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Applying the Sum of Correlated Gamma Variates in Approximate Error Probability Evaluation for MC-DS-CDMA Systems Operating in Frequency Selective Fading Channels

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Abstract. In this paper, the system performance of an MC-DS-CDMA (multi-carrier direct-sequence coded-division multiple-access) system operating in Nakagami-*m* fading channels with correlation characteristics is studied. In order to avoid the difficulty of explicitly obtaining the pdf (probability density function) for the SNR (signal-to-noise ratio) at the output of Rake receiver, an expression of distribution with the sum of Gamma varieties is applied in the derivation of closed form for system performance with BER (bit error rate) performance. Two cases, single-user and multiple-user exemplified for validating the fact that the performance degradation of the MC-DS-CDMA is sensitive to the correlation coefficient between the fading branches. Besides, the assumption of PBI (partial band interference) between the subcarriers is considered in this study.
Index Terms: Correlated Nakagami-*m* Channels, Gamma Varieties, MC-DS-CDMA system, RAKE receiver

1. Introduction

It is known that the effects of ISI (inter-symbol interference) and fading occurring at the transmission channel are two major interference sources for wireless communication systems. During the past 10 years, a number of multi-carrier modulation schemes have been proposed to overcome the aforementioned drawbacks [1]. The main objective for the future generation of wideband wireless communication systems is to support a wide area of services and a high data rate by means of a variety of techniques capable of achieving the highest possible spectrum efficiency. Hence, how to solve the problems resulting from channel fading has become a very important issue in wireless radio systems.

The CDMA (coded-division multiple-access) developed by spread-spectrum techniques has been adopted as an attractive multiple-access scheme in 3G (third-generation) wireless systems. Generally speaking, multi-carrier DS systems can be categorized into two types: (1) a combination of OFDM (orthogonal frequency division multiplexing) and CDMA, and (2) a parallel transmission-scheme of narrowband DS waveform in the frequency domain [2]. Both aforementioned modulation methods have been dedicated to analysis by combining them with several varieties of assumptions. In [3], the researchers, in order to obtain the average BER (bit error rate) performance for an MC-DS-CDMA system, employed three methods to approximate the pdf (probability density function) of the sum of i.i.d. (independent identical distribution), the Rayleigh random variable. In [4], the researchers evaluated the system performance of an MC-DS-CDMA system with MRC (maximal ratio combining) over a Rayleigh fading channel. The performance of an MC-CDMA with correlated envelopes was not only analyzed by Q. Shi, and M. Latva-aho [5], but the researchers also presented the effect of the correlated phases. Performance analyses of MC-CDMA and the MC-DS-CDMA systems operating in the presence of correlated Rayleigh fading channels were calculated, by T. Kim, et al [6], and W. Xu, and L. B. Milstein [7], respectively. Recently, the publication cited in [8] evaluated the performance of an MC-DS-CDMA system with partial band interference working in Nakagami-*m* fading channels. The same author in [9], assuming that the MC-CDMA system works in a correlated Nakagami-*m* fading environment, evaluated the average BER performance in [9]. Recently, L. –L. Yang, and L. Hanzo [10] investigated the spacing between two adjacent subcarriers of the generalized MC-DS-CDMA system over Nakagami-*m* fading channels with the BER performance. Their results given that the best BER for the MC-DS-CDMA system will be obtained after the optimum subcarrier spacing and the orthogonal between the subcarrier can be kept. The characteristics of correlated and independent subcarrier for MC-CDMA system over frequency have been studied in [11], in which the authors evaluated the average BER of an uplink MC-CDMA system with MRC reception, and proposed the relationship between the correlation of the subcarries and the fading parameters.

In this paper we aim on the evaluation of the performance of MC-DS-CDMA system which is assumed working over a correlated Nakagami-*m* fading channel. A new proposed approach, that is, the sum of correlated Gamma variates is adopted to avoid the difficulty of explicitly obtaining the pdf for the SNR (signal-to-noise ratio) at the output of MRC scheme. The paper is organized as follows. The MC-DS-CDMA system models are described in section 2. Analytical expressions of BER performance for MC-DS-CDMA in correlated Nakagami-*m* channels is derived in section 3. The numerical results from adopting the examples with single and multiple-user are presented in section 4. There is a brief conclusion was drawn in section 5.

