Proceedings of the 9th WSEAS International Conference on Applied Mathematics, Istanbul, Turkey, May 27-29, 2006 (pp39-44)

Approximated Solutions to Tow-dimensional and Axisymetric Jet Impinging Flows, Using Adomian Decomposition Method

M. NAJAFI[1, 3], M. TAEIBI-RAHNI[2, 3], KH. JAVADI[2, 3], AND S.F. HOSSEINZADEH[2, 3]
1. Mathematics Department, Kent State University, Ohio State, USA
2. Aerospace Engineering Department, Sharif University of Technology, Tehran, IRAN
3. Khayyam Research Institute, Tehran, IRAN

Abstract: Approximated solutions to two-dimensional and axisymmetric jet impinging flows have been presented here. Assumptions have been made to reduce the related full Navier-Stokes equations to a non-linear ordinary differential equation. The Admomian decomposition method (ADM) has been employed to obtain an approximated solution to this differential equation. A trial and error strategy has been used to obtain the constant coefficient in the approximated solution. The results were compared with accurate numerical results, which show that the ADM is a high performance and accurate method to solve such flow equations.

Key-Words: - Navier-Stokes, Jet Impingement, Adomian Decomposition Method, Analytic solution

1 Introduction

Achievement of exact solution for non-linear partial differential equations such as Navier-Stokes (N-S) equations is an ambitious and perfect goal for engineers and mathematicians. However, computational finite discrete approaches such as Finite Volume Method, Finite Difference Method, and Finite Element Methods have been widely used to solve the N-S equations in the past decades. Computational methodologies have been the only way to solve the N-S equation for predicting flow behavior in most flows. Some of the problems with computational methodologies are: 1- they are flow dependent. 2- To simulate real flows, they are very time consuming, 3- stability and convergence are not resolved easily in many cases, 4- we need to use modeling and approximation of some unknown in order to achieve a closed system of equation in turbulent flows. Which could be a source of error in the computational methods.

Any type of approximated solution would be very valuable for mathematicians and engineers. To this end, ADM is one the techniques which was introduced to solve non-linear ordinary and partial differential equation [1]. An advantage of this method is that, it can provide approximated or approximated solution to a rather wide class of non-linear (and stochastic) equations without linearization, perturbation, closure approximation, or discretization methods. Unlike the common methods, i.e., weak non-linearity and small perturbation which are change the physics of the problem due to simplification, ADM gives the approximated or approximated solution of the problem without any simplification. Thus, its results are more realistic [1].

During the past few years, several researchers have tried to modify the Admonian decomposition method. Zhang [2] presented a modified ADM to solve a class of non-linear singular boundary value problem, which arise as non-linear normal modal equations in non-linear conservative vibratory systems. He verified the effectiveness of his method by solving three examples. Wazwaz [3] developed a fast and accurate algorithm for solution of sixth-order boundary value problems. Luo e.al. [4] revised ADM for cases involving inhomogeneous boundary conditions using a suitable transformation. They solved inhomogeneous heat and wave equations. Jafari and Varsha [5] modified ADM to solve a system of non-linear equations, which yielded a series solution with faster accelerated convergence than the series obtained by the standard ADM. Zhu [6] et.al. presented a new algorithm for calculating Adomian polynomials for non-linear operators by parametrization. The algorithm requires less formula than the previous method developed by Adomian. Abbasbandy [7] presented some efficient numerical algorithms to solve a system of two non-linear equations (with two variables) based on Newton's method. Their modified Adomian decomposition method was applied to construct the numerical algorithms. Some numerical illustrations were given to show the efficiency of algorithms. Luo [8] proposed an efficient modification to ADM, namely two-step Adomian decomposition method (TSADM) that facilitated the calculations. He conducted a comparative study between the TSADM and previous methods with the help of several illustrative examples. Their results indicated that, the TSADM was effective and promising.

Recently, several researchers have used ADM to solve a wide range of physical phenomena in various engineering fileds such as heat and mass transfer [9], [10], [11], vibration and wave equation

[12], [13], fluid flow prediction [14, [15], [16], [17], porous media simulation [18], and other non-linear systems [19], [20], [21].

