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Multilevel Analysis in Complex Wavelet Transform Domain for Signal Processing Applications

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Abstract: - Real valued wavelets are used widely in signal processing applications. Although complex valued wavelets exists, but rarely used. Complex wavelet transform provides important phase information of the signal and it is almost shift-invariant. Due to these added advantages it can be very much useful for signal processing applications. This paper explores various properties of Daubechies complex wavelet transform. It shows that the nature of the complex wavelet coefficients does not change at multiple levels. This property provides an opportunity to apply the same function for the signal represented at multiple levels. We have applied the soft-thresholding function for denoising and deblurring of 1D and 2D signals and shown that the result gets improved at multiple levels.

We have proposed a complex wavelet based method for denoising of signals corrupted with signal dependent and signal independent noise as well as a restoration method for blurred signals corrupted with noise. The proposed method is adaptive as it uses a soft-threshold function based on the standard deviation, the absolute mean and the absolute median of wavelet coefficients. The proposed threshold is level dependent as well. The effectiveness of the complex wavelet based signal restoration method has been tested and it was found that the performance of the proposed method is better than that of other similar type of methods that uses real valued wavelets. The method can be easily extended for other applications, such as texture analysis, object tracking, registration, segmentation, etc.

Key-Words: - Complex Daubechies wavelet Transform, Shift-invariance, Denoising, Deblurring.

1 Introduction

The use of wavelet transform is becoming ubiquitous in signal processing, owing to the potential of multiresolution technique and economical representation at singularities. Recent wavelet researches are primarily focused on real-valued wavelet bases as is evident by a large number of publications on the subject. However, complex valued wavelet bases exist and recently a few authors [1,2] have studied complex-valued wavelet filter banks. The complex Daubechies wavelets and some of its applications were discussed in [4,5]. Real-valued discrete wavelet transforms are very much useful for several signal processing applications like Compression, Denoising, Deblurring etc. However, these transforms suffer from two serious disadvantages: shift-sensitivity and no phase information. Use of complex-valued wavelet transform (CxWT) can minimize these disadvantages.

The objective of the present paper is two fold: first to explore construction and properties of complex Daubechies wavelet transform, and second to show the applicability of CxWT in signal denoising and deblurring. Reduced shift-sensitivity property of CxWT is discussed in the present work and it has been shown that the nature of wavelet coefficients changes erratically at multiscale when real-valued discrete wavelet transform (DWT) is used, unlike the complex case. This is the main observation in this paper. This property indicates that for those signal processing applications, which process information at multiple scales, CxWT can be better than DWT. We have also proposed threshold based denoising and deblurring methods. The proposed threshold is adaptive in nature and it depends on the standard deviation, the absolute mean and the absolute median of wavelet coefficients at a certain scale. Thus we call this scheme as soft-thresholding. The results of our method have been compared with that of methods that use real DWT. It has been proved conclusively that CxWT yields far better results.

The rest of paper is organized as follows: Section 2 describes construction of Daubechies wavelet transform and its properties. Reduced shift-invariance property of CxWT is discussed in section 3. Section 4 deals with the application of the proposed method for denoising and deblurring of signals and the comparison of results with similar type of methods based on real DWT. In section 5 some other possible applications of CxWT is given and finally in section 6 discussions and conclusions are given.

2 Daubechies CxWT

2.1 Construction of Complex Daubechies Wavelet

The basic equation of Multiresolution theory is the scaling equation

$$\phi(t) = 2\sum_n a_n \phi(2t-n) \qquad (1)$$

where, a_n s are coefficients. The a_n s can be real as well as complex valued and $\sum a_n = 1$.

Daubechies's wavelet bases $\{\psi_{j,k}(t)\}$ in one dimension are defined through the above scaling function and multiresolution analysis of $L_2(\Re)$. One can define the Laurent series expansion on the unit circle:

$$F(Z) = \sum_{n=-N}^{N+1} a_n z^n, \text{ with } F(1) = 1 \text{ and } |z| = 1. \qquad (2)$$

For $\phi(t)$ to be Daubechies scaling function the following conditions must be satisfied:

1) ϕ is compactly supported.
2) ϕ forms orthonormal basis for some approximation space $V_0 \in L^2(\Re)$.
3) For the first N+1 moments to vanish, the regularity of the function is maximized. i.e.
   $F(-1) = F'(-1) = F''(-1) = \ldots\ldots\ldots\ldots = F^{(N)}(-1) = 0$
4) $\phi(t)$ should satisfy Multiresolution Property.

