Voronoi Tessellation based Multiscale Data Compression for Sensor Networks

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Abstract: Though several wavelet-based compressing solutions for sensor network have been proposed, those algorithms can only transform the regularly sample data. In this paper, we propose a distributed wavelet-based algorithm which can transform irregularly sample data. Considering the characteristics and location information of nodes in sensor networks, a new distributed data aggregation mode DDAM based on "subnet" is proposed firstly. On the basis of these new models, a novel wavelet-based irregularly sample data compression and data transform model DDWM is proposed for sensor networks. Theoretical analyses and simulation results show that, the above new methods have the good ability of approximation, and can compress the data efficiently and can reduce the amount of data greatly, So, it can prolong the lifetime of the whole network to a greater degree.

Key words: Sensor networks; DDAM; Voronoi tessellation; DDWM;Data compression

1 Introduction

Advances in sensor nodes, which consist of sensing, data processing, communicating and GPS (Global Positioning System) components [1], leverage the idea of wireless sensor networks. Sensor networks represent a significant improvement over traditional sensors, and are regarded as one of the challengeable works of this century [2].

In many practicable applications, The sensors are deployed by plane, assume the plane deploy a number of sensors in an area, which collect data and send to the sink node, A simple and naive design would be for each sensor to just transmit a quantized version of its own measurement to the central node. However, this approach would not be exploiting the fact that measurement originated from spatially close sensors are likely to be correlated, and energy would be wasted with the transmission of redundant data to the central node. As an alternative, since data is correlated, it would be reasonable to try to use some sort of transform in an attempt to decorrelate the information from sensors, and therefore, represent the measurement using fewer bits. However, Such transform requires the nodes to do some inter-computing, this means that extra power would be consumed in the form of local processing, In general, the cost of communication is far more than the cost of local processing, and larger transforms will tend to provide better decorrelation [7]. The key idea is that the overall performance of the system depends mainly on the local processing cost and on the communication cost. Our algorithm takes those cost into account, and has flexibility to chose the right level of transformation for each environment as proposed in [7], and our system would be capable of reducing the overall energy consumption. The most important of all, we proposed the distributed, irregular wavelet transform for sensor networks.

A good deal of prior work has addressed wavelet-based processing in sensor network, but all of them has not resolved the difficulties of working with irregularly-space data while minimize communication overhead to reduce network power consumption. A distributed lifting based data

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compression algorithm has been proposed in [7], decorrelation is achieved by means of exchange information between nodes and its neighbour. The main draw back of the algorithm can only be usd in single hop network, which require the node has the ability of transmitted the data directly to the sink node. An other lifting based algorithm was proposed in [8], which can be used in multi-hop networks. Two algorithms have been proposed in [11], the first is Haar transform multilesolution processing, the second is the Group NetHaar. Dimensions[10]propose an in-network multiscale wavelet transform and hierarchical coefficient routing with its wavRoute protocol but assumes sensor measurements lie on a square grid. While these approach assume regular sensor placement and do not apply to data sampled on irregular grids, it does not have the broad applicability of a wavelet transform to a variety of signal processing applications. It is important to propose the compression and transform algorithm suit to irregularly-spaced data. First, considering location information of node and taking inspiration from the connected dominating sets, we propose a novel distributed data aggregation mode DDAM first, and a novel wavelet-based irregularly sample data compression and data transform model DDWM is proposed then, comparing the previous work, the concept of "area" is proposed in DDAM, and the whole sensor networks was partition to connecting core by the "area", It can reduce the communication cost greatly since all the nodes only need to exchange information with the node in the connecting core. Theoretical analyses and simulation results also show DDWM have the good ability of approximation, and can compress the data efficiently and can reduce the amount of data greatly, So, it can prolong the lifetime of the whole network to a greater degree.

2 Related Work

A large number of wavelet theory suited to regularly-spaced data has emerged in recent decades, but the application of wavelet analysis to irregularly spaced sample data is a relatively challenge. The second-generation wavelet theory such as lifting scheme [13] has risen to replace traditional techniques. The pyramid-based approach of[13] represent some of the first attempts to extending lifting scheme to two dimensions. However, such approaches are typically intended to operate in a centralized fashion on an entire dataset, they are not directly applicable to the sensor network problem.