In this work, we are going to obtain an approximated solution to two-dimensional and axisymmetric jet impingement flows. For this purpose, we use self similar assumption to reduce the Navir-Stokes equations to a non-linear ordinary differential equation. Then, we use ADM to obtain an approximated solution for this problem. A trial and error strategy was implemented to obtain the constant coefficient in the approximated solution.

2 Problem Description

When a jet impinges onto a surface, very thin hydrodynamic and thermal boundary layers form in the impingement region (Figure **1**). Consequently, extremely high heat transfer coefficients are obtained within the stagnation zone.

Jet impingement arrays are generally used as an effective source of cooling on the leading edge and mid-span regions of gas turbine blades and vanes to enhance the convective heat transfer. The large rates of heat generation in integrated circuit (IC) chips pose severe thermal management challenges for the semiconductor industry. Recently, micro-jet heat sinks have been interested in by researcher. These micro-jets can be fully encapsulated, which removes the problems with larger spray cooling systems.

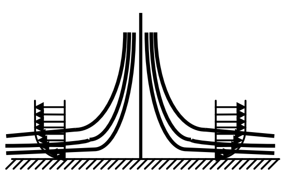

Figure 1, the Schematic of a jet impingement on a flat plate

3 Problem Formulation

Consider equation F(u(t))=g(t), where F represents a general non-linear ordinary or partial differential operator including both linear and non-linear terms. The linear terms is decomposed into $L+R$, where L is easily invertible (usually the highest order derivative) and R is the remained of the linear operator. Thus, the equation can be written as:

$$Lu + Ru + Nu = g \,, \tag{1}$$

where, Nu indicates the non-linear terms. By solving this equation for Lu, since L is invertible, we can write:

$$L^{-1}Lu = L^{-1}g - L^{-1}Ru - L^{-1}Nu \,. \tag{2}$$

Solving equation (1-2) for u, we get:

$$u = A + Bt + L^{-1}g - L^{-1}Ru - L^{-1}Nu \,, \tag{3}$$

where A and B are constants of integration and can be found from the boundary or initial conditions.

Adomian method assumes the solution u can be expanded into infinite series as:

$$u = \sum_{n=0}^{\infty} u_n \,. \tag{4}$$

Also, the non-linear term Nu will be written as:

$$Nu = \sum_{n=0}^{\infty} A_n \,, \tag{5}$$

where A_n are the special Adomian polynomials. Finally, the solution can be written as:

$$\sum_{n=0}^{\infty} u_n = u_0 - L^{-1}R \sum_{n=0}^{\infty} u_n - L^{-1} \sum_{n=0}^{\infty} A_n \,, \tag{6}$$

where u_0 is identified as: $A+Bt+L^{-1}g$ [1].

In Eq'n. (6) the Adomian polynomials can be generated by several means. Hear, we used the following recursive formulation:

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n} N\left(\sum_{i=0}^{\infty} \lambda_i u_i\right)_{\lambda=0} \,,\ n=0,\ 1,\ 2,\ \ldots \,. \tag{7}$$

Since the method does not resort to linearization or assumption of weak non-linearity, the solution generated is in general more realistic than those achieved by simplifying the model of the physical problem.

3.1 Application to Impingement Jet Flows

For plane incompressible two-dimensional flow, the governing equations are two-dimensional Navier-Stokes equations involving continuity and momentum. Using similarity solution strategy, the governing equation for this type of flows in two-dimensional and axisymmetric form can be reduced to:

$$f''' + \beta(f\,f'') + 1 - f'^2 = 0 \,, \tag{8}$$

where, $u \equiv Bxf'(\eta), v \equiv Bf(\eta)$, $\eta = y\sqrt{\frac{B}{\upsilon}}$, and B is the inverse of the flow characteristic time scale. Also, β=1 for two-dimensional and β=2 for axisymmetric flows [22].