2. System Models

2.1 Transmitter Model

In Fig. 1, we propose the block diagram of the transmitter for a MC-DS-CDMA system in which a unique spreading sequence is assumed to serve for each user. and each of the active users employs M subcarrier and binary phase shift keying (BPSK) modulation. The overall bandwidth of a MC-DS-CDMA system with all the subcarrier is given by $BW_M = (1+\mu)/MT_c$, where $0<\mu\le 1$ is roll-off factor, M is number of sub-carrier, and T_c is the chip duration. From the points described above, the total bandwidth of the MC-DS-CDMA system of the *k*-th user can be counted as $BW_T = (1+\mu)/T_c$. The transmitted signal of a MC-DS-CDMA system of the *k-th* user shown in Fig.1, can be written as [7]

$$s_k(t) = \sqrt{2E_c}\sum_{n=-\infty}^{\infty} c_{k,n} d_{k,h} h(t-nMT_c-\tau_k)\sum_{i=1}^{M}\mathrm{Re}\left[e^{j(2\pi f_i t+\theta_{k,i})}\right] \quad (1)$$

where E_c is the chip energy, $c_{k,n}$ is the pseudo-random spreading sequence of the *k-th*, $d_{k,\lfloor n/N\rfloor}\in\{+1,-1\}$ denotes the data bit of the *k-th* user, where N indicates the length of PN-sequence, *h(t)* is the impulse response of the chip wave shaping filter, τ_k is an arbitrary time delay uniformly distributed over $[0, NMT_c]$, $\mathrm{Re}[\cdot]$ denotes the real part, $\theta_{k,i}$ and f_i'$s, i=1,2,...,M$ are a random carrier phase uniformly distributed over $(0,2\pi]$ and the carrier frequency, respectively.

2.2 Receiver Model

The receiver block diagram of a MC-DS-CDMA system with BPF (band-pass filter) and LPF (low pass filter) filter is illustrated In Fig. 2. The complex lowpass equivalent impulse response of the *i-th* channel is $\{c_i = \xi_i\cdot\delta(t),\ \text{i=1...M}\}$, and $\xi_{k,i} = \alpha_{k,i}\exp(j\beta_{k,i})$, where $\alpha_{k,i}$ and $\beta_{k,i}$ correspond to represent attuation factor and phase-shift for *i-th* channel of the *k-th* userThe complex equivalent impulse response of the channel is expressed as $c(t)=\sum_{l=0}^{L-1}\alpha_l\delta(t-lT_c)$. The received signal at the receiver is given as [4]

$$r(t)=\sum_{k=1}^{K}\left\{\sqrt{2E_c}\sum_{n=-\infty}^{\infty} d_{k,h}c_{k,n}h(t-nMT_c-\tau_k)\times\sum_{i=1}^{M}\alpha_{k,i}\cos(2\pi f_i t+\psi_{k,i})\right\}+N_w(t)+N_J(t) \quad (2)$$

where K denotes the user number, $\psi_{k,i}=\theta_{k,i}+\beta_{k,i}$, $N_w(t)$ is AWGN with a double sided PSD（power spectral density）of $\eta_0/2$, $N_J(t)$ is partial band Gaussian interference with a PSD of $S_{n_J}(f)$, which is written as

$$S_{n_J}(f)=\begin{cases}\frac{\eta_J}{2}, & f_J-\frac{W_J}{2}\le|f|\le f_J+\frac{W_J}{2}\\ 0, & otherwise\end{cases} \quad (3)$$

where f_J and W_J represent the bandwidth of the interference and the center frequency, respectively. Then the interference (Jamming)-to-signal ratio, JSR, is defined as the ratio of the interference power value to signal power, and can be written as

$$\text{JSR} = \frac{\eta_J W_J}{\frac{E_b}{T}} = (1+\mu)\frac{\eta_J}{E_b}\frac{N}{M} \quad (4)$$

The output from the chip-matched filter in the branch ζ_i is give by [4]

$$\zeta_i = D_{\zeta_i} + MAI_{\zeta_i} + JSR_{\zeta_i} + N_{\zeta_i} \quad (5)$$

where the first term of the last equation denotes the desired signal of the reference case can be written as

$$D_{\zeta_i}(t) = \sqrt{E_c}\alpha_{1,i}\sum_{n=-\infty}^{\infty} d_{1,h}c_{1,n}x(t-nMT_c) \quad (6)$$

the second term in (5) is the interference comes from the other users, callas the MAI (multiple access interference), when the user number K approximates as Gaussian random variable, and can be determined as

$$MAI_{\zeta_i}(t) = \sum_{k=2}^{K}\left\{\sqrt{E_c}\xi_{k,i}\sum_{n=-\infty}^{\infty} d_{k,h}c_{k,n}\cdot x(t-nMT_c-\tau_k)\right\} \quad (7)$$

where $\xi_{k,i} \equiv \alpha_{k,i}\cos\phi_{k,i}$ and is i.i.d. (identical independent distribution) Gaussian, $\phi_{k,i} = \psi_{k,i} - \psi_{k,1}$. The third term in (5) is the JSR defined in (3), can be represented as

$$JSR_{\zeta_i}(t) = LPF\left[\sqrt{2}n'_{i,j}(t)\cos(2\pi f_i t + \psi_{1,i})\right] \quad (8)$$

where $LPF[\cdot]$ is applied to express the function of LPF, and the last term of (5) indicates the output signal caused by the fact that the AWGN passes to the low pass filter, and which can be expressed as

$$N_{\zeta_i}(t) = LPF\left[\sqrt{2}n'_{w,i}(t)\cos(2\pi f_i t + \psi_i^{(1)})\right] \quad (9)$$

where the terms $n'_{i,j}(t)$ in (8) and $n'_{w,i}(t)$ in (9) results from passing $n_J(t)$ and $n_w(t)$ in (2), respectively, through the i-th bandpass filter. It is necessary to evaluate the SNR (signal-to-noise ratio) at the output of the receiver for the reference user such that the system performance can be determined. Thus all of the statistics results of the signal at the output of the i-th correlator are to be determined and expressed as

$$\chi_i = D_{\chi_i} + MAI_{\chi_i} + JSR_{\chi_i} + N_{\chi_i} \quad (10)$$

where each terms shown in last equation is adopted as that of the same results evaluated and shown in [4].

