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THE INDIVIDUALS CONTROL CHARTS FOR BURR DISTRIBUTED DATA

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Abstract: - The setting of the control limits to utilize on a control chart supposes the assumption of the normality. However, in many situations, this condition does not hold. There are numerous studies on the control charts when the underlying distribution is non-normal. This paper examines the effects of non-normality as measured by skewness and provides an alternative method of designing individuals control charts(I Chart) for the burr distributed data with the parameter for uncorrelated and independent data. It proposes a skewness correction method for constructing the individuals control charts. The Chart is simply an adjustment of the " Skewness Correction $\bar{x}$ and R charts for Skewed Distributions" written by Lai and Cui. The results of a simulation study for the burr distributed data for n equals to five and seven are given.

Key -Words: - Quality Control, Shewhart Control Charts-(S), Skewness Correction-(SC),Normality Assumption for Control Charts, Individuals Control Charts, Zones Test.

1 Introduction

Statistical methods provide many useful applications in industrial process control. One of them is the control charts method, used to detect the occurrence of assignable causes in industry, presented by Dr.W. Shewhart in 1924. The Shewhart control charts are based on the assumption of the distribution of the quality characteristic is normal or approximately normal.

The setting of the control limits to utilize on a control chart supposes the assumption of the normality. However, in many situations, this condition does not hold. There are numerous studies on the control charts when the underlying distribution is non-normal.

For skewed populations, Type I Risk probabilities grow larger as the skewness increases. For highly skewed populations, one possible solution is to increase sample size. The conclusion of their study matches with Shewhart's assumption for the samples four or more, normal theory is satisfactory. Coobineh and Ballard suggested a weighted variance method based on the semivariance approximation and provided asymmetric control chart limits for the $\bar{x}$ and R charts. Another approach for non-normality is considered using some transformed Q statistics which are the standard normal or approximately normal by Quesenberry. Pyzdek (1992) chose to transform the data in order to make them quasi normal and Farnum (1997) used the Jonhson system of the distributions as a general tool for transforming the data to normality[1].

In this study we demonstrated the application of individuals control charts for the burr distributed data. We proposed the skewness method' s formulas for the individuals control chart for the burr distributed data. Then, a simulation study is designed as an example, for the burr distributed data, for n=5 and n=7 . Finally, we presented the results of the studies.

2 Problem Formulation

The Burr distribution is very flexible, and by appropriate selection of the parameters, the distribution can assume a wide variety of shapes. Burr proposed new $\bar{x}$ charts based on non-normality by using the Burr distribution to modify the usual symmetrical control limits. He developed the tables for constructing the modified control charts. He tabulated the expected value, standard deviation, skewness coefficient and kurtosis coefficient of the Burr distribuiton for various combinations of c and k. The user can make a standardized transformation between a Burr variate (say, Y) and another random variate (X) by these tables. The Burr system covers a wide and important range of the standardized third and fourth central moments and can be used to fit a wide variety of practical data distributions[3].

The cumulative distribution function (CDF) of the Burr distribution is

$$F(y) = 1-(1+y^c)^{-k} \qquad y>0 \tag{1}$$

where c and k are the shape parameters .

Individual control charts, which we may also observe the magnitude of skewness, are used to find and remove the assignable causes quickly from the process if the sampling is possible, although $\bar{x}$-R or $\bar{x}$-s control charts can be constructed for the same situations.

Let $x_{i1}, x_{i2, \ldots,} x_{in}$, i=1, 2,..., m, be m samples of size n from a process distribution with mean μ, standard deviation σ, and skewness k_3. When the process distribution is normal, Shewhart individuals control chart is

I_S Chart:

$$\begin{aligned} UCL_S &= \bar{\bar{x}} + 3\bar{R}/d_2 \\ CL_S &= \bar{\bar{x}} \\ LCL_S &= \bar{\bar{x}} - 3\bar{R}/d_2 \end{aligned} \tag{2}$$

Here, $\bar{\bar{x}} = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}$, $\bar{R} = \frac{1}{m}\sum_{i=1}^{m} R_i$, R_i is the range of the i th sample. d_2 depends on the sample size n and calculated when the distribution is normal[2].