As stated before, We first construct the distributed data aggregation model-DDAM, and take inspiration from irregularly spaced-smaple data compression by using Voronoi tessellation. Voronoi tessellation is a kind of data structure representing the neighborhood relationship. In general, the basic elements used to partition the space are growth point. The most general Voronoi tessellation is the planar point set p. For every growth point pi in p, there is a region Vi corresponding to it. For any growth pj in Vi, the distance between pj and pi is closer than the distance between pj and other growth point in Vi.So we can treat the sensor node in the connect core which are formed by DDAM as the growth point. According to the property of Voronoi tessellations, the Delaunay triangulation is the geometric dual of the Voronoi polygon. Given the set of of Delaunay edges to which it belongs, a sensor can easily compute the area of its Voronoi cell by simple geometric construction. A method for computing the trangulation of n-node network with an O(nlgn) communication cost is given in [15].Figure 1 to Figure 4 show the process of forming the Voronoi tessellations.







Fig 3 Form Voronoi tessellations by Delaunay triangle

Fig 4 Form Voronoi tessellations

3 Data Aggregation Model-DDAM and Data Compression Model-DDWM

3.1 Distributed Data Aggregation Mode-DDAM

Definition 1 (Simple undirected connected graph): Let graph G = (V, E), where V is the vertices set and E is the edges set, G is called a simple undirected connected graph, if and only if graph G satisfies the following two conditions:

- 1) G has no self circles, and is a undirected connected graph, and
- 2) There is at most one edge between any two nodes in graph G.

Assuming that every node in sensor networks is of the same communication radius, two nodes are called neighbors, if and only if they are in each other's communication radius. If two nodes are neighbors, then there is one link between them. Supposing the link between any two neighboring nodes is symmetric, and then the topology of a sensor network can be regarded as a simple undirected connected graph.

Definition 2 (core): The vertices set $C \subseteq V$ of graph G is called a core, if and only if C satisfies the following conditions: $\forall p \in V \Longrightarrow p \in C$ or p is neighboring to one node $q \in C$.

There are two kinds of nodes in core, one is core head, which if of the responsibility of aggregating data and routing, the other is gateway, which is responsible for routing only.

Definition 3 (connected core): As for graph G=(V, E), if The vertices set $C \subseteq V$ of graph G satisfies the following conditions: the sub graph induced from C is connected, and C is a core of graph G, then C is called a connected core of graph G.

3.1.1 The Subnets model of sensor networks

For any sensor node p, if its location is (x, y), and the sink node R and all of the sensor nodes are of the same available communication radius r, then p can know the subnet that itself belongs to, according to the following definition.

Definition 4 (subnets): For any sensor node p, if its location relative to the sink node R is (x, y), and the sink node R and all of the sensor nodes are of the same available communication radius r, then p is said to belong to the subnet (m, n), if and only if the following formulas (1) and (2) are satisfied:

$$m = [x/\frac{r}{\sqrt{2}}]$$
 -----(1), $n = [y/\frac{r}{\sqrt{2}}]$ -----(2).

Where '|' expresses the division operation, and '[x]' returns the integer that is nearest to but bigger or equal to x.

For example: If r=15 and the location of node A is (40, 40), then $A_x = 4$, $A_y = 4$, so A belongs to the subnet (4, 4). Supposing that the location of node B is (35, 37), then $B_x = 4$, $B_y = 4$, so B belongs to the subnet (4, 4) too. From the above description, it is obvious that the following theorem 1 is satisfied according to the definition 4.

Theorem 1: Supposing that the sink node R and all of the sensor nodes are of the same available communication radius r, then there is unique subnets division for any sensor networks, which satisfies the above two formulas (1) and (2).

Theorem 2: Any two nodes in same subnet are neighbors.

Proof: According to definition 4, it is easy to know that a subnet is a square area that is $\frac{r}{\sqrt{2}}$

long and $\frac{r}{\sqrt{2}}$ wide, then the largest distance between any two nodes is $\sqrt{\left(\frac{r}{\sqrt{2}}\right)^2 + \left(\frac{r}{\sqrt{2}}\right)^2} = r$. So

any two nodes are in each other's available communication radius $r.\Box$

3.1.2 Distributed energy-core generating algorithm (DDAM)

DDAM is operated on simple undirected connected graph G, and defined as follows: For any node p in G:

- 1 p can obtain its location through GPS, and then know the subnet G_p that itself belongs to and its surplus energy E_p .
- 2 *P* exchanges the following information with its neighbors periodically: (i) *p*'s current state S_p , which includes three kinds of state such as core head, gateway and member. (ii) *p*'s surplus energy E_p . (iii) *p*'s subnet G_p . (iv) *p*'s location. After that, every node can obtain its neighbors' information of state, surplus energy, subnet and the direct distances from itself to them.
- 3 Initially, the sink node in *G* is core head, but all sensor nodes belong to members. In every period, *p* computes its new state by the following rules:
 - 1 In the subnet G_p , if there is no core head, then nodes in G_p will select one node, which is of the largest surplus energy, as the core head of the subnet G_p .
 - 2 Otherwise, if p is not a core head or gateway, and p is neighboring to at least one node that belongs to subnet other than G_p , then p is set as a gateway.

