Proceedings of the 9th WSEAS International Conference on Applied Mathematics, Istanbul, Turkey, May 27-29, 2006 (pp62-67)

An ADM Closed Form Solution for Vibrating Strings

M. NAJAFI[1,2], H. KHORAMISHAD[2,3], H. MASSAH[4,5], M. MOGHIMI[3,4]
Mathematics Department
Kent State University
Ashtabulo, 3325 west street, OH 44004
USA

Abstract: - In this study, a closed form solution is obtained for a system of wave equations coupled in parallel with distributed viscous damping and springs, utilizing Adomian decomposition method (ADM). The after treatment technique is adopted to overcome shortcomings of Adomian convergency.

Key-Words: - Adomian Decomposition Method, Partial Differential Equation, coupled wave system, Adomian aftertreatment, Padé approximation.

1 Introduction

Many problems in structural dynamics deal with stabilizing the elastic energy of partial differential equations by boundary or internal energy dissipative controllers for wave equations or the Euler-Bernoulli beam equation. In this study stability of a system of wave equations coupled in parallel with distributed viscous damping and springs [1] is revisited. What comes new in this work is to find an analytical solution for the above system of partial differential equations through the application of ADM [2, 3]. Also, other measures were employed to improve the results [4].

In the recent years, there has been a great interest in ADM. The Adomian method has been applied to a wide class of linear or nonlinear, stochastic or deterministic, differential or algebraic and single or system of equations [2, 3, 5]. This method solves many types of problems without requiring linearization, discretization, perturbation or unjustified assumptions which may alter the physics of the problems. For a large number of problems, the decomposition method has shown reliable results in providing analytical approximation that converges rapidly [2, 3, 4, 5].

2 Application of Adomian Decomposition Method

The ADM consists of splitting the given equation into linear and nonlinear parts. Then the inverse of the highest-order derivative operator, usually, contained in the linear operator, is applied to the both sides of the given equation. The process is followed by decomposing the unknown function into a series whose components are to be determined. Decomposing of the nonlinear part in terms of the so called Adomian polynomials is the essential part of ADM. The successive terms of the series solution are found by recurrent relation using Adomian's polynomials.

The general form of our problem is:

$$\begin{aligned} &u_{tt} - u_{xx} + R_1(u,v) + N_1(u,v) = g_1(x), \\ &v_{tt} - v_{xx} + R_2(u,v) + N_2(u,v) = g_2(x), \\ &0 < x < 1, \quad t > 0, \end{aligned} \tag{1}$$

with initial conditions,

$$\begin{aligned} &u(x,0) = f_1(x), \quad v(x,0) = \sigma_1(x), \\ &u_t(x,0) = f_2(x), \quad v_t(x,0) = \sigma_2(x), \end{aligned} \tag{2}$$

where the two first terms in Eq. (1), $R_i(u,v)$, $N_i(u,v)$ and $g_i(x)$ are the highest-order derivative operators in respect to t and x in Eq. (1), the remainder of linear part, the nonlinear part of Eq. (1) and the function of variable x, respectively.

Eq. (1) may be rewritten in the operator form, as follows,

$$\begin{aligned} &L_t u - L_x u + R_1(u,v) + N_1(u,v) = g_1(x), \\ &L_t v - L_x v + R_2(u,v) + N_2(u,v) = g_2(x). \end{aligned} \tag{3}$$

Here L_t and L_x are operators in t and x, respectively. The inverse operator L_t^{-1} is a two-fold

[1] Associate Professor, Mathematics Department, Kent State University, USA
[2] Khayyam Research Institute, Tehran, Iran
[3] Mechanical Engineering Department, Iran University of Science and Technology, Tehran, Iran
[4] Acoustics Research Center, Institute of Applied Physics, Tehran, Iran
[5] Physics Department, Amirkabir University, Tehran, Iran

integration represented by $L_t^{-1}(.)=\int_0^t\int_0^t(.)dt\,dt$.