Let d_2^*, $d_2^* = \mu_R/\sigma_x$, be the constant for the given skewed process distribution. When k_3=0, the constant d_2^* is close to d_2 for the normal process distribution. So that, based on the skewness correction method for individuals control chart is practically the Shewhart individuals control chart. The sample size n, is determined for the samples in the control process, the estimated value for the skewness of the sample averages

$$\hat{k}_{3\bar{x}} = \hat{k}_3/\sqrt{n} \tag{3}$$

In many cases, the skewness k_3 needs to be estimated. It can be estimated by the sample skewness

$$\hat{k}_3 = \frac{1}{mn-3}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\frac{x_{ij}-\bar{\bar{x}}}{\sqrt{\frac{1}{mn-1}\sum_{i=1}^{m}\sum_{j=1}^{n}(x_{ij}-\bar{\bar{x}})^2}}\right)^3 \tag{4}$$

It is known that the estimator $\hat{k}_3$ of the k_3 is consistent. We know that the sample skewness $\hat{k}_3$ is essentially the third moment estimator[1]. Based on the skewness correction method for the individuals control chart, we propose the following

I_{SC} Chart:

$$\begin{aligned} UCL_{SC} &= \bar{\bar{x}} + \left(3 + \frac{4k_3/(3\sqrt{n})}{1+0.2k_3^2/n}\right)\frac{\bar{R}}{d_2^*} = \bar{\bar{x}} + A_{SC}^U\bar{R} \\ CL_{SC} &= \bar{\bar{x}} \\ LCL_{SC} &= \bar{\bar{x}} + \left(-3 + \frac{4k_3/(3\sqrt{n})}{1+0.2k_3^2/n}\right)\frac{\bar{R}}{d_2^*} = \bar{\bar{x}} + A_{SC}^L\bar{R} \end{aligned} \tag{5}$$

The constants of A_{SC}^U and A_{SC}^L are viewed in the next part.

It is known that the zones tests measure the autocorrelation between each observations. Here, we use those tests to compare the observations when the control limits calculated by the two methods.

The focus is on the three standart deviations are sometimes identified by zones. Each zone’s dividing line is exactly one-third the distance from the centraline to either the upper control limit or the lower control limit.

By default, Zone A is defined as the area between 2 and 3 times sigma above and below the center line; Zone B is defined as the area between 1 and 2 times sigma, and Zone C is defined as the area between the central line and 1 times sigma .When we were using the individulas control charts, we applied the following set of rules:

1- 9 points in Zone C or beyond (on one side of the centraline).
2- 6 points in a row steadily increasing or decreasing.
3- 2 out of 3 points in a row in Zone A or beyond.
4- 4 out of 5 points in a row in Zone B or beyond.
5- 15 points in a row in Zone C.

These rules serve as a signal to investigate what occured in the process.

3 Problem Solution

The constants of A_{SC}^{U} and A_{SC}^{L} are given in Table 1. If the skewness k_3 <0 **,** the A_{SC}^{U} is the same as the A_{SC}^{L} for $-k_3$ in Table 1, and vice versa.

n	**5**		**7**	
$\mathbf{k_3}$	A_{SC}^{U}	A_{SC}^{L}	A_{SC}^{U}	A_{SC}^{L}
0,0	1,30	1,30	1,11	1,11
0,4	1,41	1,19	1,19	1,03
0,8	1,52	1,12	1,27	0,98
1,2	1,65	1,03	1,38	0,93
1,6	1,77	0,98	1,48	0,87
2,0	1,90	0,94	1,56	0,85
2,4	2,03	0,89	1,67	0,79
2,8	2,12	0,87	1,75	0,77
3,2	2,24	0,85	1,83	0,77
3,6	2,33	0,83	1,90	0,74
4,0	2,39	0,83	1,98	0,71

Table 1. The constants A_{SC}^{U} and A_{SC}^{L} for n=5 ,7

If the k_3 value is not in this table, the user can take nearest k_3 value or use interpolation.

Simulation study is designed for the burr distributed data for n=5 and n=7 with different parameters to compare the Skewness Correction method wtih those Shewhart control limits and Alpha methods. Shewhart individuals control charts limits are calculated. Skewness correction method is applied to the same data set, then the limits of individuals control charts are calculated again. Lastly, we calculated the cumulative distribution function of the Burr distribution for various parameters. The results are summarized in Table2.

n=5 and k/c	**SHEWHART METHOD**	**SKEWNESS CORRECTION METHOD**	**ALPHA METHOD**	**3.Criterion for S.C Method**	**3.Criterion for Shewhart Method**
1/0,5	936	563	206	0	659
1/1	584	190	116	0	764
1/2	165	13	30	0	82
1/3	34	0	6	0	30
1/4	12	0	2	0	17
1/5	4	0	2	0	11
1/10	0	0	0	0	11
2/0.5	692	238	106	0	372
2/1	195	72	136	0	74
2/2	6	0	1	0	12
2/3	1	0	0	0	6
2/4	0	0	0	0	4
2/5	0	0	0	0	4
2/10	0	0	0	0	7
3/0.5	531	113	57	1	244
3/1	103	2	13	0	48
3/2	2	0	0	0	6
3/3	0	0	0	0	5
3/4	0	0	0	0	2
3/5	0	0	0	0	5
n=7 and k/c	**SHEWHART METHOD**	**SKEWNESS CORRECTION METHOD**	**ALPHA METHOD**	**3.Criterion for S.C Method**	**3.Criterion for Shewhart Method**
1/0,5	856	432	224	1	631
1/1	467	117	103	0	314
1/2	73	4	15	0	58
1/3	8	0	3	0	20
1/4	0	0	0	0	12
1/5	0	0	0	0	13
1/10	0	0	0	0	6
2/0.5	546	152	103	0	351
2/1	112	3	7	0	75
2/2	2	0	1	0	12
2/3	0	0	0	0	7
2/4	0	0	0	0	6
2/5	0	0	0	0	5
2/10	0	0	0	0	8
3/0.5	350	53	60	1	232
3/1	26	0	2	0	39
3/2	0	0	0	0	16
3/3	0	0	0	0	5
3/4	0	0	0	0	6
3/5	0	0	0	0	8