The general solution is given as follows:

Let $h(z) = \frac{1}{2}(1+z)$. Consider the following polynomial:

$$p_N(z) = \sum_{k=0}^{N} (-1)^k \binom{2N+1}{k} h(z)^{2N-2k} h(-z)^{2k} \qquad (3)$$

that satisfies, $p_N(z) - p_N(-z) = z$.

If we denote by Z_k, k = 1,2,……..N, the set of roots of $p_N(z)$ inside the unit circle ($|Z_k| < 1$). Any selection of R among the roots of $p_N(z)$ defines an admissible trigonometric polynomial

$$F(z) = h(z)^{1+N} \prod_{m\in R} \left(\frac{z^{-1} - Z_m}{1 - Z_m} \right) \times \prod_{n\notin R} \left(\frac{z^{-1} - \overline{Z_n}^{-1}}{1 - \overline{Z_n}^{-1}} \right) \qquad (4)$$

that satisfies the above four constraints. The solution of equation (3) and (4) will lead to Daubechies scaling functions. Some of these solutions lead to complex valued scaling function.

The construction of complex Daubechies wavelet is done as[6]. The generating wavelet $\psi(t)$ is given by,

$$\psi(t) = 2\sum_n (-1)^n \overline{a_{1-n}} \phi(2t-n) \qquad (5)$$

$\psi(t)$ and $\phi(t)$ share the same compact support [-N, N+1]. Daubechies found the solution of equation (4), where she selected R such that a_n s are real. If we do not impose this condition then we can get the a_n s as complex valued. This solution is known as complex Daubechies scaling function and this leads to complex Daubechies wavelet function. The complex solutions exist for all values of N ≥ 2. Symmetry is only possible with even N [2].

Any function $f(t)$ can be decomposed into complex scaling function and mother wavelet as:

$$f(t) = \sum_k c_k^{j_0} \phi_{j_0,k}(t) + \sum_{j=j_0}^{j_{\max}-1} d_k^j \psi_{j,k}(t) \qquad (6)$$

where, j_0 is a given resolution level, $\{c_k^{j_0}\}$ and $\{d_k^j\}$ is known as approximation and detail coefficients.

2.2 Properties of Complex Daubechies Wavelets

All the usual properties of real Daubechies wavelet bases are derived from the amplitude $|F(z)|^2 = z^{-1} p_N(z)$ [6]. Thus those properties do not depend on the particular factorization of $p_N(z)$ and are maintained in the complex solution. However complex Daubechies wavelets exhibit some other important properties:

2.2.1 Symmetricity and Linear Phase Property

In [2] it has been shown that complex Daubechies wavelet can be symmetric. The symmetry property of filter makes it easy to handle the boundary problems for finite length signals. Recently in [7], a method to achieve both symmetry and approximate linear phase on a complex Daubechies wavelet is proposed. The linear phase response of the filter precludes the nonlinear phase distortion and keeps the shape of the signal. In the present work we have used Symmetric complex Daubechies wavelets (SDW), endowed with linear phase property.

2.2.2 Relationships between real and imaginary components of scaling and wavelet functions

Let $\phi(t) = k(t) + i\,l(t)$ be a scaling function and $\psi(t) = u(t) + i\,v(t)$ be a wavelet function. Let $\hat{l}(\omega)$ and $\hat{k}(\omega)$ are Fourier transforms of $l(t)$ and $k(t)$. Consider the ratio

$$\alpha(\omega) = -\frac{\hat{l}(\omega)}{\hat{k}(\omega)} \qquad (7)$$

Clonda[5] observed that $\alpha(\omega)$ is strictly real-valued and behaves as ω^2 for $|\omega| < \pi$. This observation relates the imaginary and real components of scaling function: $l(t)$ accurately approximates the second derivative of $k(t)$, up to some constant factor.

Similarly for wavelet function $\psi(t)$, the ratio

$$\beta(\omega) = -\frac{\hat{v}(\omega)}{\hat{u}(\omega)} \qquad (8)$$

is also real valued.

