

# Some Methods for Computing RMS Values and Phase Differences of Currents and Voltages

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*Abstract:* - The data primary processing in electro-energetic system is based on algorithms that perform the numerical filtration of input signals or the computation of some characteristic values (root-mean-square values and phase differences) of currents and voltages. This paper presents possible methods for computing these characteristic values: method of the average value, modified A4 method, method of the direct root-mean-square value, Fourier method. The main advantages and disadvantages of each method and experimental results obtained using the last method are also presented.

*Key-Words:* - method, periodical signal, root-mean-square value, phase difference, sample, computational load.

## 1 Introduction

The signals acquired from electro-energetic plants are primary processed so that some variables result, necessary for the process command. The signal primary processing consists in filtration, testing the bordering with pre-established limits, correction of the repetitive errors, computation of some characteristic values [7].

The data acquisition equipment in electro-energetic systems [4,5,6,7] performs the data primary processing based on algorithms that assure the following operations:

- numerical filtration of analogue input signals; this operation enables the rejection of the continuous component and upper harmonics from the spectrum of the processed currents and voltages;
- numerical filtration of digital input signals; this operation eliminates the undesirable effect of the input contact vibrations;
- computation of the root-mean-square (rms) values and phase differences for some analogue input signals (currents or voltages).

There are many methods for computing the rms values and phase differences; such method must satisfy the following conditions:

- it assures the imposed measurement accuracy;
- it doesn't suppose a laborious mathematical calculus;

- the computational time must be less than a specified value (small enough for enable a result interpretation in real time).

## 2 Methods for Computing RMS Values and Phase Differences

This section presents four methods (the most important) for computing rms values and phase differences of currents and voltages. A discussion of each method is also presented and the adequate method in electro-energetics is recommended.

### 2.1 Method 1 (Method of the Average Value)

If the analogue input signal  $y(t)$  is alternative sinusoidal, then the rms value  $Y_{ef}$  can be computed [7]:

$$Y_{ef} = 1.11Y_M; \quad (1)$$

$Y_M$  is the average value of the full-wave rectified signal.  $Y_M$  can be computed as the arithmetic mean of the modulus of  $n$  sampled values,  $|y(t_0)|, |y(t_1)|, |y(t_2)|, \dots, |y(t_{n-1})|$ ; these values are acquired at the moments  $t_0, t_1, t_2, \dots, t_{n-1}$ , equidistant distributed during one period  $T$  of the signal  $y(t)$ . So, the rms

value computed at  $t_i$ , denoted  $Y_{ef}|_{t_i}$ , depends on the values  $y(t_i), y(t_{i-1}), \dots, y(t_{i-n+1})$ :

$$Y_{ef}|_{t_i} = \frac{1.11}{n} \sum_{k=0}^{n-1} |y(t_{i-k})|. \quad (2)$$

The computational load decreases if  $Y_{ef}|_{t_{i+1}}$  is computed with relation

$$Y_{ef}|_{t_{i+1}} = Y_{ef}|_{t_i} + \frac{1.11}{n} [|y(t_{i+1})| - |y(t_{i-n+1})|]. \quad (3)$$

The main advantages of this method are the simplicity, reduced computational load, correction of some possible errors of the analog-to-digital conversion.

The method of the average value has the following disadvantages:

- it doesn't enable the phase difference computation;
- it introduces a method error for the signals that are not sinusoidal or they have a non zero continuous component;
- the method relative error is 5...7% even for a sinusoidal signal and  $n=10...15$  samples.

## 2.2 Method 2 (Modified A4 Method)

The modified A4 method [7] performs the computation of rms values ( $I_{ef}$  and  $V_{ef}$ ) of two alternative sinusoidal signals: the current  $i(t)$  and the voltage  $v(t)$ .

We consider an equivalent dipole with  $v(t) = V_{max} \sin(\omega t + \alpha)$  (4)

its terminal voltage and

$i(t) = I_{max} \sin(\omega t + \beta)$  (5)

the current through the terminals. It is well known that its equivalent impedance  $Z$  and the phase difference  $\varphi$  characterize the dipole,

$$Z = \frac{V_{ef}}{I_{ef}}, \quad \varphi = \alpha - \beta, \quad (6)$$

in the stabilized sinusoidal regime. The equivalent resistance of the dipole is

$$R = Z \cos \varphi, \quad (7)$$

and the equivalent reactance is

$$X = Z \sin \varphi. \quad (8)$$

If  $R$  and  $X$  are known, the phase difference can be computed:

$$\varphi = \arctg \frac{X}{R}. \quad (9)$$

Four consecutive times ( $t_1, t_2, t_3, t_4$ ) are established; they are equidistant distributed at