Table2. Results of the simulation study

In Table 2, the number of observations that lies out of the limits calculated by the Shewart control limits are demonstrated in the first column. In the second column, the number of observations that lies out of the limits evaluated by the Skewness correction method are displayed. In the third column, it is attained the results that are calculated by the cumulative distribution of the Burr distribution. The results of the zone tests are displayed in the fourth and the fifth column. In the fourth column, the number of observations that could not pass the criterion 3 are shown. In the last paragraph, the number of observations that could not pass the criterion 3 are demonstrated.

In this simulation study, the Alpha method is called as the Type I Risks (probabilities of a false alarm) of the individuals chart. Then, it is calculated by

$$\alpha_x = E\left[1 - P(LCL \le \overline{X} \le UCL \mid LCL, UCL)\right] \quad (6)$$

where E, is the expected value and P is the probability that a new subgroup is from this distribution.

For the Burr distribution , numerous values of the shape parameters were chosen for convenience since the skewness depends on the two shape parameters only. For each subgroup of size, n=5 and n=7, m=20 subgroups with μ_0, σ_0 and k_3 were generated to calculate the control limits. Those samples were carried out to see if their individuals control limits fell outside the control limits. This procedure was repeated for 1000 times. We note that the constant d_2=2,326 for n=5 and d_2=2,704 for n=7 were used while calculating the Shewhart control limits. Then, The Type I Risk was estimated by the percentage of 0,005 of the individuals control limits .

The results of the simulation study are:

1- The number of observations that fell outside the control limits when Shewhart method is used is much more than that is the Skewness correction method and Alpha method is used, for n=5 and for k=1 c=0.5,1,2,3,4,5 ,k=2 c=0.5,1,2,3 and k=3 c=0.5,1,2 . This implies that Shewhart Control method is the most conservative method for the chosen parameters above, especially when c=0.5. Skewness correction method is less conservative than the Alpha test.

2- For the different parameters (k=1 c=10 , k=2 c=4,5,10 and k=3 c=3,4,5) and for n=5, Skewness correction method is compatible with the Shewhart method and the Alpha method.

3- What is observed when n=7 is chosen is for the Burr distributed data :

The number of observations that fell outside the control limits when Shewhart method is used is much more than that is the Skewness correction method and Alpha method is used, for k=1 c=0.5,1,2,3 ,k=2 c=0.5,1,2 and k=3 c=0.5,1. This implies that Shewhart Control method is the most conservative method for the chosen parameters above. Skewness Correciton method is less conservative than the alpha test.

4- For the different parameters (k=1 c= 4,5,10, k=2 c=3,4,5,10 and k=3 c=2,3,4,5) and for n=7, Skewness Correction Method is compatible with the Shewhart method and the Alpha method. In fact, the three methods are compatible when the larger values of shape parameter of c.

5- All the three methods passes the zone criterion, except 3. criterion. The observations are restricted by the Shewhart control limits.

6- Skewness correciton method for n=5 and n=7 , is closer to the percentage of 0.5% of the alpha test.

4 Conclusion

This paper has shown the development of skewness correction method for the individuals control charts for burr distributed data. The function used here, depends on Chan and Cui ‘s model. The Burr distribution is applied to derive the control limits. A simulation study is designed for n=5 and n=7.

For n=7, the individuals control chart limits obtained by the skewness correction method are larger than the limits of Shewhart method and smaller than the limits of the alpha test. The three methods are compatible when the larger values of shape parameter of c .The skewness correction method is more conservative for the smaller values of shape parameter of c.

In this study, according to the control charts which are compared, when the distribution of the data set is burr distributed, the individuals control chart limits calculated by the skewness correction method are close to individuals control chart limits calculated by the Alpha method for the larger values of sample size n and especially c increases. Hence if the data set is Burr distributed, researchers may use individuals control chart with skewness correction instead of Shewhart individuals control charts.

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