Theorem 3: If G=(V, E) is a simple undirected connected graph, then the nodes set $\Psi = \{p \mid p \text{ is } e \in V\}$

a core head or gateway, and $p \in V$ obtained by algorithm DDAM, is a connect core. Ψ is called

a energy core because nodes in Ψ are all of the locally highest surplus energy.

Proof: From theorem 2 and the step 3).(1) of algorithm DDAM, we can know that every node in G is a core head or neighboring to a core head. So Ψ is a core of graph G. Next we will prove that Ψ is connected also.

Let p and q are any two core heads in Ψ , so p, $q \in \Psi$. Since nodes are of the same available communication radius r, in order to simplify the description, we can set r = 1, then the distance between p and q equals to the hops from p to q, written as d(p, q). Since G is connected, then d(p, q) is a limited integer.

1) (a) If $d(p, q) = 1 \Longrightarrow p$ and q are neighbors, so they can reach each other directly. $\Longrightarrow \Psi$ is connected.

(b) If d(p, q) = 2, then there exists a path (p, r, q). Because d(p, q) = 2, it is obvious that p and q are not neighbors. From theorem 2, it is easy to know that p and q belong to different subnets. Since r is neighboring to both p and q, then we can know that r is a gateway or a core head according to the step 3).(2) of algorithm DDAM. So $r \in \Psi \implies \Psi$ is connected.

(c) If d(p, q) = 3, then there exists a path (p, r_1, r_2, q) . Because d(p, q) = 3, it is obvious

that p and q are not neighbors. From theorem 2, it is easy to know that p and q belong to different subnets. Since r_1 is neighboring to both p and r_2 , then we can know that r_1

is a gateway or a core head according to the step 3).(2) of algorithm DDAM. So $r_1 \in \Psi$.

Since $d(r_1, q)=2$, we can know that $r_2 \in \Psi$ according to the previous step (b) of this

proof. $\Rightarrow \Psi$ is connected.

- 2) Supposing that Ψ is connected when d(p, q) = m (m>3).
- 3) If d(p, q) = m+1, then there exists a path $(p, r_1, r_2, ..., r_m, q)$. Because Ψ is a core

of $G \Rightarrow r_2$ is a core head or neighboring to a core head $r \in \Psi \Rightarrow d(p, r) \leq 3$. From the

step 1) of this proof, we can know that p can reach r through Ψ . And since $d(r, q) \le m$, then it is obvious that r can reach q through $\Psi : \Longrightarrow \Psi$ is connected. \Box

3.2 The Data Transform Model and Data Gathering Model

In section 3.1, we get the connect core by running the DDAM, now we discuss the data transform model and data gathering model, we first give the definition as follows:

Definition 5 *partner nodes* Given any two node A and B in the Connect core, if the cost of communication between A and B is fewer than A and other node in the connect core, Then A and B is call the partner nodes

Definition 6(*Voronoi* cell): Assume $S = \{s_1, s_2, ..., s_n\}$ represent n sensor node deployed in two

dimensions space R^2 and a_i is the place of sensor node s_i , then $Cell(s_i) = \prod_{j=1, j \neq i}^n \{x \mid dist(a_i, x) \le dist(a_j, x), \forall x \in R^2\}$ is Voronoi cell of sensor node $s_i.dist(p, q)$

repsent the *Euclidea* distance of *p*, *q*.

Clearly, , Assume $S = \{s_1, s_2, ..., s_n\}$ represent n sensor node deployed in two dimensions space R^2 , then $Cell(s_1), Cell(s_2), ..., Cell(s_n)$ is the *Partition of* R^2 according to definition 8

Definition 7(*Voronoi Tessellation*): Assume $S = \{s_1, s_2, ..., s_n\}$ represent n sensor node deployed in two dimensions space R^2 , then the partition of R^2 defined by $Cell(s_1), Cell(s_2), ..., Cell(s_n)$ is the *Voronoi tessellation* of S.