$\Delta t = \frac{T}{N}$ ;  $T$  is the period of  $i(t)$  and  $v(t)$ , and usually  $N = 20$  and  $\Delta t = 1ms$ . The samples

$$i(t_p) = i_p = I_{max} \sin \left[ \frac{2\pi}{N} (k - 4 + p) \right], \quad (10)$$

$$v(t_p) = v_p = V_{max} \sin \left[ \frac{2\pi}{N} (k - 4 + p) \right], \quad (11)$$

with  $3 \leq k \leq N$  and  $p = 1, 2, 3, 4$ , are acquired at the moments  $t_1, t_2, t_3, t_4$ .

$R, X$  and  $\varphi$  can be computed [7] based on these samples:

$$R = \frac{v_2(i_4 - i_2) - v_3(i_3 - i_1)}{i_2(i_4 - i_2) - i_3(i_3 - i_1)}, \quad (12)$$

$$X = 2 \frac{v_3 i_2 - v_2 i_3}{i_2(i_4 - i_2) - i_3(i_3 - i_1)} \sin \frac{2\pi}{N}, \quad (13)$$

$$\varphi = \arctg \frac{X}{R} = \arctg \left[ \frac{2(v_3 i_2 - v_2 i_3)}{v_2(i_4 - i_2) - v_3(i_3 - i_1)} \sin \frac{2\pi}{N} \right] \quad (14)$$

The denominator in relation (12) can be now evaluated:

$$i_3 - i_1 = 2I_{max} \sin \frac{2\pi}{N} \cos \left[ \frac{2\pi}{N} (k - 2) \right], \quad (15)$$

$$i_4 - i_2 = 2I_{max} \sin \frac{2\pi}{N} \cos \left[ \frac{2\pi}{N} (k - 1) \right], \quad (16)$$

$$i_3(i_3 - i_1) = I_{max}^2 \cos \left[ \frac{2\pi}{N} (k - 2) \right] \cdot \left\{ \cos \left[ \frac{2\pi}{N} (k - 2) \right] - \cos \left( \frac{2\pi}{N} k \right) \right\}, \quad (17)$$

$$i_2(i_4 - i_2) = I_{max}^2 \cos \left[ \frac{2\pi}{N} (k - 1) \right] \cdot \left\{ \cos \left[ \frac{2\pi}{N} (k - 3) \right] - \cos \left[ \frac{2\pi}{N} (k - 1) \right] \right\} = I_{max}^2 \sin \left[ \frac{2\pi}{N} (k - 2) \right] \left\{ \sin \left( \frac{2\pi}{N} k \right) - \sin \left[ \frac{2\pi}{N} (k - 2) \right] \right\} \quad (18)$$

$$i_3(i_3 - i_1) - i_2(i_4 - i_2) = I_{max}^2 \left( 1 - \cos \frac{4\pi}{N} \right) = 2I_{ef}^2 \left( 1 - \cos \frac{4\pi}{N} \right). \quad (19)$$

So, the rms value of the current can be computed if  $i_1, i_2, i_3, i_4$  are known:

$$I_{ef} = \sqrt{\frac{i_3(i_3 - i_1) - i_2(i_4 - i_2)}{2 \left( 1 - \cos \frac{4\pi}{N} \right)}}. \quad (20)$$

A similar procedure enables the computation of the rms value of the voltage, if the samples  $v_1, v_2, v_3, v_4$  are known:

$$V_{ef} = \sqrt{\frac{v_3(v_3 - v_1) - v_2(v_4 - v_2)}{2\left(1 - \cos\frac{4\pi}{N}\right)}} \quad (21)$$

$I_{ef}$ ,  $V_{ef}$  and  $\phi$  are computed with the following relations [7] for the usual value  $N=20$ :

$$I_{ef} = 1.618\sqrt{i_3(i_3 - i_1) - i_2(i_4 - i_2)}, \quad (22)$$

$$V_{ef} = 1.618\sqrt{v_3(v_3 - v_1) - v_2(v_4 - v_2)}, \quad (23)$$

$$\phi = \arctg\left[0.618\frac{v_3i_2 - v_2i_3}{v_2(i_4 - i_2) - v_3(i_3 - i_1)}\right]. \quad (24)$$

The advantages of this method:

- it enables the computation both rms values and phase difference;
- it supposes some simple mathematical calculus, so, a reduced computational load;
- it can perform the mean of 10...20 consecutive rms values; the resulted relative error is less 5%.

