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Continues Model for Vertical Vibration of Tension Leg Platform

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Abstract: -In this paper the dynamic response of the leg of a tension leg platform (tether) subjected to the load simulated as ocean wave at the top of the leg is presented. The structural model is very simple but several complicated factors such as foundation effect, buoyancy and simulated ocean wave load are considered. Two continuous models are proposed to present the structural system and the mentioned effects. The problem is solved by means of non-harmonic Fourier expansion in terms of eigenfunctions obtained from a non-regular Sturm-Liouville system.

Key-Words: - TLP, Axial vibration, Continues model, Wave load

1 Introduction

Tension leg platform (TLP) is a well-known structure for oil exploitation in deep water. Many studies have been carried out to understand the structural behavior of TLP and to determine the effect of several parameters on dynamic response and average life time of the structure [1-4]. The most important point in the design of TLP is the pretension of the legs. The pretension causes that the platform behaves like a stiff structure with respect to the vertical degrees of freedom (heave, pitch and roll), whereas with respect to the horizontal degrees of freedom (surge, sway and yaw) it behaves as a floating structure. Among the various degrees of freedom, vertical motion (heave) is very important because of the direct effect on the stress fluctuation that leads to fatigue and fracture of tethers. Therefore the conceptual studies to understand the dynamic vertical response of TLP can be useful for designers.

Rossit et al. (1996) presented an analytical solution for the dynamic response of the leg of TLP subjected to an axial suddenly applied load at one end [5]. The applied load was constant and the effect of the buoyancy was not considered. The aim of this paper is the solution of the mentioned problem using two models. The structural models are very simple but several complicated factors such as foundation, buoyancy and simulated ocean wave loading are considered. At the first model the foundation assumed rigid but at the second model it is assumed that the foundation is embedded in the ocean bottom, which acts as a Winkler-type foundation. The buoyancy is modeled as a spring at the top of the leg. A concentrated force is applied at the top of the leg as simulated ocean wave load. The problem is solved by means of non-harmonic Fourier expansion in terms of eigenfunctions obtained from a non-regular Sturm-Liouville system [6, 7]. Tabeshpour et al., (2004a) have investigated the effect of added mass fluctuation on the heave response of tension leg platform for a discrete model by using perturbation method [8]. A continues model for vertical motion of TLP considering the effect of continues foundation has been reported by Tabeshpour et al., (2004b), [9]. Also Tabeshpour et al., (2004c) have presented a closed form formulation for the effect of added mass fluctuation on the heave response of tension leg platform considering continues model [10]. General configuration of TLP is shown in Fig. 1.

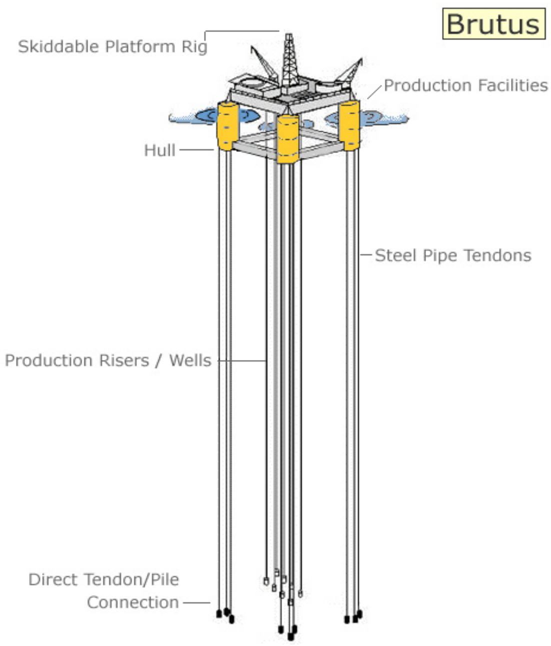

Fig. 1 - Configuration of TLP

2 Analytical Solution of the Model

The structural model of the system is shown in Fig. 2. The behavior of the system is described by the following differential equation

$$\begin{pmatrix} [u(y)-u(y-l_f)]E_f A_f + \\ [u(y-l_f)-u(y-l)]E_t A_t \end{pmatrix} \frac{\partial^2 v}{\partial y^2} + F_h(t)\delta(y-l) = \begin{pmatrix} [u(y)-u(y-l_f)]\rho_f A_f + \\ [u(y-l_f)-u(y-l)]\rho_t A_t + \\ M_f\,\delta(y-l_f)+M\,\delta(y-l) \end{pmatrix} \frac{\partial^2 v}{\partial t^2} \qquad (1)$$

where u is step function, v is the axial deformation, E is the Young modulus of the tether material, A_t and A_f are the cross sectional areas of the tether and foundation respectively, ρ_t and ρ_f are the density of tether and

