theorem:

Consider the closed loop system (1), with the following ILC controller:

$$u^{i+1}(t) = u^i(t) + q\Delta e^i(t) \quad (3)$$

Where t represents time, i shows the iteration in the index axis, u is the controller and Δe^i is the difference between the derivative of nth state and the particular linear combination of all of states in the index axis, expressed as below:

$$\Delta e^i(t) = \dot{x}_n^{\,i+1} - (-\alpha_1 x_1{}^i - \alpha_2 x_2{}^i -,\ldots - \alpha_n x_n{}^i) \quad (4)$$

where α_i 's (i=1,..,n) comes from the selected desired negative matrix Q which is defined as follows:

$$Q = \begin{bmatrix} 010 & \ldots & 0 \\ 0010 & \ldots & 0 \\ . & & \\ . & & \\ . & & \\ -\alpha_1 - \alpha_2 \ldots - \alpha_n & & \end{bmatrix},$$

and q is a constant which is called learning factor.
Then (1) will be asymptotically stable around the origin by the below convergence condition:

$$\|1 + qg(X^i)\| < 1$$

Proof:

By defining the Lyapunov function as below and following the proof procedure, a quoted controller will be derived:

$$V = \frac{1}{2} X^T X \Rightarrow \dot{V} = X^T \dot{X} \quad (5)$$

now if we can design a controller which satisfies the below relation:

$$\dot{X} = Q \times X \quad (6)$$

where Q is a negative definite matrix in the above mentioned form, then (5) changes to:

$$\dot{V} = X^T \dot{X} = X^T Q X < 0 \quad (7)$$

Therefore the necessary and sufficient conditions for Lyapnunov Theorem are satisfied and the stability of system (1) is achieved.
The relation (6) is expressed again in matrix form as below:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ . \\ . \\ . \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 010 & \ldots & 0 \\ 0010 & \ldots & 0 \\ . & & \\ . & & \\ . & & \\ -\alpha_1 - \alpha_2 \ldots - \alpha_n & & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ x_n \end{bmatrix} \quad (8)$$

According to above equation if only the last row is satisfied then all other rows will follow automatically which means that the below relation should be gratified:

$$\dot{x}_n = -\alpha_1 x_1 - \alpha_2 x_2 -,\ldots - \alpha_n x_n$$

To establish the last relation our first order ILC method is defined and the updating law for its controller is introduced as (3). Where in relation (3), “i” represents moving in the virtual index axis between two sequent step time. In other word, if the system is in the time “t”, it is frozen in this time and moves to update in index term until the error which is defined by relation (4) reaches to the desired selected value.
Now the convergence of our mentioned algorithm is evaluated as below:

$$\begin{aligned} u^{i+1}(t) &= u^i(t) + q\Delta e^i(t) \\ &= u^i(t) + q[\dot{x}_n{}^{i+1} - (-\alpha_1 x_1{}^i - \alpha_2 x_2{}^i -,\ldots - \alpha_n x_n{}^i)] \\ &= u^i(t) + q\{[f(X^i) + g(X^i) u^i(t)] + \\ &\quad [\alpha_1 x_1{}^i + \alpha_2 x_2{}^i +,\ldots + \alpha_n x_n{}^i]\} \\ &= [1 + qg(X^i)] u^i(t) + \\ &\quad q[f(X^i) + \alpha_1 x_1{}^i + \alpha_2 x_2{}^i +,\ldots + \alpha_n x_n{}^i] \end{aligned}$$

For simplicity the new function h(X) is defined as below:

$$h(X^i) = f(X^i) + \alpha_1 x_1{}^i + \alpha_2 x_2{}^i +,...+ \alpha_n x_n{}^i$$

and by following the procedure it can be shown that:

$$u^{i+1}(t) = [1 + qg(X^i)]u^i + qh(X^i) \qquad (9)$$

If the system is stable then it can be written:

If $i \to \infty \Rightarrow x_i \to 0, \forall i \ (i = 1,2,...n)$, according to assumption for f function that $f(0,t) = 0$ it can be resulted $h(X^i) \to 0$, so according to the above mentioned lemma, the algorithm would be converged by the below condition:

$$\left\| 1 + qg(X^i) \right\| < 1$$

now if the mentioned algorithm converges, the relation (7) is satisfied and based on the Lyapunov Stability Theorem, the system would be stable.