This method has some disadvantages:

- a) it imposes the acquisition of the same number of samples during a period, even a frequency variation  $\Delta f$  exists ( $f$  is the frequency of the analyzed signals  $i(t)$  and  $v(t)$ );
- b) the response time is too large if the averaging of the consecutive rms values is performed; for example, the delay time is 30ms if  $T=20ms$  ( $f=50Hz$ ) and the arithmetic mean is computed for 10 consecutive values.

### 2.3 Method 3 (Method of the Direct RMS Value)

It is a method based on the relation that defines the rms value  $Y_{ef}$  of a periodical signal  $y(t)$  with  $T$  its period:

$$Y_{ef} = \sqrt{\frac{1}{T} \int_0^T y^2(t) dt}. \quad (25)$$

$n$  samples are acquired at the moments  $t_0, t_1, t_2, \dots, t_{n-1}$ , equidistant distributed during one period  $T$ ; usually  $n=20$ . The rms value at the moment  $t_i$ , denoted  $Y_{ef}|_{t_i}$ , is computed depending on the last  $n$  sampled values of the analogue signal  $y(t)$ :

$$Y_{ef}|_{t_i} = \sqrt{\frac{1}{n} \sum_{k=0}^{n-1} y^2(t_{i-k})}. \quad (26)$$

This relation can be written in the following form:

$$\left(Y_{ef}|_{t_{i+1}}\right)^2 = \left(Y_{ef}|_{t_i}\right)^2 + \frac{1}{n} \left[y^2(t_{i+1}) - y^2(t_{i-n+1})\right]. \quad (27)$$

The main advantages of this method are the reduced computational load and the correction of some possible errors of the analog-to-digital conversion.

The method of the direct rms value performs only the computation of the rms value and this isn't computed with a high precision (the relative method error is between 4% and 6%).

### 2.4 Method 4 (Fourier Method)

It is based on the decomposition in Fourier series of the analogue input signal  $y(t)$ ; the coefficients in the resulted expression are used for the computation of the rms value and phase difference of each harmonic of the analyzed signal  $y(t)$ .

The Fourier method can be applied if the fundamental harmonic has a fixed frequency ( $f=50Hz$ ) and  $y(t)$  satisfies the Dirichlet conditions. In mathematics, the Dirichlet conditions [2] are the following conditions that must be met for a function  $g(x)$ , to have a Fourier transform:

- $g(x)$  must be single valued;
- $g(x)$  must have a finite number of extrema in any given interval;
- $g(x)$  must have a finite number of discontinuities in any given interval;
- $g(x)$  must be absolutely integrable.

The decomposition in Fourier series of the alternative sinusoidal input signal  $y(t)$  is [2]

$$y(t) = A_0 + \sum_{k=1}^{\infty} [M_k \cos(k\omega t) + N_k \sin(k\omega t)]. \quad (28)$$

The coefficients  $M_k$  and  $N_k$  are used for computing the rms value of the  $k$ -harmonic,

$$Y_{k,ef} = \sqrt{M_k^2 + N_k^2} \quad (29)$$

and phase difference related to a virtual reference system

$$\phi_k = \arctg\left(\frac{M_k}{N_k}\right). \quad (30)$$

The coefficients  $M_k$  and  $N_k$  are computed as follow:

$$M_k = \frac{2}{T} \int_0^T y(t) \cos(k\omega t) dt, \quad (31)$$

$$N_k = \frac{2}{T} \int_0^T y(t) \sin(k\omega t) dt. \quad (32)$$

A method of numerical integration [1,3] enables the computation of Fourier coefficients  $M_k$  and  $N_k$ . Hence, can be used the rectangle method, trapezoid method or Simpson's method for computing  $M_k$  and  $N_k$ . A comparison of these methods can be realized in according to the number of function evaluations, scaling or accuracy. Because the Simpson's method is the most accurate, it is recommended for this application in electro-energetics.

The main advantages of the Fourier method are the following:

- it performs the computation both the rms value and phase difference for each harmonic of the analyzed signal;
- it assures a high precision of computation (the relative error is less 2%).

The Fourier method supposes an important computational load.

### 2.5 Discussion

Four methods for computing the rms values and/or phase differences of currents and voltages are presented in this paper:

1. method of the average value,
2. modified A4 method,
3. method of the direct rms value,
4. Fourier method.

The first three methods suppose simple mathematical calculus, so, a reduced computational load.