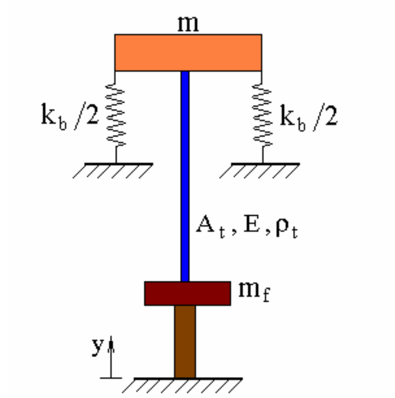

Fig. 2 - Dynamic structural model

foundation material respectively, l_t and l_f are the length of tether and foundation respectively and δ denotes the Dirac delta function. The applied vertical load subjected to the mass m, is the generated wave load, $F_h(t)=\sum_{j=1}^{N} F_j \sin(\Omega_j t+\phi_j)$ obtained from the wave spectrum. The system is linear, therefore the solution of the equation (1) is carried out considering a single term input, $F_h(t)=F_0\sin(\Omega t)$, and then the overall response of the system is evaluated by summation of all responses. The initial conditions are

$$v(y,0)=0\ ,\ \frac{\partial v}{\partial t}(y,0)=0 \qquad (2)$$

The mass distribution functions are defined as

$$m(y)=[u(y)-u(y-l_f)]\rho_f A_f + [u(y-l_f)-u(y-l)]\rho_t A_t + M_f\,\delta(y-l_f) + M\,\delta(y-l) \qquad (3)$$

In the case of free vibration, Eq. (1) becomes

$$\begin{pmatrix} [u(y)-u(y-l_f)]E_f A_f + \\ [u(y-l_f)-u(y-l)]E_t A_t \end{pmatrix} \frac{\partial^2 v}{\partial y^2} = m(y)\frac{\partial^2 v}{\partial t^2} \qquad (4)$$

Equation (4) can be solved assuming $m(y)=[u(y)-u(y-l_f)]\rho_f A_f + [u(y-l_f)-u(y-l)]\rho_t A_t$

subjected to the boundary conditions

$$v(0,t)=0 \qquad (5)$$

$$-E_t A_t \frac{\partial v}{\partial y}(l_f,t) - M_f \frac{\partial^2 v}{\partial t^2}(l_f,t) = E_f A_f \frac{\partial v}{\partial y}(l_f,t) \tag{6}$$

$$-k_b v(l,t) - M \frac{\partial^2 v}{\partial t^2}(l,t) = E_t A_t \frac{\partial v}{\partial y}(l,t) \tag{7}$$

Equation (4) can be solved for two parts of the bar. In $0 \le y = y_1 \le l_f$, one has

$$E_f A_f \frac{\partial^2 v}{\partial y^2} = m(y)\frac{\partial^2 v}{\partial t^2} \quad \text{or} \quad \frac{\partial^2 v}{\partial y^2} = \frac{1}{c_f^2}\frac{\partial^2 v}{\partial t^2} \tag{8}$$

where $m(y) = \rho_f A_f$, $c_f^2 = E_f / \rho_f$.

Using separation of variables, the eigenfunctions are determined as

$$Y_{n1} = B \sin \alpha_{nf} y_1 \tag{9}$$

where α_{nf} is the separation constant and and $c_f \alpha_{nf} = \omega_{nf}$ is the angular frequency.

In $l_f \le y = y_2 \le l$, one has

$$E_t A_t \frac{\partial^2 v}{\partial y^2} = m(y)\frac{\partial^2 v}{\partial t^2} \quad \text{or} \quad \frac{\partial^2 v}{\partial y^2} = \frac{1}{c_t^2}\frac{\partial^2 v}{\partial t^2} \tag{10}$$

where $m(y) = \rho_t A_t$, $c_t^2 = E_t / \rho_t$.

Similarly using separation of variables, the eigenfunctions are determined as

$$Y_{n2} = A\left[\cos \alpha_{nt} y_2 + \left(\frac{k_b}{k_t}\frac{1}{\alpha_{nt} l_t} - \frac{M}{m_t}\alpha_{nt} l_t\right)\sin \alpha_{nt} y_2\right] \tag{11}$$

where α_{nt} is the separation constant and and $c_t \alpha_{nt} = \omega_{nt}$ is the angular frequency.

