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A Fuzzy Goal Programming Approach for a Single Machine Scheduling Problem

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Abstract: - This paper presents the fuzzy mixed-integer goal programming model for a single machine scheduling problem with bi-objectives consisting of the minimization of the total weighted flow time and total weighted tardiness. This model is solved by a fuzzy goal programming approach to verify and validate the proposed approach for the above problem. Two test problems in small and large sizes are generated at random and then the computational results demonstrate the effectiveness of the proposed approach.

Key-Words: - Single machine scheduling, Fuzzy goal programming, Weighted flow time, Weighted tardiness

1 Introduction

Scheduling consists of planning and arranging jobs in an orderly sequence of operations in order to meet the customer's requirements [1]. The schedule of jobs and the control of their flows through a production process are the most significant role in any modern manufacturing systems. In a single-machine scheduling, there is only one machine to process all jobs to optimize the objective function, say minimizing the sum of the maximum earliness and tardiness [2]. It is well known that the optimal solution of single objective models can be quite different to those models consist of multi objectives. In fact, the decision maker often wants to minimize the earliness/tardiness penalty or total flow time. Each of these objectives is valid from a general point of view. Since these objectives conflict with each other, a solution may perform well for one objective or it gives inferior results for others. For this reason, scheduling problems have a multi-objective nature. In decision making situations, the high degree of fuzziness and uncertainties is included in the data set. The fuzzy set theory provides a framework for handling the uncertainties of this type [3].

Bellman and Zadeh [4] presented some applications of fuzzy theories to the various decision-making processes in a fuzzy environment. Zimmerman [5 and 6] presented a fuzzy optimization technique to a linear programming (LP) problem with single and multi objectives. The fuzzy set theory has been applied to formulate and solve problems in various areas such as artificial intelligence, image processing, robotics, pattern recognition, and the like (Hannan, [7] and Yager, [8]). Different approaches to multi-objective single machine problems with fuzzy parameters have been presented in the literature during the last decade. Ishii and Tada [9] considered a single machine scheduling problem minimizing the maximum lateness of jobs with fuzzy precedence relations. A fuzzy precedence relation relaxes the crisp precedence relation and represents the satisfaction level with respect to precedence between two jobs. Therefore, the problem to be solved considered an additional objective to maximize the minimum satisfaction level obtained by the fuzzy precedence relations. An algorithm for determining non-dominated solutions is proposed based on a graph representation of the precedence relations.

Adamopoulos and Pappis [10] presented a fuzzy-linguistic approach to multi-criteria sequencing problem. They considered a single machine, in which each job is characterized by fuzzy processing times. The objective is to determine the processing times of jobs and the common due date as well as to sequence the jobs on the machine where penalty values are associated with due date assigned, earliness, and tardiness. Another approach to solve multi-criteria single machine scheduling was presented by Lee, *et al.* [11]. The proposed approach is used linguistic values to evaluate each criterion (e.g. very poor, poor, fair, good, and very good) and to represent its relative weight (e.g. very unimportant, unimportant, medium important, important, and very important). A tabu search method is used as a stochastic tool to find the near-optimal solution with an aggregated fuzzy objective function.

Ishibuchi and Murata [12] presented a flow shop scheduling problem with fuzzy parameters such as fuzzy due dates and fuzzy processing times, in which the objectives are to minimize the total flow time, makespan, and the maximum earliness and tardiness of all jobs. A multi-objective genetic algorithm is developed to handle these fuzzy scheduling objectives. Thereafter, a number of researches have extended the fuzzy set theory to the field of goal programming proposed by Narsimhan [13]. In fact, the fuzzy goal and multi-objective programming has a very extensive application. For example, Sinha, *et al.* [14] presented a fuzzy goal programming in multi-criteria decision systems. Rao, *et al.* [15] proposed a fuzzy goal programming approach for the structural optimization problem. Kumar, *et al.* [16] proposed a fuzzy goal programming approach for a vendor selection problem in supply chain. Mishra, *et al.* [17] presented a fuzzy goal programming model of a machine-tool selection and operation allocation problem in flexible manufacturing systems.

2 Problem Formulation

The following notations and definitions are used to describe the single machine scheduling problem with multi objectives.