Method of the average value and method of the direct rms value assure only the computation of the rms value, not with a high precision (the relative method error is between 4% and 7%). An advantage of these two methods is the correction of some possible errors of the analog-to-digital conversion.

The modified A4 method imposes the acquisition of the same number of samples during a period, even a frequency variation  $\Delta f$  exists;  $f$  is the frequency of the analyzed signals  $i(t)$  and  $u(t)$ . The response time is too large for this method if the mediation of the consecutive rms values is performed.

The Fourier method assures a high precision of computation and performs the computation both the rms value and phase difference for each harmonic of the analyzed signal. This method supposes an important computational load, but this bad point is eliminated if an adequate microprocessor is used.

A comparison of the presented methods enables the choice of the proper method for each application. For example, the Fourier method (and Simpson's method numerical integration) is recommended for the computation of the rms values and phase differences of the currents and voltages in electro-energetic systems; some experimental results obtained using this method are presented in this paper.

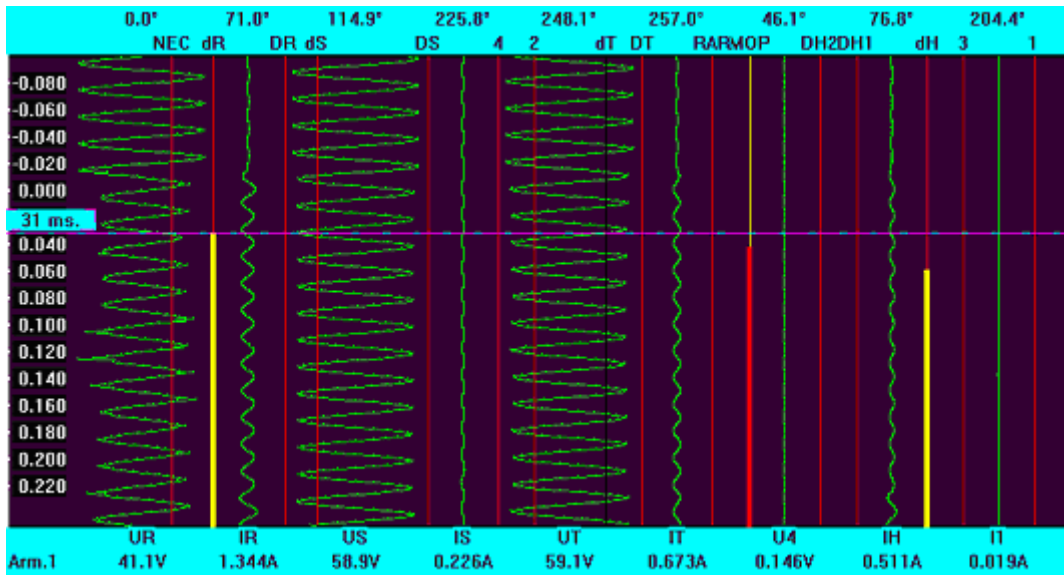


Fig. 1. Experimental results in after-defect analysis.

### 3 Experimental Results

There are PDM equipments that assure the data acquisition and graphically recording of the electric

events in the high and mean voltage networks of the energetic system. Such equipment is useful in after-defect analysis, during the putting into service works

for electrical plants etc.; it also performs the measuring of some characteristic parameters of electric lines.

For example, Fig. 1 represents the waveforms of currents and voltages obtained before and after a real event detected and graphically recorded in a power transformation station, endowed with PDM equipment. The time scale is in the left side of the image. The time (in milliseconds) is related to the start of the defect:  $t < 0$  in the preliminary-defect zone, and  $t > 0$  in the after-defect zone. The cursor selects  $t = 31\text{ms}$  in Fig. 1; the characteristic values of currents and voltages are computed at this moment. The currents and voltages remain between normal limits and the waveforms of these signals are not distorted in the preliminary-defect zone; this is the optimal zone for measuring the rms values and phase differences in normal operating conditions. The current and voltage rms values and phase differences of currents and voltages for each phase (R, S or T) are measured using the Fourier method (and Simpson's method for numerical integration). The rms values are displayed below the waveforms (the line Arm.1) in Fig. 1, and the phase differences are specified above the waveforms.

#### 4 Conclusion

This paper presents four methods for computing rms values and phase differences of currents and voltages: method of the mean value, modified A4 method, method of the direct RMS value and Fourier method; a discussion for each method is also realized. By comparing the particularities, advantages and disadvantages of the presented methods, we can recommend the Fourier method for computing the rms values and phase differences of currents and voltages in electro-energetics. Some

experimental results (obtained using the Fourier method) are also presented in this paper.

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