Applying the operator L_t^{-1} on (3) yields,

$$\begin{aligned}&u(x,t)=f_1(x)+tf_2(x)+L_t^{-1}L_xu\\&-L_t^{-1}R_1(u,v)-L_t^{-1}N_1(u,v)+L_t^{-1}g_1(x),\\&v(x,t)=\sigma_1(x)+t\sigma_2(x)+L_t^{-1}L_xv\\&-L_t^{-1}R_2(u,v)-L_t^{-1}N_2(u,v)+L_t^{-1}g_2(x).\end{aligned}\tag{4}$$

The first two terms in (4) are constants of integration which can be calculated by (2).

According to ADM, unknown functions u and v are decomposed as,

$$u(x,t)=\sum_{n=0}^{\infty}u_n(x,t),\quad v(x,t)=\sum_{n=0}^{\infty}v_n(x,t).\tag{5}$$

Also, the nonlinear terms $N_1(u,v)$ and $N_2(u,v)$ are decomposed into infinite series by use of Adomian polynomials,

$$N_1(u,v)=\sum_{n=0}^{\infty}A_n,\quad N_2(u,v)=\sum_{n=0}^{\infty}B_n.\tag{6}$$

The Adomian polynomials A_n and B_n can be calculated by [7],

$$\begin{aligned}A_n&=\frac{1}{n!}\left[\frac{d^n}{d\lambda^n}N_1\Big(\sum_{i=0}^{n}\lambda^iu_i,\sum_{i=0}^{n}\lambda^iv_i\Big)\right]_{\lambda=0},\\B_n&=\frac{1}{n!}\left[\frac{d^n}{d\lambda^n}N_2\Big(\sum_{i=0}^{n}\lambda^iu_i,\sum_{i=0}^{n}\lambda^iv_i\Big)\right]_{\lambda=0}.\end{aligned}\tag{7}$$

Introducing (5) and (6) into (4) gives,

$$\begin{aligned}&\sum_{n=0}^{\infty}u_n(x,t)=f_1(x)+tf_2(x)+\\&L_t^{-1}\left(L_x\left(\sum_{n=0}^{\infty}u_n\right)\right)-L_t^{-1}R_1\left(\sum_{n=0}^{\infty}u_n,\sum_{n=0}^{\infty}v_n\right)-\\&L_t^{-1}\left(\sum_{n=0}^{\infty}A_n\right)+L_t^{-1}g_1(x),\end{aligned}\tag{8a}$$

$$\begin{aligned}&\sum_{n=0}^{\infty}v_n(x,t)=\sigma_1(x)+t\sigma_2(x)+\\&L_t^{-1}\left(L_x\left(\sum_{n=0}^{\infty}v_n\right)\right)-L_t^{-1}R_2\left(\sum_{n=0}^{\infty}u_n,\sum_{n=0}^{\infty}v_n\right)-\\&L_t^{-1}\left(\sum_{n=0}^{\infty}B_n\right)+L_t^{-1}g_2(x).\end{aligned}\tag{8b}$$

Now, according to ADM, the system of equations (8) is transformed into a set of recursive relations given by,

$$\begin{aligned}&u_0(x,t)=f_1(x)+tf_2(x)+L_t^{-1}g_1(x),\\&u_{n+1}(x,t)=L_t^{-1}L_xu_n-L_t^{-1}R_1(u_n,v_n)-L_t^{-1}A_n,\end{aligned}\tag{9a}$$

$$\begin{aligned}&v_0(x,t)=\sigma_1(x)+t\sigma_2(x)+L_t^{-1}g_2(x),\\&v_{n+1}(x,t)=L_t^{-1}L_xv_n-L_t^{-1}R_2(u_n,v_n)-L_t^{-1}B_n.\end{aligned}\tag{9b}$$

The ($k+1$)-term approximate solutions for u and v can be determined respectively by,

$$\varphi_{k+1}=\sum_{n=0}^{k}u_n(x,t),\quad \psi_{k+1}=\sum_{n=0}^{k}v_n(x,t).\tag{10}$$

Finally, the analytical solution can be formed by summing up the power series:

$$\begin{aligned}u(x,t)&=\sum_{n=0}^{\infty}u_n(x,t),\\v(x,t)&=\sum_{n=0}^{\infty}v_n(x,t).\end{aligned}\tag{11}$$

3 Coupled Wave System in One Dimension

Let $\Omega_1=\Omega_2=\Omega=(0,1)$ be open sets in IR. Also, let $\partial\Omega_1,\partial\Omega_2$ be the boundaries of Ω_1 and Ω_2, respectively. The coupling constants $\beta>0$ and $\alpha>0$ are damping and spring coefficients, respectively. We assume that the projection of Ω_1 into Ω_2, denotes as Ω. Also, $u(x,t)$ and $v(x,t)$ are the displacements of two vibrating strings measured from their equilibrium position.