2.1 Indices and Parameters

N = number of jobs.
P_i = processing time of job i ($i=1, 2\ldots N$).
R_i = release time of job i ($i=1, 2\ldots N$).
D_i = due date of job i ($i=1, 2\ldots N$).
W_i = weight of job i ($i=1, 2\ldots N$).
M = a large positive integer value.

2.2 Decision Variables

$$X_{ij} = \begin{cases} 1 & \text{if job } j \text{ is scheduled after job } i\,; \\ 0 & \text{otherwise.} \qquad \forall i,j \text{ and } i \neq j \end{cases}$$

C_i = completion time of job i
T_i = tardiness of job i

2.3 Mathematical Model

The mixed-integer programming (MIP) formulation of the single machine scheduling problem for bi-objectives and a set of constraints can be written as follows:

$$\min \; Z_1 = \sum_{i=1}^{N} W_i C_i \qquad (1)$$

$$\min \; Z_2 = \sum_{i=1}^{N} W_i T_i \qquad (2)$$

s.t.

$$C_i \geq R_i + P_i \qquad \forall i \qquad (3)$$

$$X_{ij} + X_{ji} = 1 \qquad \forall i,j \;\; ; i \# j \qquad (4)$$

$$C_i - C_j + M\, X_{ij} \geq P_i \quad \forall i,j \;\; ; i \# j \qquad (5)$$

$$T_i = \max\{0, C_i - D_i\} \quad \forall i \qquad (6)$$

$$X_{ij} \in \{0,1\} \qquad \forall i,j \; ; i \# j \qquad (7)$$

The objective functions (1) and (2) minimize the total weighted flow time and total weighted tardiness respectively. Constraint (3) ensures that the completion time of the job is greater than its release time plus processing time. Constraint (4) specifies the order relation when any two jobs have already scheduled. Constraint (5) stipulates the completion time relativity of any two jobs. M should be large enough for Constraint (6) so that it is always feasible. Constraint (7) specifies the tardiness of each job.

2.4 Fuzzy Mixed-Integer Goal Programming Model

When vague information related to the objectives is presented, then the problem can be formulated as a fuzzy goal-programming problem. A typical fuzzy mixed-integer goal programming problem (f-MIGP) formulation can be stated as follows:

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$$\begin{aligned}
&\text{Find } x_i && i=1,2,..,n \\
&\text{to satisfy } Z_l(x_i) \cong \tilde{Z}_l && l=1,2,...,L \\
&h_j(x_i) \le d_j && j=1,2,...,J \\
&S_k(x_i) = c_k && k=1,2,...,K \\
&x_i \ge 0 \text{ and integer} && i=1,2,...,n
\end{aligned} \quad (8)$$

where,

$Z_l(x_i)$ is the l^{th} goal constraint.

$h_j(x_i)$ is the j^{th} inequality constraint.

$S_k(x_i)$ is the k^{th} equality constraint.

$\tilde{Z}_l$ is the target value of the l^{th} constraint.

d_j is the available resource of inequality constraint.

c_k is the available resource of equality constraint.

In relations given in Eq. (8), the symbol '$\cong$' indicates the fuzziness of the goal. It represents the linguistic term 'about' and it means that $Z_l(x_i)$ should be in the vicinity of the aspiration $\tilde{Z}_l$. The l^{th} fuzzy goal signifies that the decision maker will be satisfied even for values $Z_l(x_i) \cong \tilde{Z}_l$ slightly greater than (or lesser than) up to a stated deviations signified by the tolerance limit. The j^{th} system constraint $h_j(x_i) \le d_j$ and the k^{th} system constraint $S_k(x_i) = c_k$ are assumed to be crisp.

The fuzzy set theory [3] is based on the extension of the classical definition of the set. In the classical set theory, each element of a universe *X* either belongs to a set *A* or not, whereas in the fuzzy set theory, an element belongs to a set *A* with a certain membership degree.

Definition: A fuzzy set *A* in *X* is defined by:

$A = \{(x, \mu_A(x)) \mid x \in X\}$

where, $\mu_A(x): X \to [0,1]$ is called the membership function of *A* and $\mu_A(x)$ is the degree of membership to which *x* belongs to *A*.