Another unexpected relationship is between the real component of complex wavelet and scaling function.

$$\mu(\omega) = -i\,\frac{\hat{u}(\omega)}{\hat{k}(\omega)} \qquad (9)$$

This quantity is strictly real-valued and behaves as ω^{N+1} for $|\omega| < \pi$ [5].

2.2.3 Multiscale Edge Information

From the above property 2.2.2, equations (7) and (8) indicate $l(t) \approx \alpha\,\Delta k(t)$ and $v(t) \approx \beta\,\Delta u(t)$. This gives multiscale projections as,

$$\begin{aligned}\left\langle f(t),\phi_{j,k}(t)\right\rangle &= \left\langle f(t),k_{j,k}(t)\right\rangle + i\left\langle f(t),l_{j,k}(t)\right\rangle \\ &\approx \left\langle f(t),k_{j,k}(t)\right\rangle + i\,\alpha\left\langle \Delta f(t),k_{j,k}(t)\right\rangle\end{aligned} \qquad (10)$$

$$\begin{aligned}\left\langle f(t),\psi_{j,k}(t)\right\rangle &= \left\langle f(t),u_{j,k}(t)\right\rangle + i\left\langle f(t),v_{j,k}(t)\right\rangle \\ &\approx \left\langle f(t),u_{j,k}(t)\right\rangle + i\,\alpha\left\langle \Delta f(t),u_{j,k}(t)\right\rangle\end{aligned} \qquad (11)$$

From the above equation (10), it can be concluded that the real component of complex scaling function carries averaging information and the imaginary component carries Laplacian (i.e. edge information). Similarly from equation (11), it can be concluded that the imaginary component of complex wavelet function also carries edge information. In our recent work [8], we have presented a method for detecting strong edges, using only imaginary component of complex wavelet coefficients at multiscale.

2.3 Advantages of using Daubechies CxWT

Due to the following advantages Daubechies complex wavelet can be very useful for several applications:

1. It is approximate shift-invariant [4,8].
2. Perfect Reconstruction
3. No redundancy: Other popular complex wavelet transform like DTCWT [1] has a redundancy of 2^m:1 for m-D signal, while Daubechies CxWT have no such redundancy.
4. Number of computations in Daubechies CxWT (although it involves complex calculations) is same as that of DWT, while DTCWT have 2^m times computation as that of DWT for m-D signals.
5. It provides true phase information [8].

3 Shift-Invariance Revisited

A transform is shift sensitive if an input signal shift causes an unpredictable change in transform coefficients. In DWT the shift sensitivity arises from downsamplers in the implementation. Daubechies CxWT has reduced shift sensitivity. Fig. 1 illustrates the reduced shift-sensitivity of Daubechies CxWT. Fig. 1(a) shows an input signal and shifted form of input signal by one sample. Fig. 1(b) shows high-pass wavelet coefficients of the original and the shifted signals using DWT while fig. 1(c) shows the corresponding high-pass wavelet coefficient magnitudes. From the figure it is quite clear that the real wavelet transform is highly shift-sensitive whereas CxWT is approximate shift-invariant in nature. Our observation on Daubechies CxWT coefficient reveals that the magnitudes of wavelet coefficients vary slowly with input shift while the phase vary rapidly i.e. there is an unpredictable change in the phase of wavelet coefficients with input shift. This is illustrated in Fig. 1(d).

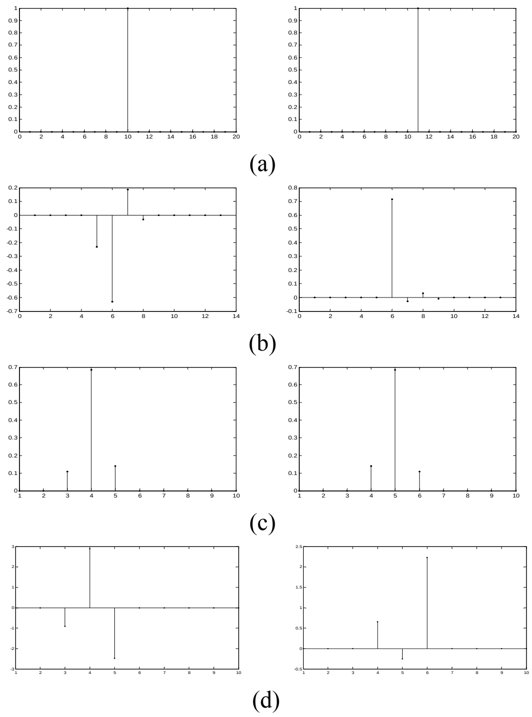
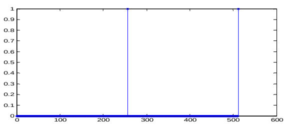