The governing equations prescribed the above systems are [1]:

$$\begin{aligned}&u_{tt}-c_1^2u_{xx}=\alpha(v-u)+\beta(v_t-u_t),\ \text{ in } \Omega_1\times(0,\infty),\\&v_{tt}-c_2^2v_{xx}=\alpha(u-v)+\beta(u_t-v_t),\ \text{ in } \Omega_2\times(0,\infty),\end{aligned}\tag{12}$$

with the initial conditions,

$$\begin{aligned}&u(0)=f_1,\quad u_t(0)=g_1,\quad \text{in } \Omega_1,\\&v(0)=f_1,\quad v_t(0)=g_2,\quad \text{in } \Omega_2,\end{aligned}\tag{13}$$

and we have Dirichlet boundary condition,

$$u=v=0,\quad on\ \partial\Omega\times(0,\infty).\tag{14}$$

Here, c_1 and c_2 are wave propagation speeds, also the distributed springs and dampers linking two vibrating springs are the coupling terms; that is, $\alpha(u-v)$ and $\beta(u_t-v_t)$. Energy can flow from one object to another through this parameter (α) and damp via shock absorber (β). Also $u(x,t)$ and

$v(x,t)$ are the displacement of two vibrating strings measured from their equilibrium positions.

3.1 Application of ADM in Eq. (12)

The essence of this paper depends on the following theorem.

Theorem 1, the system (12) along with boundary conditions (17) is:

(a) unstable if the initial and boundary conditions and also the system parameters are identical.

(b) stable if the initial conditions are different, regardless of the system parameters.

Proof of (a): without loss of generality let $\alpha = \beta = 1$, and $c_1 = c_2 = c$,

$$\begin{aligned} u_{tt} - c^2 u_{xx} &= (v-u) + (v_t - u_t), \\ v_{tt} - c^2 v_{xx} &= (u-v) + (u_t - v_t), \end{aligned} \qquad (15)$$

with similar initial conditions,

$$\begin{aligned} u(x,0) &= \sin \pi x, & v(x,0) &= \sin \pi x, \\ u_t(x,0) &= 0, & v_t(x,0) &= 0, \end{aligned} \qquad (16)$$

and boundary conditions,

$$\begin{aligned} u(0,t) &= 0, & v(0,t) &= 0, \\ u(1,t) &= 0, & v(1,t) &= 0. \end{aligned} \qquad (17)$$

Rewriting Eq. (15) in operator form as follows,

$$\begin{aligned} L_t u &= c^2 L_x u + (v-u) + (v_t - u_t), \\ L_t v &= c^2 L_x v + (u-v) + (u_t - v_t), \end{aligned} \qquad (18)$$

where L_t and L_x are second order partial differential operators in respect to t and x, respectively. Now, applying the inversed operator L_t to system (18) yields,

$$u(x,t) = f_1(x) + t f_2(x) + c^2 L_t^{-1} L_x u + L_t^{-1}(v-u) + L_t^{-1}(v_t - u_t), \qquad (19a)$$

$$v(x,t) = \sigma_1(x) + t\sigma_2(x) + c^2 L_t^{-1} L_x v + L_t^{-1}(u-v) + L_t^{-1}(u_t - v_t), \qquad (19b)$$

where L_t^{-1} is a two-fold integration. According to Eq. (11), Eq. (19) can be decomposed as follows,

$$\sum_{n=0}^{\infty} u_n(x,t) = f_1(x) + t f_2(x) + c^2 L_t^{-1}\left(L_x\left(\sum_{n=0}^{\infty} u_n \right)\right) + L_t^{-1}\left(\sum_{n=0}^{\infty} v_n - \sum_{n=0}^{\infty} u_n \right) + L_t^{-1}\left(\frac{\partial}{\partial t}\left(\sum_{n=0}^{\infty} v_n - \sum_{n=0}^{\infty} u_n \right)\right), \qquad (20a)$$

$$\sum_{n=0}^{\infty} v_n(x,t) = \sigma_1(x) + t\sigma_2(x) + c^2 L_t^{-1}\left(L_x\left(\sum_{n=0}^{\infty} v_n \right)\right) + L_t^{-1}\left(\sum_{n=0}^{\infty} u_n - \sum_{n=0}^{\infty} v_n \right) + L_t^{-1}\left(\frac{\partial}{\partial t}\left(\sum_{n=0}^{\infty} u_n - \sum_{n=0}^{\infty} v_n \right)\right), \qquad (20b)$$