The boundary conditions are $u=v=0.$ at the wall (η=0) and u=Bx at large distances from the wall. This means:

$$f(0) = f'(0) = 0 \ \ \& \ \ f'(\eta) = 1 \text{ as } \ \eta \to \infty \,. \tag{9}$$

Equation (8) contains two non-linearities and no analytic solution has ever been found for it [22].

To apply the decomposition method, we write equation (8) in an operator form as:

$$L_\eta f = 1 - \beta(f\,f'') + f'^2 \,, \tag{10}$$

where, $L = \frac{d^3}{d\eta^3}$. Assume, the inverse of the operator as

$$L_\eta^{-1} = \int_0^\eta \int_0^\eta \int_0^\eta (.)\, d\eta\, d\eta\, d\eta \,. \tag{11}$$

Applying the inverse operator to Eq'n. (10) yields:

$$L_\eta^{-1} L_\eta f = L_\eta^{-1} 1 - \beta L_\eta^{-1} (f f'') + L_\eta^{-1} f'^2 \qquad (12)$$

3.1.1 Two-dimensional Impinging Jet Flow

If we consider β=1 in Eq'n. (12) the two-dimensional jet impinging flow will be obtained as:

$$f(\eta) = f(0) + f'(0)\,\eta + f''(0)\,\eta^2/2 - \eta^3/6 + L_\eta^{-1}(-f f'' + f'^2). \qquad (13)$$

Applying the boundary condition (9) at η=0, we get:

$$f(\eta) = f''(0)\,\eta^2/2 - \eta^3/6 + L_\eta^{-1}(-f f'' + f'^2). \qquad (14)$$

The ADM consider the following expression for $f(\eta)$:

$$f(\eta) = \sum_{n=0}^{\infty} f_n(\eta). \qquad (15)$$

Also, the method assumes the non-linear function F($f(\eta)$)as an infinite power series of polynomials as:

$$F(f(\eta)) = \sum_{n=0}^{\infty} A_n. \qquad (16)$$

In Eq's. (15) and (16) A_n's are calculated from (7) and f_n's can be obtained by substituting (15) and (16) into (14) to get:

$$\sum_{n=0}^{\infty} f_n(\eta) = f''(0)\,\eta^2/2 - \eta^3/6 + L_\eta^{-1}\Big(\sum_{n=0}^{\infty} A_n\Big). \qquad (17)$$

In Eq'n. (17) to calculate the components of A_n and f_n, we need $f_0(\eta)$ as follows:

$$f_0(\eta) = f''(0)\,\eta^2/2 - \eta^3/6. \qquad (18)$$

To find $f''(0)$ we will utilize the boundary condition at infinity (see Eq'n. 9). To this end, let $f''(0)$ =α. Thus, Eq'n. (18) becomes:

$$f_0(\eta) = \alpha\,\eta^2/2 - \eta^3/6. \qquad (19)$$

Now, the remaining components of $f(\eta)$ in Eq'n. (17) can be determined recurrently as:

$$f_{n+1}(\eta) = L^{-1}(A_n). \qquad (20)$$

Recall that, A_k's can be generated from Eq'n. (7) as:

$$A_0 = -f_0'' f_0 + f_0'^2,$$

$$A_1 = -f_0'' f_1 - f_0 f_1'' + 2 f_0' f_1',$$

$$A_2 = -f_0'' f_2 - f_1 f_1'' - f_0 f_2'' + 2 f_0' f_2' + f_1'^2,$$

$$A_3 = -f_0'' f_3 - f_2 f_1'' - f_1 f_2'' - f_0 f_3'' + 2 f_1' f_2' + 2 f_0' f_3',$$

Similarly f_n can be determined from equations (19) and (20) as:

$$f_0 = \alpha\,\eta^2/2 - \eta^3/6,$$

$$f_1 = \frac{1}{2520}\eta^7 - \frac{1}{360}\alpha\eta^6 + \frac{1}{120}\alpha\eta^5,$$

$$f_2 = \frac{1}{2494800}\eta^{11} - \frac{1}{226800}\alpha\eta^{10} + \frac{1}{90720}\alpha^2\eta^9 - \frac{1}{40320}\alpha^3\eta^8,$$

$$\ldots.$$

Therefore,

$$f = f_0 + f_1 + f_2 + f_3 + f_4 + \ldots \qquad (21)$$

Here, α is yet to be determined (see results and discussion in section 4).