Remark1: by using the mentioned method and according to the Lyapunov Theorem, all of the states move to the origin and the stability in view point of states are achieved.

Remark 2: our designed method is not only capable in such systems as (1), but also any system in the form of

$$\dot{x} = f(x,t) + g(x,t) \times u \quad x \in R^n \ , u \in R^m$$

Which could be converted to system (1) by using input-state linearization method.

Remark 3: In view point of sliding mode control, the sliding surface is known as a special case of lyapunov surface. By using our algorithm, one can claim that has found a sliding surface. Consider the following relations:

$$Y = CX, \quad C = [c_1,...,c_n] \quad c_i \neq 0, \quad X = [x_1,...x_n]^T$$

where Ci's are selected in such a way that CX would be hurwitz.

By defining the Lyapunov function and following the procedure below, we will have:

$$V = \frac{1}{2} Y^T Y \Rightarrow \dot{V} = Y^T \dot{Y} = (CX)^T (CX)' = X^T C^T C\dot{X}$$

$$\dot{V} = X^T C^T CQX, \quad C^T C \geq 0, \quad Q < 0$$

$$\dot{V} < 0$$

Based on the Lyapunov theorem Y moves to the origin and $CX \longrightarrow 0$ so,

$$c_1 x_1 + ... + c_n x_n = 0 \quad or \quad c_1 x + c_2 x^{(1)} ... + c_n x^{(n-1)} = 0$$

Because of Y is Hurwitz, then all of the states moves to origin. In this case *S=CX* is called sliding surface which has been claimed to find by our method.

Remark 4: this method can be used in online control. In this case according to our designed method we can write the below relations:

$$e = r - y = r - x_1$$

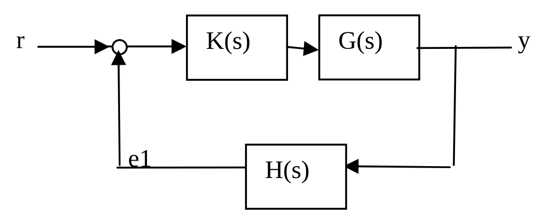
$$e1 = \dot{x}_n - (-\alpha_1 x_1 - \alpha_2 x_2 -,... - \alpha_n x_n)$$

$$e1(s) = s^n (r(s) - e(s)) + \alpha_1 (r(s) - e(s))$$

$$+ \alpha_2 s(r(s) - e(s)) +,...+ \alpha_n s^{n-1} (r(s) - e(s))$$

$$e1 = H(s)(r(s) - e(s))$$

$$e1 = H(s) y(s)$$



where K(s) is our designed controller which is defined as:

$$u(t+1) = u(t) + q\Delta e(t)$$

where in the above relation q is a learning factor and the corrective term is described as below:

$$\Delta e(t) = \dot{x}_n - (-\alpha_1 x_1 - \alpha_2 x_2 -,... - \alpha_n x_n)$$

It is obvious that the above relation changes to a simple PI controller but in fact our designed controller is the combination of the blocks, H(s) and K(S) which is distinguished from a PI controller and gives us more options for designing.

By usin the Theories as Small Gain Theorem in the closed loop system which is shown in the previous figure, the condition for the stability is achieved, but as it is known, this is more conservative theorem.

4 Experimental Results

Now the results of our designed method are illustrated in three nonlinear systems with our problem formulation. The results are shown in both off-line and on-line cases and following that compared with sliding mode control method.