Here, Eq. (20) is linear, so the nonlinear terms, $N_i(u,v);\ i = 0,1$, are not appeared. Hence, Eq. (7) can be ignored. Now what follows from Eq. (20) is:

$$u_0 + \sum_{n=0}^{\infty} u_{n+1}(x,t) = f_1(x) + t f_2(x) + c^2 L_t^{-1}\left(L_x\left(\sum_{n=0}^{\infty} u_n \right)\right) + L_t^{-1}\left(\sum_{n=0}^{\infty} v_n - \sum_{n=0}^{\infty} u_n \right) + L_t^{-1}\left(\frac{\partial}{\partial t}\left(\sum_{n=0}^{\infty} v_n - \sum_{n=0}^{\infty} u_n \right)\right), \qquad (21a)$$

$$v_0 + \sum_{n=0}^{\infty} v_{n+1}(x,t) = \sigma_1(x) + t\sigma_2(x) + c^2 L_t^{-1}\left(L_x\left(\sum_{n=0}^{\infty} v_n \right)\right) + L_t^{-1}\left(\sum_{n=0}^{\infty} u_n - \sum_{n=0}^{\infty} v_n \right) + L_t^{-1}\left(\frac{\partial}{\partial t}\left(\sum_{n=0}^{\infty} u_n - \sum_{n=0}^{\infty} v_n \right)\right), \qquad (21b)$$

Each of the equation in (21) can be rewritten in a set of the following recursive relations:

$$\begin{aligned} u_0(x,t) &= f_1(x) + t f_2(x), \\ u_{n+1}(x,t) &= c^2 L_t^{-1}\left(L_x\left(\sum_{n=0}^{\infty} u_n \right)\right) + L_t^{-1}\left(\sum_{n=0}^{\infty} v_n - \sum_{n=0}^{\infty} u_n \right) + L_t^{-1}\left(\frac{\partial}{\partial t}\left(\sum_{n=0}^{\infty} v_n - \sum_{n=0}^{\infty} u_n \right)\right) \quad n \ge 0. \end{aligned} \qquad (22)$$

Similarly,

$$
\begin{aligned}
&v_0(x,t)=\sigma_1(x)+t\sigma_2(x),\\
&v_{n+1}(x,t)=c^2L_t^{-1}\left(L_x\left(\sum_{n=0}^{\infty}v_n\right)\right)+\\
&L_t^{-1}\left(\sum_{n=0}^{\infty}u_n-\sum_{n=0}^{\infty}v_n\right)+\\
&L_t^{-1}\left(\frac{\partial}{\partial t}\left(\sum_{n=0}^{\infty}u_n-\sum_{n=0}^{\infty}v_n\right)\right)\quad n\geq 0.
\end{aligned}
\tag{23}
$$

The ($k+1$)-term approximant results for u and v can be found respectively as,

$$
\begin{aligned}
&\varphi_{k+1}=\sum_{n=0}^{k}u_n(x,t),\\
&\psi_{k+1}=\sum_{n=0}^{k}v_n(x,t).
\end{aligned}
\tag{24}
$$

One-term approximation for u and v are,

$$
\begin{aligned}
&\varphi_1(x,t)=u_0=f_1(x)+tf_2(x),\\
&\psi_1(x,t)=v_0=\sigma_1(x)+t\sigma_2(x).
\end{aligned}
\tag{25}
$$