3.1.2 Axisymmetric Impingement Jet Flow

If β=2 in Eq (8), the governing equation for axisymmetric jet impinging flow are obtained as [22]:

$$f(\eta) = f(0) + f'(0)\,\eta + f''(0)\,\eta^2/2 - \eta^3/6 + L_\eta^{-1}(-2 f f'' + f'^2) \qquad (22)$$

Using ADM and applying the same approach as discussed for two-dimensional case, the Adomian polynomial and closed form solution in the axisymetric case can be obtained as:

$$A_0 = -2 f_0'' f_0 + f_0'^2,$$

$$A_1 = -2 f_0'' f_1 - 2 f_0 f_1'' + 2 f_0' f_1', \qquad (23)$$

$$A_3 = -2 f_0'' f_3 - 2 f_2 f_1'' - 2 f_1 f_2'' - 2 f_0 f_3'' + 2 f_1' f_2' + 2 f_0' f_3',$$

$$\ldots$$

Similarly, f_n's can be determined from Eq (19) and (20) as:

$$f_0 = \alpha\,\eta^2/2 - \eta^3/6,$$

$$f_1 = -\frac{1}{2520}\eta^7 + \frac{1}{360}\alpha\eta^6,$$

$$f_2 = -\frac{1}{277200}\eta^{11} + \frac{1}{25200}\alpha\eta^{10} - \frac{1}{9072}\alpha^2\eta^9,$$

$$f_3 = -\frac{1051}{2724321600}\eta^{15} + \frac{1051}{1816214400}\alpha\eta^{14} - \frac{223}{77837760}\alpha^2\eta^{13} + \frac{1}{213840}\alpha^3\eta^{12} + \qquad (24)$$

$$\ldots.$$

Finally,

$$f = f_0 + f_1 + f_2 + f_3 + f_4 + \ldots \qquad (25)$$

Now, to find the complete solution to Eqn's. (21) and (3-20), we need to find α. Accurate prediction of α is considerably important, since it affects the accuracy of the final solution. Recall, f'' is related to the shear stress and thus $f''(0)$ is related to the shear stress at the wall. Therefore, its accurate prediction is very important from skin friction view point.

Wang [23], mentioned, that Pade approximation can not be applied for this type of equation. He transformed the Blasius equation into a new space and calculated α which was rough. Later, Hashim [24] showed that, ADM Pade approach can be used and gives a more accurate α for Blasius equation than Wang's.

However, in this work, neither we used Pade approximation nor Wang's transformation. We employed a trial and error approach. To achieve this goal, we relied on the physics of the problem and we looked at the physical meanings of f' and f''. Recall that f' is the dimensionless velocity inside the boundary layer and f'' is related to the shear stress. As expected and from the boundary conditions (Eq'ns. 9), f' approaches unity as η becomes very large. Here, one must ask "how large is infinity". To answer this question, we should look at f'' which approaches zero outside the boundary layer. Thus, the answer is: when f'' becomes very small. Now, if we assume η_∞ as a new variable and consider $f''(\eta)\big|_\infty = 0$, as a new equation, then we would have a system of two non-linear algebraic equations with unknowns α and η (see below) which can be solved by the trial and error strategy mentioned above:

$$\begin{cases} f'(\eta)=1 & as\,\eta\to\infty \\ f''(\eta)=0 & as\,\eta\to\infty \end{cases} \quad , \quad . \qquad (26)$$

4 Results and Discussion

ADM was used to achieve an approximated solution to two-dimensional and axisymmetric jet impingement flows. As mentioned earlier, α and η_∞ were obtained by a trial and error approach from Eq'n. (3-25) as:

Two-dimensional Jet Impingement**:**

$f''(0)=\alpha=1.23197$ and $\eta_\infty=3.485$

Axisymmetric Jet Impingement:

$f''(0)=\alpha=1.31195$ and $\eta_\infty=2.3$.