By using the approach proposed by Yang, *et al.* [18], the f-MIGP formulation may be solved to determine the decision set and then to maximize the set. This approach is based on a piecewise linear approximation with the min-operator for aggregating the fuzzy goals. Once the membership functions of the fuzzy objectives $\mu_{Z_l(x_i)}$ are known, the fuzzy optimization problem (f-MIGP) formulation is transformed into an equivalent crisp formulation (c-MIGP) for the optimization problem. An equivalent crisp mathematical programming (c-MIGP) formulation is given as follows:

$$\begin{aligned}
&\max \ \alpha \\
&\text{s.t.} \\
&\quad \alpha \le \mu_{Z_l}(x_i)\ , \quad l=1,2,...,L \\
&\quad h_j(x_i) \le d_j \ , \quad j=1,2,...,J \\
&\quad S_k(x_i) = c_k \ , \quad k=1,2,...,K \\
&\quad x_i \ge 0 \text{ and integer}
\end{aligned} \quad (9)$$

2.5 Application of f-MIGP Model

An f-MIGP for a single machine scheduling problem formulation is presented as follows:

$$\sum_{i=1}^{N} W_i C_i \cong \tilde{Z}_1 \quad (10)$$

$$\sum_{i=1}^{N} W_i T_i \cong \tilde{Z}_2 \quad (11)$$

s.t.

Constraints (3) to (7).

3 Problem Solution

In this paper, the effectiveness of the FGP technique for the single machine scheduling problem in a small size is demonstrated through a data set as shown in Table 1. For each job, the processing time, release time, and weight of jobs are chosen at random between 0 and 10. The corresponding due date is also computed by $D_i = P_i N(1-M)$ as given in [1]. *N* is the number of jobs and *M* the uniformly random number between 0 and 1.

Table 1 Input data for a small-sized problem

Job	P_i	R_i	D_i	W_i
1	5	2	13	3
2	7	5	18	5
2	8	8	20	4
4	3	10	8	6
5	2	10	5	4

The following solution procedure is employed to solve the above numerical example.

Step 1. One objective is taken at a time and the rest of the formulation is solved by using the Lingo 8 software as shown in Table 2.

Table 2 Intermediate computational results

Individual objective function for minimization	Z^*
Z_1^* Total weighted flow time	377
Z_2^* Total weighted tardiness	130

Step 2. Suitable membership functions for all objective functions are decided on the basis of intermediate results in the solutions set of the individual objective function.

The membership functions of the two fuzzy goals consisting of minimizing the total weighted flow time and the total weighted tardiness of jobs are constructed as given in Eqs. (12) to (13).

$$\mu(Z_1)=\begin{cases}1 & \text{, if } Z_1 \le 377\\ 1-\dfrac{(Z_1-377)}{50} & \text{, if } 377 \le Z_1 \le 427\\ 0 & \text{, if } Z_1 \ge 427\end{cases} \tag{12}$$

$$\mu(Z_2)=\begin{cases}1 & \text{, if } Z_2 \le 130\\ 1-\dfrac{(Z_2-130)}{10} & \text{, if } 130 \le Z_2 \le 140\\ 0 & \text{, if } Z_2 \ge 140\end{cases} \tag{13}$$

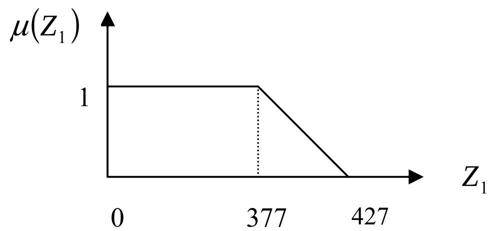

Fig. 1 Membership function of Z_1

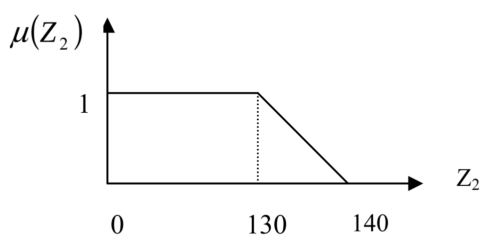

Fig. 2 Membership function of Z_2

To solve the multi-objective formulations, the proposed model for the above-mentioned problem is written as follows:

max α

s.t.

$$\alpha \le 1+\frac{377}{50}-\frac{1}{50}Z_1$$

$$\alpha \le 1+\frac{130}{10}-\frac{1}{10}Z_2$$

Constraints (3) to (7).