The integration constants in (25) are evaluated by (16), as follows;

$$
\begin{aligned}
&\varphi_1(x,0)=\sin\pi x\rightarrow f_1(x)=\sin\pi x,\\
&\frac{\partial}{\partial t}\varphi_1(x,0)=0\rightarrow f_2(x)=0,\\
&\psi_2(x,0)=\sin\pi x\rightarrow\sigma_1(x)=\sin\pi x,\\
&\frac{\partial}{\partial t}\psi_2(x,0)=0\rightarrow\sigma_2(x)=0,
\end{aligned}
\tag{26}
$$

using (26) leads to:

$$
\begin{aligned}
&\varphi_1(x,t)=u_0=\sin\pi x,\\
&\psi_1(x,t)=v_0=\sin\pi x,
\end{aligned}
\tag{27}
$$

By using (22), proceeding terms for u_n are evaluated, in the following forms,

$$
\begin{aligned}
&u_1=-\frac{(\pi ct)^2}{2!}\sin\pi x, \qquad u_2=\frac{(\pi ct)^4}{4!}\sin\pi x,\\
&u_3=-\frac{(\pi ct)^6}{6!}\sin\pi x, \qquad u_4=\frac{(\pi ct)^8}{8!}\sin\pi x,\\
&u_5=-\frac{(\pi ct)^{10}}{10!}\sin\pi x, \qquad \ldots
\end{aligned}
\tag{28}
$$

use of (24) yields,

$$
u(x,t)=\sin\pi x\left(1-\frac{(\pi ct)^2}{2!}+\frac{(\pi ct)^4}{4!}-\frac{(\pi ct)^6}{6!}+\frac{(\pi ct)^8}{8!}-\ldots\right)
\tag{29}
$$

From Eq. (29), one can reach to the exact solution,

$$
u(x,t)=\sin\pi x\sum_{n=0}^{\infty}\frac{(-1)^n(\pi ct)^{2n}}{(2n)!}.
\tag{30}
$$

Where the summations in right hand side speaks of Maclaurin series of $\cos(\pi ct)$. Hence the above equation leads to the following exact solution,

$$
u(x,t)=\sin(\pi x)\cos(\pi ct),
\tag{31}
$$

similarly, one can get the exact solution for $v(x,t)$ as follows,

$$
v(x,t)=\sin(\pi x)\cos(\pi ct).
\tag{32}
$$

Equations (31) and (32) represent an oscillatory system, that is as $t\rightarrow\infty$, the above system, Eq. (15) along with Eqs. (16) and (17), will never get to rest.

Another evidence to support our first claim (see Theorem 1, part (a)) is to deal with the energy of the system as t goes to infinity. According to [1], the energy of the system is defined by,

$$
\begin{aligned}
E(t)=\frac{1}{2}\int_0^1\Big\{&|u_t|^2+c_1^2|u_x|^2+\\
&|v_t|^2+c_2^2|v_x|^2+\alpha|u-v|^2\Big\}dx,
\end{aligned}
\tag{33}
$$

Now, by introducing (31) and (32) into (32), one can get that, as $t\rightarrow\infty$, $E(t)$ will be constant, that is,

$$
E(t)=\frac{\pi^2}{2},
\tag{34}
$$

Eq. (34) implies that the energy of the system is conserved, see Fig.1, therefore the proof of part (a) of the above theorem is completed.

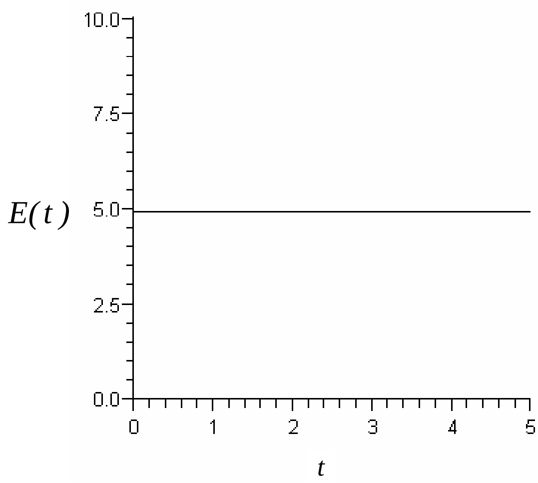

Fig.1 The energy Eq. (32) versus t, along with (16) and (17) is conservative. Theorem 1 part (a).