Fig 5 show the result of forming the Voronoi tessellation by some sensor nodes in R^2





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Fig6 a The cluster have 7 sensors b data flow in the cluster c data flow from cluster to super cluster
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We get the connect core by the set of node Ψ according theorem 3, the data compression mode and the data gathering model are built on it. Such a transform model must be well matched to the routing protocol employed in the networks. For this purpose, we first propose a new distributed hierarchical routing, then a new distributed irregular wavelet transform related to the haar wavelet basis is proposed on this hierarchical routing.

Such a hierarchical routing topology will typically consist of clusters of sensors. so, we treat the connect core as cluster, and chose the node with the most energy as the cluster head, then found all the partner nodes in the connect core, For instance in Fig 6(a), there are 7 sensor nodes deployed in irregular space and formed into a cluster. Without loss general, we chose node 5 as the cluster head, and then chose partner nodes such as 1 and 2,3 and 4. As for the partner nodes, the node closer to the cluster head sends their measurements to the nodes far away from the cluster head. Then the node closer the head compute the HP and LP coefficients and send the HP to cluster head, while the LP coefficient is sent to the next level transform by routing hierarchy. For example, in Fig 6(b) node 1 send measurement to node 2, node 3 send measurement to node 4. After compute the HP and LP

coefficient in node 2 and 4, node 2 send the HP coefficient to node 5, while LP coefficient is sent to node 4 for next level transform. Until all LP coefficient flowing up to cluster head.

The cluster head stores all the HP coefficient, but the LP coefficient flow up to the next level of transform by the routing hierarchy as show in figure 6(c), clusters c_1 through cluster c_3 are grouped together to form a super cluster s c_1 . For every super cluster, we also chose the super cluster head, all the cluster head send the LP coefficient along with the area of the cluster to the super cluster head for next level transform. Then the super cluster head compute the scaling coefficient and the wavelet coefficient using the LP coefficient which is sent by the lower lever cluster head, and then send the scaling coefficient to the sink node. As for the head of the super cluster, it performs double duty. As for the cluster head, it store the set of HP coefficient, and as for the super cluster head, it store the set of wavelet coefficient.

Given this transform model, we can discuss the details of the issue of data gathering .The piecewise-constant signal is adopted in the haar wavelet in many regular settings. We adopt a similar piecewise-constant model for the data gathered in the network, assume that measurements hold constant over the Voronoi cells surrounding each sensors. We assume that each sensor in the cluster know the area of Voronoi cell surrounding the sensor. Given the set of areas and measurements, we can conceptual map the 2-D transform problem to 1-D as shown in Figure7. Where sensor measurements are taken to constant over intervals whose lengths correspond to sensor's Voronoi cell areas.



Fig7 the mapping of piecewise-constant function over 2-D areas to 1-D intervals

The method to computer LP and HP coefficient based on piecewise-constant signal in regular spaced sample data has been proposed in[11], which is over the intervals [0,0.5] and [0.5,1], and the scaling function merely sums measurements in the two regions while the wavelet function differences those measurements. As for the irregular settings, we must consider both the measurement and the area of the Voronoi cell. Similar to the LP and HP of piecewise-constant signal, we present a technique and form a set of basis functions which span the piecewise-constant measurement in a cluster of sensors as shown in Equation (1).

$$f_{1} = \frac{1}{s_{1} + s_{2}} \sqrt{\frac{s_{2}}{s_{1}}}, f_{2} = -\frac{1}{s_{1} + s_{2}} \sqrt{\frac{s_{1}}{s_{2}}}, f_{0} = \sqrt{\frac{1}{s_{1} + s_{2}}}$$
(1)

$$LP = (m_{1}s_{1} + m_{2}s_{2}) \sqrt{\frac{1}{s_{1} + s_{2}}}, \quad HP = \frac{m_{1}s_{1}}{s_{1} + s_{2}} \sqrt{\frac{s_{2}}{s_{1}}} - \frac{m_{2}s_{2}}{s_{1} + s_{2}} \sqrt{\frac{s_{1}}{s_{2}}}$$
(2)