It is clear that displacement and force are continues at $y_1 = l_f$ or $y_2 = 0$. From the continuity in displacement one has

$$Y_{n1}(l_f) = Y_{n2}(l_t)$$

or

$$A\left[\cos \alpha_{nt} l_t + \left(\frac{k_b}{k_t}\frac{1}{\alpha_{nt} l_t} - \frac{M}{m_t}\alpha_{nt} l_t\right)\sin \alpha_{nt} l_t\right] - B \sin \alpha_{nf} l_f = 0 \tag{12}$$

Also the continuity in force gives

$$E_t A_t \frac{\partial v_1}{\partial y_1}(l_t,t) = -E_f A_f \frac{\partial v_2}{\partial y_2}(l_f,t) - M_f \frac{\partial^2 v_1}{\partial t^2}(l_t,t) \tag{13}$$

or

$$A\frac{k_t}{k_f}\alpha_{nt} l_t\left[\sin \alpha_{nt} l_t - \left(\frac{k_b}{k_t}\frac{1}{\alpha_{nt} l_t} - \frac{M}{m_t}\alpha_{nt} l_t\right)\cos \alpha_{nt} l_t\right] + B\left(\frac{M_f}{m_f}\alpha_{nf}^2 l_f^2 \sin \alpha_{nf} l_f - \alpha_{nf} l_f \cos \alpha_{nf} l_f\right) = 0 \tag{14}$$

The coefficients A and B can be determined by solving Eqs. (12) and (14). There are nonzero solutions if

$$\begin{vmatrix} \cos \alpha_{nt} l_t + \left(\frac{k_b}{k_t}\frac{1}{\alpha_{nt} l_t} - \frac{M}{m_t}\alpha_{nt} l_t\right)\sin \alpha_{nt} l_t & -\sin \alpha_{nf} l_f \\ \frac{k_t}{k_f}\alpha_{nt} l_t\left[\sin \alpha_{nt} l_t - \left(\frac{k_b}{k_t}\frac{1}{\alpha_{nt} l_t} - \frac{M}{m_t}\alpha_{nt} l_t\right)\cos \alpha_{nt} l_t\right] & \frac{M_f}{m_f}\alpha_{nf}^2 l_f^2 \sin \alpha_{nf} l_f - \alpha_{nf} l_f \cos \alpha_{nf} l_f \end{vmatrix} = 0 \tag{15}$$

Now the frequency equation is resulted as

$$\left[\left(\frac{k_b}{k_t}\frac{1}{\alpha_{nt}l_t}-\frac{M}{m_t}\alpha_{nt}l_t\right)\tan\alpha_{nt}l_t+1\right]\left(-\frac{M_f}{m_f}\alpha_{nf}l_f\tan\alpha_{nf}l_f+1\right)-\frac{k_t}{k_f}\frac{\alpha_{nt}l_t}{\alpha_{nf}l_f}\tan\alpha_{nf}l_f\left[\tan\alpha_{nt}l_t-\left(\frac{k_b}{k_t}\frac{1}{\alpha_{nt}l_t}-\frac{M}{m_t}\alpha_{nt}l_t\right)\right]=0 \qquad (16)$$

where:

$\rho_t A_t l_t = m_t$: total mass of the tether;

$\rho_f A_f l_f = m_f$: total mass of the foundation;

$E_t A_t / l_t = k_t$: the axial stiffness of the tether, and

$E_f A_f / l_f = k_f$: axial stiffness of the foundation.

The response of the tether subjected to axial load, can be expressed in terms of normal modes of the system

$$v(y,t)=\sum_{n=1}^{\infty}Y_n(y)T_n(t) \qquad (17)$$

$$Y_n(y)=\left[u(y)-u(y-l_f)\right]Y_1(y)+\left[u(y-l_f)-u(y-l)\right]Y_2(y) \qquad (18)$$

$$M(y)=\left[u(y)-u(y-l_f)\right]\rho_f A_f+\left[u(y-l_f)-u(y-l)\right]\rho_t A_t+M_f\,\delta(y-l_f)+M\,\delta(y-l) \qquad (19)$$

Because of the orthogonality of the normal modes, it can be shown that

$$\int_0^l M(y)Y_n(y)Y_r(t)=0 \ (n\neq r) \qquad (20a)$$

$$\int_0^l M(y)Y_n(y)Y_r(t)=H_r \ (n=r) \qquad (20b)$$

Defining

$$M_1(y_1)=\rho_f A_f+M_f\,\delta(y_1-l_f) \qquad (21)$$

and

$$M_2(y_2)=\rho_t A_t+M\,\delta(y_2)+M_f\,\delta(y_2-l_t) \qquad (22)$$

Equation (20) can be rewritten as

$$\int_0^{l_f}M_1(y_1)Y_{n1}(y_1)Y_{r1}(y_1)+\int_0^{l_t}M_2(y_2)Y_{n2}(y_2)Y_{r2}(y_2)=0 \ (n\neq r) \qquad (23a)$$