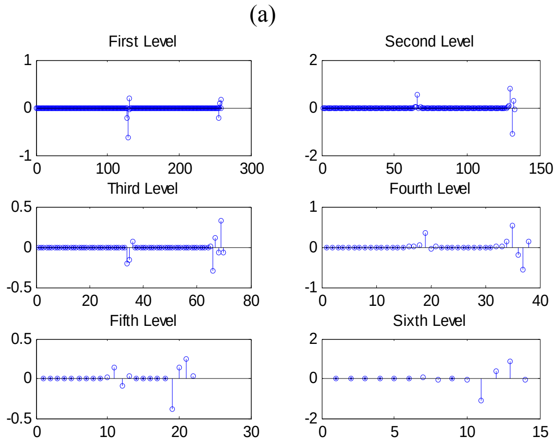
Fig.1. (a).Original signal and the shifted signal by one sample, (b).High-pass wavelet coefficients of the original and the shifted signal using db4 wavelet, (c).High-pass wavelet coefficient magnitude and (d). phase (in radian) of original and shifted signal using SDW6 complex wavelet.

Shift-invariance property also affect the informational content at multilevel. In DWT as one moves towards higher level, the nature of wavelet coefficients at different subbands changes unpredictably while in the case of CxWT the nature of wavelet coefficients at different subbands remains preserved. This is shown in fig.2. Thus applying any operation at multiscale wavelet coefficients will work in uniform way for complex wavelet coefficients, unlike its real counterpart.

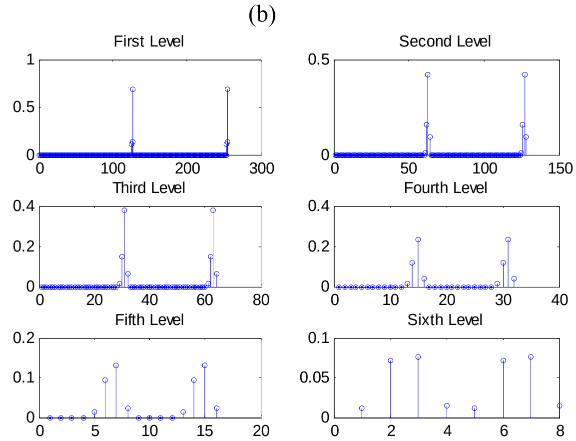
We have observed that for localized features such as sharp edges, there is strong local phase coherence across scales. The phase coherence increases with the strength of features. This is shown in figure 3. Fig. 3(a) and 3(e) shows two signals, one has a sharp feature and other has blurred features and Fig. 3(b)-3(d) and Fig.3(f)-3(h) show phases of wavelet coefficients of Fig.3(a) and 3(e) at 1st, 2nd and 3rd levels. From this figure, it is clear that there exists a strong phase coherence for sharp features.



(a)



(b)



(c)

Fig.2 (a). Original Input signal, (b). Detail real wavelet transform coefficients of signal at several levels, (c). Magnitude of detail complex wavelet coefficients at several levels.