Proof of (b): without loss of generality, let $\alpha=\beta=1$, and $c_1=c_2=c$,

$$u_{tt} - c^2 u_{xx} = (v-u) + (v_t - u_t),$$
$$v_{tt} - c^2 v_{xx} = (u-v) + (u_t - v_t), \tag{35}$$

with initial conditions,

$$u(x,0) = \sin \pi x, \qquad v(x,0) = -\sin \pi x,$$
$$u_t(x,0) = 0, \qquad v_t(x,0) = 0, \tag{36}$$

and boundary conditions,

$$u(0,t) = 0, \qquad v(0,t) = 0,$$
$$u(1,t) = 0, \qquad v(1,t) = 0. \tag{37}$$

The integration constants in (25) are evaluated by (36), as follows;

$$\varphi_1(x,0) = \sin \pi x \rightarrow f_1(x) = \sin \pi x,$$
$$\frac{\partial}{\partial t}\varphi_1(x,0) = 0 \rightarrow f_2(x) = 0,$$
$$\psi_2(x,0) = -\sin \pi x \rightarrow \sigma_1(x) = -\sin \pi x,$$
$$\frac{\partial}{\partial t}\psi_2(x,0) = 0 \rightarrow \sigma_2(x) = 0, \tag{38}$$

Now, the first term approximation of $u(x,t)$ and $v(x,t)$ are,

$$\varphi_1(x,t) = u_0 = \sin \pi x,$$
$$\psi_1(x,t) = v_0 = -\sin \pi x. \tag{39}$$

By using (22), the proceeding terms for u_n are evaluated:

$$u_1 = -\left(1 + \frac{1}{2}c^2\pi^2\right)t^2 \sin \pi x,$$
$$u_2 = \left[\left(\frac{2}{3} + \frac{1}{3}c^2\pi^2\right)t^3 + \left(\frac{1}{6} + \frac{1}{6}c^2\pi^2 + \frac{1}{24}c^4\pi^4\right)t^4\right]\sin \pi x,$$
$$\vdots \tag{40}$$

Similarly, using Eq.(23), the proceeding terms for u_n are,

$$v_1 = \left(1 + \frac{1}{2}c^2\pi^2\right)t^2 \sin \pi x,$$
$$v_2 = -\left[\left(\frac{2}{3} + \frac{1}{3}c^2\pi^2\right)t^3 + \left(\frac{1}{6} + \frac{1}{6}c^2\pi^2 + \frac{1}{24}c^4\pi^4\right)t^4\right]\sin \pi x,$$
$$\vdots \tag{41}$$

By (24) one can find,

$$\varphi_3(x,t) = \sum_{n=0}^{3} u_n(x,t) = \left[1 - \left(1 + \frac{1}{2}c^2\pi^2\right)t^2 + \left(\frac{2}{3} + \frac{1}{3}c^2\pi^2\right)t^3 - \left(\frac{1}{6} + \frac{1}{24}c^4\pi^4\right)t^4\right]\sin \pi x, \tag{42}$$

similarly,

$$\psi_3(x,t) = \sum_{n=0}^{3} v_n(x,t) = -\left[1 - \left(1 + \frac{1}{2}c^2\pi^2\right)t^2 + \left(\frac{2}{3} + \frac{1}{3}c^2\pi^2\right)t^3 - \left(\frac{1}{6} + \frac{1}{24}c^4\pi^4\right)t^4\right]\sin \pi x. \tag{43}$$

There are variety of choices to improve the radius of convergence in (42) and (43). The simplest one is to compute more terms, but it is tedious. Since the above system starts with oscillations, an extension to ADM is employed. The use of this extension, Aftertreatment (AT) technique, leads to a closed form solution [4, 8]. This technique uses Laplace transform and Padé approximation [2] which approximates a function by ratio of two polynomials.

Due to AT technique, Laplace transform is applied to the coefficient of $\sin(\pi x)$ in equation (42). yields,

$$\ell(\varphi_3(x,t)) = \left[\frac{1}{s^7}\left(s^6 - c^2\pi^2 s^4 - 2s^4 + 4s^3 + 2c^2\pi^2 s^3 + c^4\pi^4 s^2 - 4s^2 - 4c^4\pi^4 s - 16s - . \; 16c^2\pi^2 s - 6c^4\pi^4 - 12c^2\pi^2 - 8 - c^6\pi^6\right)\right]\sin \pi x \tag{44}$$