The final computational results for the proposed model are optimal as shown in Table 3.

Table 3 Final computational results

Out put parameters	values
Degree achievement of fuzzy goal	0.76
Optimum of total weighted flow time	389
Optimum of total weighted tardiness	132

In addition, the associated optimal sequence of jobs is given bellow:

J_1	J_5	J_4	J_2	J_3

Table 4 shows the input data for the large-sized problem. The single machine scheduling problem is solved to validate the effectiveness of the FGP approach. The intermediate computational results for the individual objective functions are shown in Table 5.

The membership functions of the two fuzzy goals based on the intermediate computational results of the individual objective function are designed as given in Eqs. (14) and (15).

Table 4 Input data for a large-sized problem

Job	P_i	R_i	D_i	W_i
1	5	8	25	3
2	6	9	30	4
3	4	5	20	5
4	8	10	40	2
5	7	10	35	6
6	2	4	10	5
7	3	8	15	2
8	5	8	25	3
9	9	5	45	4
10	8	10	40	6

Table 5 Intermediate computational results

Individual objective function for minimization	Z^*
Z_1^* Total weighted flow time	1107
Z_2^* Total weighted tardiness	104

$$\mu(Z_1)=\begin{cases}1 & ,\text{if } Z_1 \le 1107\\ 1-\dfrac{(Z_1-1107)}{300} & ,\text{if } 1107 \le Z_1 \le 1407\\ 0 & ,\text{if } Z_1 \ge 1407\end{cases} \quad (14)$$

$$\mu(Z_2)=\begin{cases}1 & ,\text{if } Z_2 \le 104\\ 1-\dfrac{(Z_2-104)}{50} & ,\text{if } 104 \le Z_2 \le 154\\ 0 & ,\text{if } Z_2 \ge 154\end{cases} \quad (15)$$

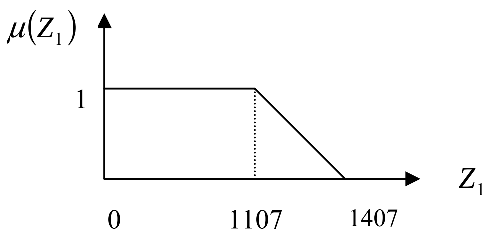

Fig. 3 Membership function of Z_1

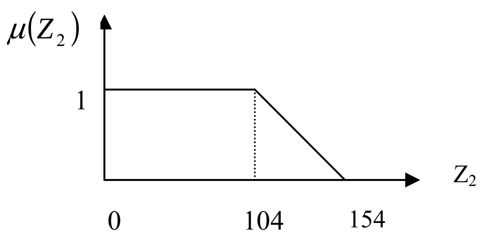

Fig. 4 Membership function of Z_2

The proposed formulation of the large-sized problem is transferred and rewritten into the following model:

max α

s.t.

$$\alpha \le 1+\frac{1107}{300}-\frac{1}{300}Z_1$$

$$\alpha \le 1+\frac{104}{50}-\frac{1}{50}Z_2$$

Constraints (3) to (7).

The final computational results of the large-sized problem are shown in Table 6.

Table 6 Final computational results

Out put parameters	values
Degree achievement of fuzzy goal	0.83
Optimal of total weighted flow time	1159
Optimal of total weighted tardiness	113

4 Conclusion

This paper has proposed the new fuzzy mixed-integer goal programming model for a single machine scheduling problem with two objectives. In this paper, these two objectives are to minimize the total weighted flow time and total weighted tardiness simultaneously. This work has been done for the first time in solving a bi-criteria single machine scheduling problem. Due to the real-world situation and satisfaction of the decision maker for the above objectives, the proposed model is solved by a fuzzy goal programming approach. The associated computational results have been reported to show the effectiveness of the proposed approach.

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