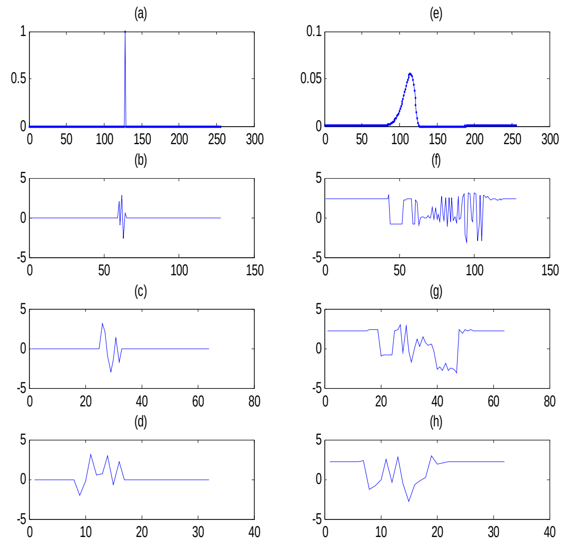


Fig.3 (a). A sharp featured signal, (b)-(d). Phase of wavelet coefficients of the signal at 1st, 2nd and 3rd level, (e). A blurred featured signal, (f)-(h). Phase of wavelet coefficients of the blurred signal at 1st, 2nd and 3rd level.

4 Applications of CxWT in Denoising and Deblurring

4.1 Signal Denoising

For denoising of signals, the threshold based denoising scheme is used. The proposed threshold is adaptive in nature. Donoho [9] has used the median estimator for the estimation of noise variance and based on this estimator, a threshold which depends on the median of absolute wavelet coefficients is used. In our recent work [3], we have found that a soft-threshold which depends on the standard deviation (σ), the absolute mean (μ) and the absolute median (M) of wavelet coefficients performs better. The value of soft-threshold is calculated as

$$\text{Threshold} = \frac{1}{2^{j-1}}\left(\frac{\sigma}{\mu}\right)M \qquad (12)$$

where j is the level number for which the threshold is computed.

The computed threshold is applied only on the magnitude of wavelet coefficients. Thresholding is applied at several levels. The signal-independent noise gets removed at lower levels while the signal-dependent noise is removed up to higher levels [3]. Inverse wavelet transform gives the reconstructed signal. Comparative results of denoising for one representative case by the proposed method and other denoising methods using real DWT[9,10] is given in fig. 4.

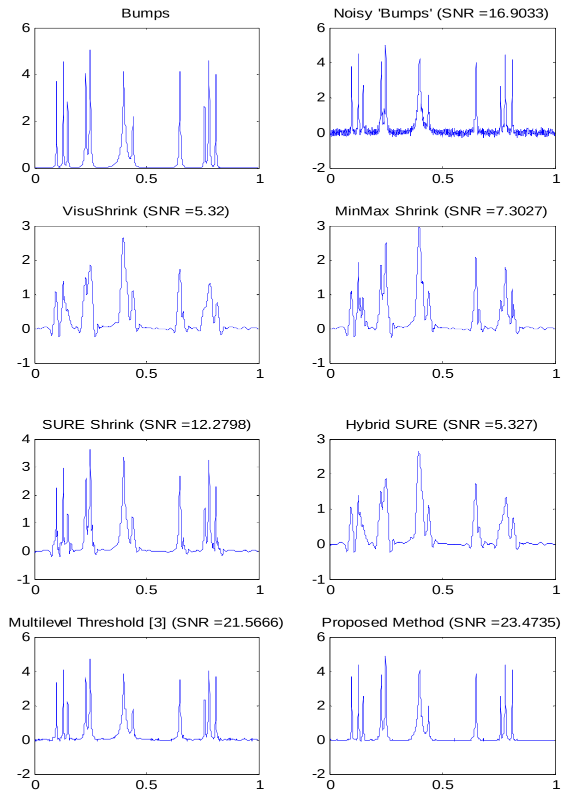


Fig.4. Denoising performance

4.2 Signal Deblurring

The observation model for any general system is

$$y = h * x + n \qquad (13)$$

where h, the linear operator, causes the blur in the image and n is the amount of noise.

In Fourier domain, the observation model is

$$Y = HX + N \qquad (14)$$

where H, X, N and Y are DFT's of h, x, n and y. Applying the Pseudo inverse filter,

$$\tilde{X} = \begin{cases} X + \dfrac{N}{H}, & if|H| > 0 \\ 0 & , \quad otherwise \end{cases} \qquad (15)$$

Here the term $H^{-1}N$ creates the colored noise in the deblurred signal. Neelamani et.al.[11] has attempted a shrinkage function in Fourier domain to remove the colored noise, but we observed that the colored noise can be removed by applying soft-thresholding of wavelet coefficients at several levels.