$$\int_0^{l_f}M_1(y_1)Y_{n1}(y_1)Y_{r1}(y_1)+\int_0^{l_t}M_2(y_2)Y_{n2}(y_2)Y_{r2}(y_2)=H_r \ (n=r) \qquad (23b)$$

$$H_r=H_{r1}+H_{r2} \qquad (24a)$$

$$H_{r1}=\frac{m_f}{2}+Y_{r1}^2(l_f)\left(\frac{M_f}{2}\right) \qquad (24b)$$

$$H_{r2}=\left(\frac{k_b}{k_t}\frac{1}{\alpha_{nt}l_t}-\frac{M}{m_t}\alpha_{nt}l_t\right)^2\frac{m_t}{2}+\frac{m_t}{2}+Y_{r2}^2(l_t)\left(\frac{M_f}{2}\right)+Y_{r2}^2(0)\left(\frac{k_b}{2\alpha_{rt}^2c_t^2}+\frac{M}{2}\right) \qquad (24c)$$

Multiplying Eq.(1) by $Y_n(y)dy=\left[u(y)-u(y-l_f)\right]Y_1(y)dy+\left[u(y-l_f)-u(y-l)\right]Y_2(y)dy$ and integrating between 0 and l, one obtains

$$\begin{pmatrix}\left[u(y)-u(y-l_f)\right]E_f A_f+\\ \left[u(y-l_f)-u(y-l)\right]E_t A_t\end{pmatrix}\times\int_0^l Y_r\left(\sum Y_n''T_n\right)dy+F_h(t)\int_0^l Y_r\delta(y)dy=\int_0^l M(y)Y_r\left(\sum Y_n\ddot{T}_n\right)dy \qquad (25)$$

or

$$E_f A_f\int_0^{l_f}Y_{r1}\left(\sum Y_{n1}''T_n\right)dy_1+E_t A_t\int_{l_f}^{l}Y_{r2}\left(\sum Y_{n2}''T_n\right)dy_2+F_h(t)Y_{r2}(0)=H_r\ddot{T}_r \qquad (26)$$

since Y_{n1} satisfies (8) and Y_{n2} satisfies (10), one has

$$Y''_{n1} = -\frac{M_1(y_1)}{E_f A_f} c_f^2 \alpha_{nf}^2 Y_{n1} \tag{27}$$

$$Y''_{n2} = -\frac{M_2(y_2)}{E_t A_t} c_t^2 \alpha_{nt}^2 Y_{n2} \tag{28}$$

Substituting (27) and (28) in (26) and applying (23) results in

$$\ddot{T}_n + \left(c_f^2 \alpha_{nf}^2 \frac{H_{n1}}{H_n} + c_t^2 \alpha_{nt}^2 \frac{H_{n2}}{H_n} \right) T_n = \frac{Y_{n2}(0)}{H_n} F_0 \sin(\Omega t) \tag{29}$$

Solving the above differential equation, one has

$$T_n(t) = A_n \cos k_n t + B_n \sin k_n t + \frac{F_0}{k_n^2 - \Omega^2} \frac{Y_{n2}(0)}{H_n} \sin(\Omega t) \tag{30}$$

where

$$k_n^2 = c_f^2 \alpha_{nf}^2 \frac{H_{n1}}{H_n} + c_t^2 \alpha_{nt}^2 \frac{H_{n2}}{H_n} \tag{31}$$

Initial conditions results in

$$A_n = 0 \; ; \; B_n = -\frac{F_0}{k_n^2 - \Omega^2} \frac{\Omega}{k_n} \frac{Y_{n2}(0)}{H_n} \tag{32}$$

and

$$T_n(t) = \frac{F_0}{k_n^2 - \Omega^2} \frac{\Omega}{k_n} \frac{Y_{n2}(0)}{H_n} \times \left(-\frac{\Omega}{k_n} \sin k_n t + \sin(\Omega t) \right) \tag{33}$$

or

$$T_n(t) = \frac{\dfrac{F_0}{k_n^2 - \Omega^2} \dfrac{\Omega}{k_n} \dfrac{Y_{n2}(0)}{H_n} \left(-\dfrac{\Omega}{k_n} \sin k_n t + \sin(\Omega t) \right)}{\dfrac{m_f}{2} + \left(Y_{r1}^2(l_f) + Y_{r2}^2(l_t) \right) \left(\dfrac{M_f}{2} \right) + \left(\dfrac{k_b}{k_t} \dfrac{1}{\alpha_{nt} l_t} - \dfrac{M}{m_t} \alpha_{nt} l_t \right)^2 \dfrac{m_t}{2} + \dfrac{m_t}{2} + Y_{r2}^2(0) \left(\dfrac{k_b}{2\alpha_{rt}^2 c_t^2} + \dfrac{M}{2} \right)} \tag{34}$$