Table (1) compares our approximated ADM solution with the reliable numerical data of reference [22]. As shown, there is an excellent agreement between the two different results.

Table 1. Comparison of $f''(0)$ with the reliable numerical data of reference [22].

	Numerical Data	ADM	Error
2-D	$f''(0)$=1.23259	$f''(0)$ 1.23197	5×10^{-4}
Axisymmetric	$f''(0)$=1.31194	$f''(0)$=1.31195	1×10^{-5}

Recall, another boundary layer effect is the displacement thickness, as the distance the outer inviscid flow is pushed away from the wall by the retarded viscous layer. The formal definition of the boundary layer displacement thickness is [22]]:

$$\eta^* = \int_0^\infty (1-f'^2)d\eta = \lim_{\eta\to\infty}(\eta - f) = \eta_\infty - f(\eta)\big|_\infty \qquad (27)$$

As listed in Table (2), the predicted boundary layer displacement thickness in our solution, is considerably close to the numerical predictions of Ref. [22].

Table 2. Comparison of η^* with the accurate numerical data of reference [22].

	Numerical Data	ADM	Error
2-D	η^*=0.6479	η^*=0.6481	2×10^{-4}
Axisymmetric	η^*=0.5689	η^*=0.5684	5×10^{-4}

Tables (3) and (4) compare our solution and the numerical data of reference [22] for two-dimensional and axisymmetric jet impingement flows, respectively. As is seen from these tables, there is an excellent agreement between the two results.

4.1 Sensitivity of the Solution to the Number of Polynomial Terms Used

Obviously, as the number of terms in the expansion polynomial series of ADM method increases, the closed form solution gets closer to the exact solution. Also using infinite number of terns, the ADM closed form solution coincides with the exact solution. However, when ADM is used, one must first answer the question "how many terms are needed". It is not so easy to find a general answer to the question, since it is problem dependent. We have shown that, as η increases and as the sensitivity of the flow variables become higher, we need to keep more terms in the polynomial expansion series.

Figure **2** shows the behavior of the solution with respect to the number of terms in the expansion polynomial series. Note from this figure that:

1-As η increases, the difference between our solution and the exact solution increases and thus it is necessary to keep more terms in our closed form solution.

2-As the sensitivity of the variables increases (e.g. f'') the sensitivity to the number of terms in the closed form solution increases. As shown in Tables 3 and 4, we selected f'' (which has the highest sensitivity with respect to f and f'') for comparison with other numerical results.

Thus, when using ADM to solve such problems, having a good physical knowledge of the problems is very important. Since, it is practically impossible to consider infinte number of terms in the polynomial series, it needs to be determined how far from the wall (in η direction) our ADM results are physically acceptable. This matter is more important for problems having at least one boundary at infinity. However, obviously, the infinity boundaries are different in mathematics and in physics. So that, a closed form solution obtained for a physical problem which is very accurate inside the infinity boundaries may not be true outside of it. In this work, we obtained η_∞=3.485 and η_∞=2.3 for two-dimensional and axisymmetric flows, respectively and as Tables 1 to 4 show, the results are very accurate for $\eta \leq \eta_\infty$. However for $\eta_\infty > \eta$, the results are not physical. If we are interested in achieving physical and accurate results for $\eta_\infty > \eta$ (3.48 or 2.3 in this work) we must consider more terms in ADM. This means that, by adding more terms in the ADM the range of infinity boundary (η_∞) goes farther away from the wall. In other words, adding more terms the results are acceptable for a wider range

5 Conclusion

Admomian decomposition method has been employed to obtain an approximated solution to two-dimensional and axisymmetric jet impingement flows. A trial and error strategy has been used to obtain the constant coefficient (α) in the closed form approximated solution. To verify our solution, the results were compared with reliable numerical data. In this regard, the sensitive variable f'' and also the displacement thickness (η^*) were compared with the numerical results. These comparisons showed excellent agreements.