As for every partner nodes in the given cluster, Using the f0,f1 and f2 expressions, it is easily verified that the pair of basis functions are mutually orthogonal and have unit-norm, and span the field of functions constant over the s1 and s2. Pairing measurements to generate HP and LP coefficients according to Equation 2 give us roughlyN/2 LP coefficient over the cluster of N

sensors. Note that to fit into our transform model, we must describe the whole cluster with a single LP value for transport to the next layer of the hierarchy. To do so, we merely iterate the pairing and transform process as show on Figure 6(b).Given pairings that form the binary tree in Figure 6(b) and repeated application of Equation (2) to the partners, we can form a set of N-1 HP coefficients and a LP coefficient for a cluster of N sensors. Note during the iterating operation, we must input the correspond areas of the Voronoi cells which surrounding the partner nodes. When the iterating operation get to the root node in Figure 6(b), the cluster head send the LP value together with the area of the whole cluster to the super cluster head. Thus there is a set of LP coefficients and areas in the super cluster head. Since the formula (2) can not fit for the super cluster. We give another formula (3) for computing the LP and HP of super cluster.

$$LP = \frac{\sum_{i=1}^{n} s_i m_i}{\sqrt{\sum_{i=1}^{n} s_i}}, \quad HP_i = \frac{\sqrt{s_i}}{\sum_{i=1}^{n} s_i} \left\{ \sum_{j=i+1}^{n} s_j m_j + \sum_{j=i+1}^{n} s_j m_i \right\}$$
(3)

Once we get the formulation of computer the LP and HP coefficient, we can implement each transform as detailed in Algorithm 1 and 2, which respectively provide pseudo code for the compression of cluster and the super cluster

Algorithm1 MultiscaleCprs(measurements, areas)

Input: measurements and the areas of Voronoi cell

Output LP and the areas of the cluster

- 1. if the number of measurements is 1 then
- 2. return the measurements and areas
- 3. newMS $\leftarrow \phi$ newAR $\leftarrow \phi$
- 4. $Match(areas) \rightarrow Partners //found the partner node by the area of voronoi cell which surrounding the sensors$
- 5. for each partner in Partners do
- 6. computer HP and LP according formula (2)
- 7. LP \rightarrow newMS

п

- 8. sum(partner areas) \rightarrow newAR
- 9. end for
- 10. Multiscale(newMS, newAR)

Algorithm2 superCompress(LPs,areas)

Input LPs, areas// the LPs and areas are the LP coefficient and area of the cluster which in the super cluster Output LP HP_i

- 1. computer LP and H P_i according formula (3)
- 2. return LP and HP_i

4 Simulation and Results

To evaluate the effectives of our proposed transform, we randomly assign 3600 sensor nodes on the square. Sensors are assigned to 5-level hierarchy, with 600,100,20,4 clusters in each of the fine to coarse level and 1 root cluster. The hierarchy is allowed to form randomly. We adopt two

simulated sensor measurement fields which were used in in [16] as shown in Figure 8 and Figure 9. The first consists of a piecewise-constant smooth quadratic signal, the second of a noisy quadratic with a discontinuity. Figure 10 and Figure 11 display the nonlinear approximation results for each. Mean square approximation error on a log scale is plotted on the y-axis with the number of coefficients used in the approximation. Both curves exhibit the smooth rapidly decreasing decay as we expacted, which indicate the transform model have the good performance of the reconstruction . Second, as a sanity chick, we also compare the performance of our proposed transform in the regular setting to transforms using regular Daubechies-2, Daubechies-4, Daubechies-6, and Daubechies-8 as done in [16]. We compare the approximation curves, as shown in Figure 13.





Fig8 smooth quadratic piecewise-planar signal







Fig10approximation-error versus coefficient count curves in smooth quadratic Fig11 approximation-error versus coefficient count curves in noisy quadratic with discontinuity





Fig12the comparison of average energy cost

Fig13 comparison of DDWM with D-2,D-4,D-6,D-8in a regular setting

Though our transform is out-performed by the Daubechies wavelets, as we expected, their performance is clearly comparable. It indicates our transform are capable of performing well in both the regular and irregular domains, while Daubechies wavelet cannot make. Third, In order to test the performance of the data compression, we provide a comparison of the average energy cost of the node in the network as shown in Figure 12. After running the transform, the average energy cost of node change from 4.48 to 2.36. So the transform can effectively reduce the average energy cost of node and prolong the lifetime of the whole network to a greater degree.

5 Conclusion

Based on Voronoi tessellation, We have developed a fully distributed, irregular-grid wavelet transform and protocol for sensor networks that is capable of piecewise-constant multiscale approximation. Theoretical analysis and simulation results show that the transform model has good performance of nonlinear approximation, and can reduce the amount of transmitted data in the sensor networks. In the future work, we intend to accommodate higher orders of approximation than piecewise constant, enabling to greater data compression.

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