and

$$T_{nj}(t) = \frac{\dfrac{F_j}{k_n^2 - \Omega_j^2} \dfrac{\Omega_j}{k_n} \dfrac{Y_{n2}(0)}{H_n} \left(-\dfrac{\Omega_j}{k_n} \sin k_n t + \sin(\Omega_j t + \phi_j) \right)}{\dfrac{m_f}{2} + \left(Y_{r1}^2(l_f) + Y_{r2}^2(l_t) \right) \left(\dfrac{M_f}{2} \right) + \left(\dfrac{k_b}{k_t} \dfrac{1}{\alpha_{nt} l_t} - \dfrac{M}{m_t} \alpha_{nt} l_t \right)^2 \dfrac{m_t}{2} + \dfrac{m_t}{2} + Y_{r2}^2(0) \left(\dfrac{k_b}{2\alpha_{rt}^2 c_t^2} + \dfrac{M}{2} \right)} \tag{35}$$

and

$$Y_n(y) = \left[u(y) - u(y - l_f) \right] \sin \alpha_{nf} y_1 + \left[u(y - l_f) - u(y - l) \right] \times \left[\cos \alpha_{nt} y_2 + \left(\frac{k_b}{k_t} \frac{1}{\alpha_{nt} l_t} - \frac{M}{m_t} \alpha_{nt} l_t \right) \sin \alpha_{nt} y_2 \right] \tag{36}$$

Now the dynamic response of the tether is

$$v(y,t) = \sum_{n=1}^{\infty} \sum_{j=1}^{N} Y_n(y) T_{nj}(t) \tag{37}$$

The dynamic stress of the tether becomes

$$\sigma(y,t) = EA \frac{\partial v}{\partial t}(y,t) \tag{38}$$

3 Conclusion

The analytical solution of the tether response of TLP was presented for a simple continuous model. The applied load is a simulation of ocean wave. Some complicated factors such as foundation effect and buoyancy were considered. The presented solutions give a conceptual view of the heave response of TLP under sea wave loads. The formulation presented herein can be used in analytical study on fatigue life of tethers.

References

[1] Ahmad, S., 1996, Stochastic TLP response under long crested random sea, *Journal of Computers and Structures*, 61(6), pp. 975-993.

[2] Jain, A.K., 1997, Nonlinear coupled response of offshore tension leg platforms to regular wave forces, *Ocean Engineering*, 24 (7), pp. 577-592.

[3] Faltinsen, O.M., Van Hooff, R.W., Fylling, I.J., and Teigen, P.S., 1982, Theoretical and experimental investigations tension leg platform behavior, *Proceedings of BOSS 1*, pp. 411-423.

[4] Lee, C.P., 1994. Dragged surge motion of a tension leg structure. *International Journal of Ocean Engineering* 21 (3), 311-328.

[5] Rossit, C.A., Laura, P.A.A., and Bambill, D.V., 1996, Dynamic response of the leg of a TLP subjected to an axial suddenly applied load at one end, *Ocean Engineering*, 23(3), pp. 219-224.

[6] Benedek, A., Guichal, E., and Panzone, R., 1974, *On certain non-harmonic Fourier expansions as eigenfunctions of non-regular Sturm-Liouville system*s, Instituto de Matematica, Publication No. 4, Universidad Nacional del Sur, Bahia Blanca, Argentina.

[7] Laura, P.A.A., Reyes, J.A., and Rossi, R.E., 1974, Analysis of a cable-like system suddenly stopped at one end, *J. Sound Vibr*. 37, pp. 195-204.

[8] Tabeshpour, MR, Golafshani, AA, and Seif, MS (2004a). The Effect of Added Mass Fluctuation on Heave Response of Compliant Offshore Structures, *Proc 8th International Conference on Mechanical Engineering*, ISME, Iran, (In Persian).

[9] Tabeshpour, MR, Golafshani, AA, and Seif, MS (2004b). Simple Models for Heave Response of TLP under Harmonic Vertical Load, *Proc 8th International Conference on Mechanical Engineering*, ISME, Iran.

[10] Tabeshpour, M.R., Seif, M.S. and Golafshani, A.A., (2004c), Vertical response of TLP with the effect of added mass fluctuation, *16th Symposium on Theory and Practice of Shipbuilding*, Crovatia.