Also, the results showed that, as the sensitivity of the flow variables rises the need for keeping more terms in the closed form polynomial solution is increased. For the problems which involve at least one boundary condition at infinity, the applicable range of the solution in η direction (infinity boundary) must indicated. For η greater than the boundary layer thickness, more terms needs to be considered in our ADM series solution.

Table 3. Comparison of our results with those of Ref [22] for two-dimensional flow.

η	f'' Num.	f'' ADM	Error
0.1	0.11826	0.11819	7E-05
0.2	0.22661	0.22648	0.00013
0.3	0.32524	0.32504	0.0002
0.4	0.41446	0.41419	0.00027
0.5	0.49465	0.49431	0.00034
0.6	0.56628	0.56587	0.00041
0.7	0.62986	0.62939	0.00047
0.8	0.68594	0.68538	0.00056
0.9	0.73508	0.73444	0.00064
1.0	0.77787	0.77714	0.00073
1.1	0.81487	0.81405	0.00082
1.2	0.84667	0.84575	0.00092
1.3	0.87381	0.87279	0.00102
1.4	0.89681	0.89567	0.00114
1.5	0.91617	0.91491	0.00126
1.6	0.93235	0.93095	0.0014
1.7	0.94578	0.94423	0.00155
1.8	0.95684	0.95514	0.0017
1.9	0.96588	0.96401	0.00187
2.0	0.97322	0.97117	0.00205
2.2	0.98386	0.981412	0.002448
2.4	0.99055	0.987663	0.002887
2.6	0.99464	0.991293	0.003347

Table 4. Comparison of our results with those of Ref [22] for axisymmetric flow.

η	f'' Num.	f'' ADM	Error
0.1	0.12619	0.12619	0
0.2	0.24239	0.24239	0
0.3	0.34863	0.34863	0
0.4	0.44499	0.44499	0
0.5	0.53160	0.53160	0
0.6	0.60871	0.60871	0
0.7	0.67663	0.67663	0
0.8	0.73577	0.73578	-1E-05
0.9	0.78666	0.78666	0
1.0	0.82987	0.82988	-1E-05
1.1	0.86608	0.86608	0
1.2	0.89598	0.89599	-1E-05
1.3	0.92032	0.92033	-1E-05
1.4	0.93983	0.93984	-1E-05
1.5	0.95522	0.95524	-2E-05
1.6	0.96718	0.96719	-1E-05
1.7	0.97631	0.97632	-1E-05
1.8	0.98316	0.98318	-2E-05
1.9	0.98822	0.98823	-1E-05
2.0	0.99190	0.99188	2E-05
2.2	0.99635	0.99598	0.00037

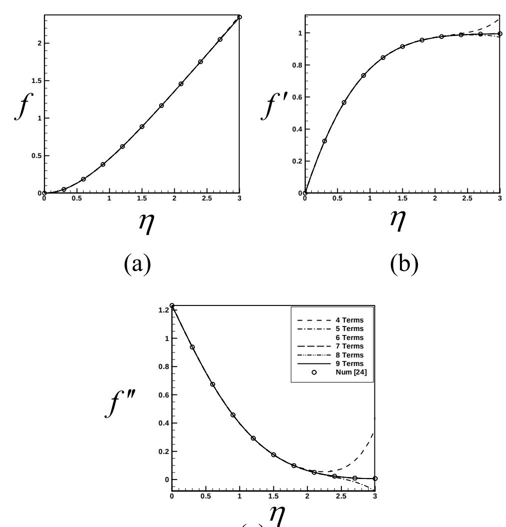

Fig. 2, of solution to the number of ADM terms used, (a) f , (b) f', and (c) f''.

References:

[1]. Adomian, G., "A review of the decomposition method in applied mathematics," J. Math. Anal. Appl., Vol. 135 (1988) 501–544.