Our proposed algorithm for deblurring is as follows –

Step 1: Apply pseudo-inverse filter in Fourier domain to get estimate $\tilde{X}$. Inverse Fourier transform of $\tilde{X}$ gives an estimate of x as $\tilde{x}$.

Step 2: Compute Daubechies CxWT of $\tilde{x}$, say w. Soft-threshold the wavelet coefficients w by the computed multiscale threshold as in equation (12) at several scales. Inverse wavelet transform gives the deblurred image.

Fig.5 shows a blurred image corrupted with noise and restored by Weiner filter, ForWaRD[11], TI-WaveD[12] and the proposed method. Here in the proposed method we have removed colored noise upto 6 level. From the figure it is clear that the blocking artifacts present in ForWaRD and TI-WaveD is not dominantly present in the proposed method

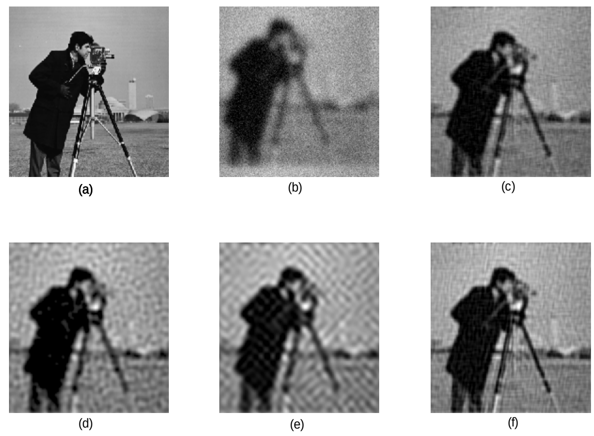


Fig.5 (a). Original Image, (b). Blurred and noisy Image (SNR=7.01dB), (c). Weiner filtered image (SNR=14.82 dB), (d). ForWaRD restored image (SNR=14.56dB), (e). WaveD restored image (SNR=14.85dB), (f). Restored image by the proposed method (SNR=16.42dB).

5 Other Possible Applications

Some of the applications, where the present method of multilevel complex wavelet analysis can be helpful, are mentioned below.

The multiscale property of CxWT make it well suited for texture analysis [13]. Shift-invariance property is important too, as it makes the texture feature vectors independent of precise texture location. It makes possible to make texture features rotationally invariant.

Since CxWT is Shift-invariance in nature, therefore, if the object is moving (equivalently shifted in frames), then the nature of wavelet coefficients in the region where the object is placed will not change. This helps to design efficient methods for tracking of moving objects.

Since the nature of complex wavelet coefficients at multilevel remains same, so it can be used for designing efficient methods for image registration as in real wavelet domain [14]. Efficient representation of multiscale wavelet coefficients allows to develop hierarchical segmentation algorithm as well.

6 Discussion and Conclusions

The application and usability of CxWT is almost an unexplored area. In this paper, we have explored some properties of CxWT and its suitability for various signal processing applications. Symmetric Daubechies wavelet transform handles the boundary problems of signal efficiently and its linear phase property allows it to retain the shape of the signal. Daubechies complex wavelet coefficients carry edge information inherently, while in the case of real wavelet it requires to be computed.

Shift-invariance nature of CxWT is an important property. Nature of real wavelet coefficients at multiple scales is not same while the nature of magnitude of complex wavelet coefficients at several scales is approximately same as discussed in section 3. Thus applying the same function at multiple scales will work well in complex wavelet domain, unlike the real domain. Thus the thresholding of complex wavelet coefficients is much effective.

Most of signal independent noise is represented at a few lower levels, while signal dependent noise is represented at several scales. So for removal of signal independent noise, the improvement is better than that of real-domain denoising. But for signal dependent noise, the use of complex-domain denoising gives a large improvement.

Removal of the colored noise in signal deblurring application is a difficult task. In this paper it has been shown that multilevel signal representation in complex wavelet domain helps in the removal of colored noise significantly by the application of soft-thresholding at multiple levels as discussed in section 4.2. Since for localized features, strong phase coherence across the scale is present in complex-domain, so by using this property one can have an idea about the blur quantity.

For all other applications that need translation invariance such as tracking of objects, texture analysis, signal registration, segmentation, etc, CxWT may be useful.

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