For the sake of simplicity, let $s = 1/\xi$; then,

$$\bar{\ell}(\varphi_3(x,t)) = \left[\xi - \left(2 + c^2\pi^2\right)\xi^3 + \left(4 + 2c^2\pi^2\right)\xi^4 + (-4 + c^4\pi^4)\xi^5 - \left(16 + 16c^2\pi^2 4c^4\pi^4\right)\xi^6 - \left(8 + 12c^2\pi^2 + 6c^4\pi^4 + c^6\pi^6\right)\xi^7\right]\sin \pi x, \tag{45}$$

Now, Eq. (45) is approximated by Padé approximation $\left[\frac{2}{2}\right]$,

$$\left[\frac{2}{2}\right]_\xi = \frac{\xi(1+2\xi)}{1 + 2\xi + \left(2 + c^2\pi^2\right)\xi^2}. \tag{46}$$

Now, let $\xi = 1/s$, then Eq. (46) becomes

$$\left[\frac{2}{2}\right]_s = \frac{2+s}{\left(2 + c^2\pi^2\right) + 2s + s^2}. \tag{47}$$

Finally, applying the inverse Laplace transform to (47), results to:

$$u(x,t) \cong e^{(-t)}\left[\cos\left(t\sqrt{1+\pi^2}\right) + \frac{\sin\left(t\sqrt{1+\pi^2}\right)}{\left(\sqrt{1+\pi^2}\right)}\right]\sin \pi x. \tag{48}$$

Similarly, one can also get the approximant solution for *v(x,t)*,

$$v(x,t) \cong -e^{(-t)}\left[\cos\left(t\sqrt{1+\pi^2}\right) + \frac{\sin\left(t\sqrt{1+\pi^2}\right)}{\left(\sqrt{1+\pi^2}\right)}\right]\sin \pi x. \tag{49}$$

Therefore the stability claim of the system, (see Theorem1 part (b)), is established by introducing (48) and (49) into (33) from which the result is plotted in Fig.2, and that completes the proof of part (b) of the theorem.

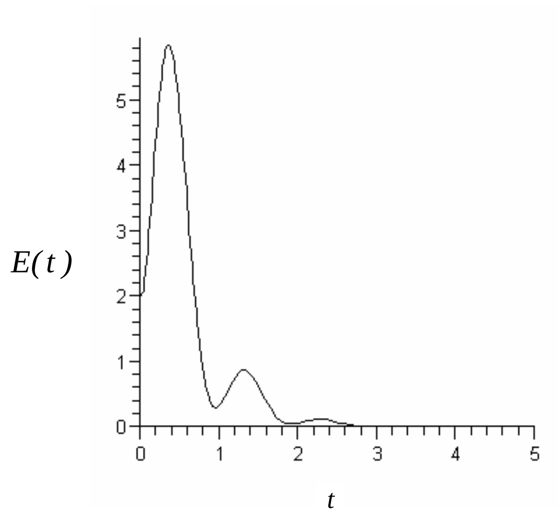

Fig.2 The energy Eq. (32) versus *t*, along with (36) and (37) decays. Theorem 1, part (b).

4 Conclusion

The main goal of this work has been achieved by studying the stability of wave equations using extended Adomian decomposition method and the results agree reasonably well with numerical computations [1]. It is important to note that, unlike common methods e.g., numerical and perturbation, this method gives a closed form solution to our problem.

References:

[1] M. Najafi, Study of exponential stability of coupled wave system via distributed stabilizer, International Journal of Mathematics and Mathematical Science, 1999.

[2] G. Adomian, Solving Frontier Problems of Physics: The Decomposition Method, Kluwer Academic Publishers, Boston, 1994.

[3] G. Adomian, A review of the decomposition method in applied mathematics, J. Math. Anal. Appl. 135, 1988, pp. 501–544.

[4] Y. C. Jiao, et al., An Aftertreatment Technique for Improving the Accuracy of Adomian's Decomposition Method, Computers Math. Applic. 43, 2002, pp. 783–798.

[5] G. Adomian, stochastic systems, ACADEMIC PRESS, New York, 1983.

[6] Zhu Yonggui, Chang Qianshun, Wu Shengchang, a new algorithm for calculating Adomian polynomials, Applied Mathematics and Computation, In Press, 2004.