Duffing Equation:

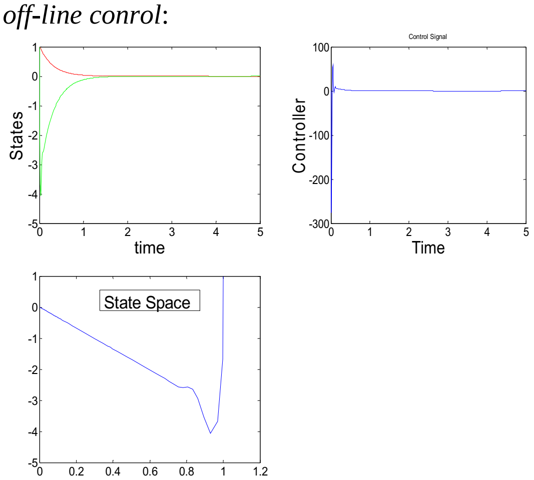
The Duffing equation is described by:

$$\ddot{x} + p_2\dot{x} + p_1 x + x^3 = q\cos(wt) + u$$

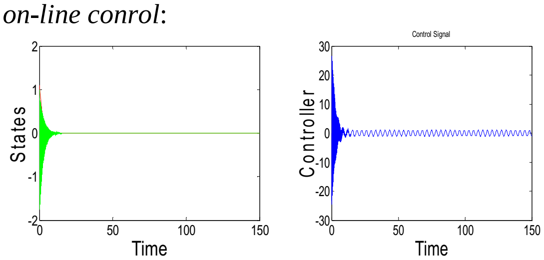
where the parameters p1, p2, q, w are constants. It is well known that the Duffing equation exhibits chaotic behaviors in certain parameter regions.
The above dynamical equation can be written as the following state equation:

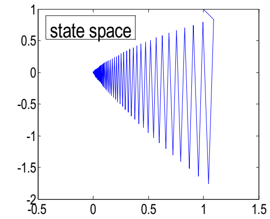
$$\begin{bmatrix}\dot{x}\\ \ddot{x}\end{bmatrix}=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\end{bmatrix}+\begin{bmatrix}0\\ -p_1x-p_2\dot{x}-x^3+q\cos(wt)\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u$$

off-line conrol:



on-line conrol:





Using Sliding mode control:

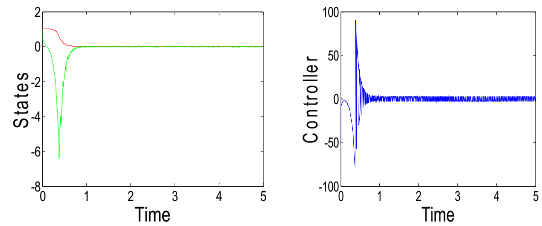


Fig 1- off-line, online, and sliding mode control: states, controller, and state space

One-Link Rigid Robot Manipulator:

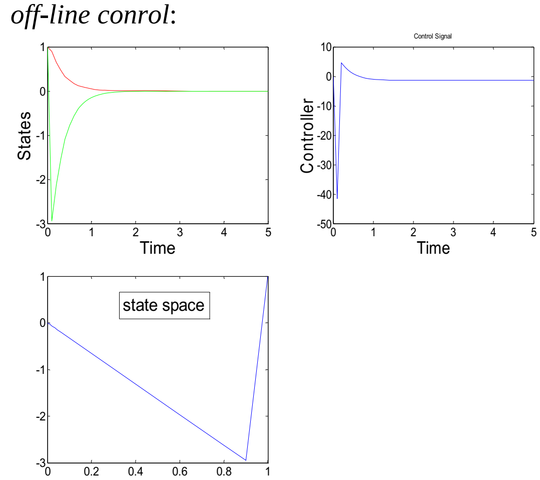
The dynamic equation of the one-link rigid manipulator is given by:

$$m\ell^2\ddot{x} + d\dot{x} + m\ell g\cos(x) = u$$

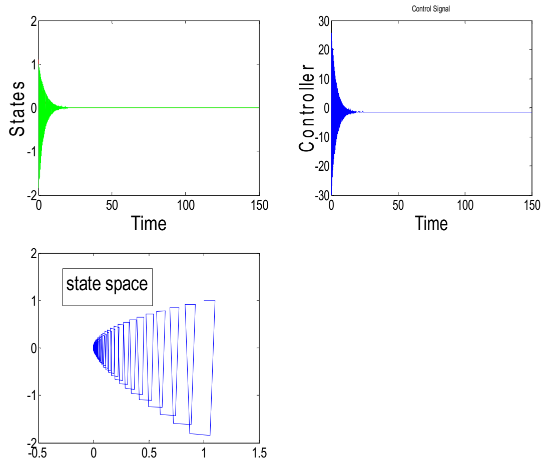
Where the link is of length l and mass m, and q is the angular position.
The above dynamical equation can be written as the following state equation:

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\end{bmatrix}=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\end{bmatrix}+\begin{bmatrix}0\\ -(ml^2)^{-1}d\,\dot{x}-gl^{-1}\cos(x)\end{bmatrix}+\begin{bmatrix}0\\ (ml^2)^{-1}\end{bmatrix}u$$

off-line conrol:



on-line conrol:



Using Sliding mode control:

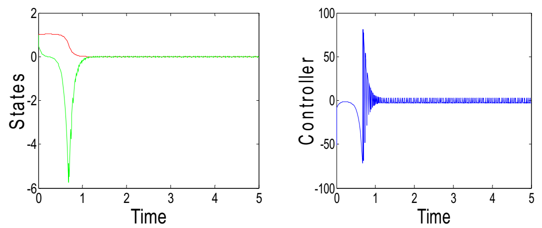


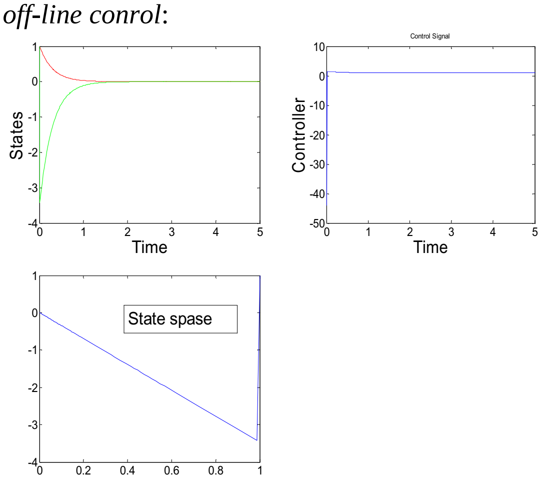
Fig 2- off-line, online, and sliding mode control: states, controller, and state space

Inverted pendulum:

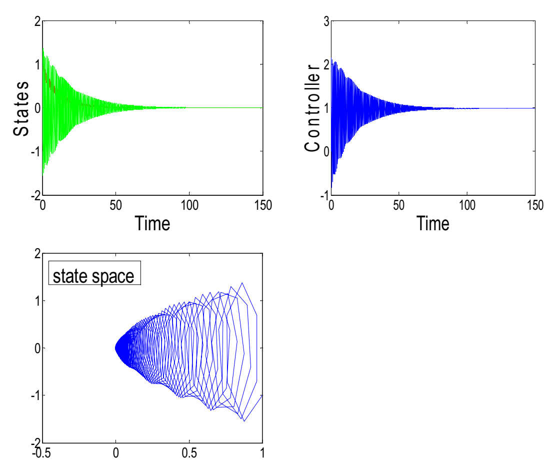
The dynamic equation of the inverted pendulum is given by:

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\end{bmatrix}=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\end{bmatrix}+\begin{bmatrix}0\\ -\frac{g_0}{l}\sin(x+\pi/2)-\frac{k_0}{m}\dot{x}\end{bmatrix}+\begin{bmatrix}0\\ (ml^2)^{-1}\end{bmatrix}u$$

off-line conrol:



on-line conrol:



Using Sliding mode control:

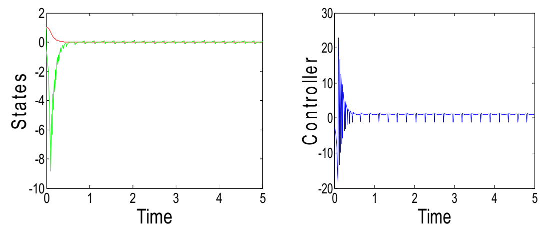


Fig 3- off-line, online, and sliding mode control: states, controller, and state space

4 Conclusion

Achieving a proper Lyapunov function which satisfies the condition of the Lyapunov Theorem, is the most important factor in the stability of control systems.

In this paper, for a class of nonlinear systems, using the ILC method and defining the Lyapunov surface, it has been attempted to provide the necessary condition for Lyapunov Theorem, and subsequently the stability of such systems.

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