[2]. Zhang, X., "A modification of the Adomian decomposition method for a class of non-linear singular boundary value problems," J. Comp. and Appl. Math., Vol. 180 (2005) 377–389.

[3]. Wazwaz, A.M., "The numerical solution of sixth-order boundary value problems by the modified decomposition method," Appl. Math. and Comput., Vol. 118 (2001) 311–325.

[4]. Luo, X.G., "Revisit on partial solutions in the Adomian decomposition method," Appl. Math. and Comput., Vol. 170 (2005) 570–583

[5]. Jafari, H. and Daftardar-Gejji V., "Revised Adomian decomposition method for solving a system of non-linear equations" Accepted for Pub. in Appl. Math. and Comput.

[6]. Zhu, Y., Chang, Q., and Wu, S., "A new algorithm for calculating Adomian polynomials," Appl. Math. and Comput., Vol. 169 (2005) 402–416

[7]. Abbasbandy, S., "Extended Newton's method for a system of non-linear equations by modified Adomian decomposition method," Appl. Math. and Comput., Vol. 170 (2005) 648–656.

[8]. Luo, X.G., "A two-step Adomian decomposition method" Appl. Math. and Comput., Vol. 170 (2005) 570–583.

[9]. Pamuk, S. "an Application for linear and non linear equation by Adomian's decomposition method," Appl. Math. and Comput., Vol. 163 (2005) 89-96.

[10]. Al-Khaled, K., "Numerical solutions of the Laplace equation," Appl. Math. and Comput., Vol. 170 (2005) 1271–1283.

[11]. Saha Ray, S. and Bera, R.K., "Analytical solution of a fractional diffusion equation by Adomian decomposition method," Accepted for Pub. in Appl. Math. and Comput.

[12]. Helal, M.A. Mehanna, M.S., "A comparison between two different methods for solving KdV–Burgers equation" Chaos, Solitons and Fractals, Vol. 28 (2006) 320–326

[13]. Momani, S., "Non-perturbative analytical solutions of the space- and time-fractional Burgers equations," Chaos, Solitons and Fractals, Vol. 28 (2006) 930–937

[14]. Wang, L., "A new algorithm for solving classical Blasius equation" Appl. Math. Comput., Vol. 157 (2004) 1–9.

[15]. Bulut, H., Ergut, M., Asil, V., and Bokor, R.H., "Numerical solution of a viscous incompressible flow problem through an orifice by Adomian decomposition method," Appl. Math. and Comput., Vol.153 (2004) 733–741.

[16]. Fathi, MA,, Kamel, A.K., "An approximation of the analytic solution of the shock wave equation," Accepted for Pub. in Comput. and Appl. Math.

[17]. Soh, C.W., "Non-perturbative semi-analytical source-type solutions of thin-film equation," Accepted for Pub. in Appl. Math. and Comput.

[18]. Pamuk, S. "Solution of the porous media equation by Adomian's decomposition method" Physics Letters A, 344 (2005) 184–188.

[19]. Hashim, I., Noorani, M.S.M., Ahmad, R., Bakar, S.A., Ismail, E.S., and Zakaria A.M., "Accuracy of the Adomian decomposition method applied to the Lorenz system," Accepted for Pub. in Chaos, Solitons and Fractals.

[20]. J. Biazar, J., "Solution of systems of integral–differential equations by Adomian decomposition method," Appl. Math. and Comput., Vol. 168 (2005) 1232–1238.

[21]. Jafari, H. and Daftardar-Gejji V., "Solving a system of non-linear fractional differential equations using Adomian decomposition" Accepted for Pub. in Appl. Math. and Comput.

[22]. White, M.F. "Viscous fluid flow" 2nd Ed., McGraw-Hill, Inc., 1991.

[23]. Wang, L. "A new algorithm for solving classical Blasius equation" Appl. Math. Comput. Vol. 127 (2004)1-9.

[24]. Hashim, I., "Comments on ''A new algorithm for solving classical Blasius equation'' by L. Wang," Accepted for Pub. in Appl. Math. and Comput.