2.3 Correlated Nakagami-m Channel

The sum of correlated Gamma variates [12] is going to be introduced briefly in his subsection. Let $\{q_i\}_{i=1}^{L}$ be a set of L correlated not necessarily identically distributed gamma variates parameters with m and Ω_i, respectively, [i.e., $q_i \sim G(m,\Omega_i)$] and let { ρ_{ij} , $i,j=1,2,...,L$ } denotes the correlation coefficient between the branch of q_i and q_j, where $i \neq j$, i.e.

$$\rho_{ij} = \rho_{ji} = \frac{Cov(q_i,\, q_j)}{\sqrt{Var(q_i)Var(q_j)}},\ 0 \le \rho_{ij} \le 1,\ \ i,j=1,2,...,L \quad (11)$$

where $Var(\cdot)$ and $Cov(\cdot)$ are the variance and the covariance operators, respectively. The pdf of $\gamma = \sum_{i=1}^{L} q_i$ can be expressed as

$$p_\gamma(\gamma) = \prod_{i=1}^{L}\left(\frac{\lambda_1}{\lambda_i}\right)^m \sum_{k=0}^{\infty}\frac{\delta_k \gamma^{Lm+k-1}e^{-\gamma/\lambda_1}}{\lambda_1^{Lm+k}\Gamma(Lm+k)}U(\gamma) \quad (12)$$

where $\Gamma(\cdot)$ is the gamma function and $U(\cdot)$ is the unit step function. The $\lambda_1 = \min_i\{\lambda_i\}$, $\{\lambda_i\}_{i=1}^{L}$ are the eigenvalues of the matrix $A=DC$, where the D is a $L\times L$ diagonal matrix with the entries $\{\Omega_i\}_{i=1}^{L}$, and C is an $L\times L$ positive definite matrix defined by

$$C = \begin{bmatrix} 1 & \rho_{12}^{1/2} & \cdots & \rho_{1L}^{1/2} \\ \rho_{21}^{1/2} & 1 & \cdots & \rho_{2L}^{1/2} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{L1}^{1/2} & \cdots & \cdots & 1 \end{bmatrix}_{L\times L} \quad (13)$$

where ρ_{ij} , $i,j=1,2,...,L$ were expressed in (11), and the recursive parameter , δ_k, in (12) can be calculated by the formula shown as

$$\begin{cases} \delta_0 = 1 \\ \delta_{k+1} = \dfrac{m}{k+1}\displaystyle\sum_{i=1}^{k+1}\left[\sum_{j=1}^{L} m_j\left(1-\frac{\lambda_1}{\lambda_j}\right)^i\right]\delta_{k+1-i}, \quad k=0,\ 1,\ 2,......, \end{cases} \quad (14)$$

where $\{\lambda_i\}_{i=1}^{L}$ are the eigenvalues of the matrix $A=DC$, and $\lambda_1 = \min[\lambda_i]$ [12].

3. Performance Analysis

The conditional mean of χ_i shown in (10), condition upon the channel attenuation factor $\alpha_{1,i}$ are given by

$$E\left[\chi_i \mid \alpha_i, d_{1,h}\right] = E\left\{\chi_i \mid \alpha_{1,i}, \{d_{1,h}\}\right\}$$
$$= \sqrt{E_c}\alpha_{1,i}\sum_{n'=0}^{N-1}\sum_{n=-\infty}^{\infty} d_{1,h}c_{1,n}c_{1,n'}\cdot x[(n'-n)MT_c]$$

$$= \pm N\sqrt{E_c}\alpha_{1,i} \tag{15}$$

Note that the $x\left[(n'-n)NT_c\right]=0$ for $n' \neq n$. The conditional variance of χ_i can be represented as

$$\begin{aligned} Var\{\chi_i \mid \alpha_{k,1}\} &\equiv \sigma_i^2 \\ &= Var\{MAI_{\chi_i} + JSR_{\chi_i} + N_{\chi_i} \mid \alpha_{1,i}\} \\ &= Var\{MAI_{\chi_i}\} + Var\{JSR_{\chi_i}\} + Var\{N_{\chi_i}\} \end{aligned} \tag{16}$$

where the results of each terms shown in (15) can be calculated as given in [4].

All signals at the output of the correlators are combined with the MRC scheme, and the result can be expressed as

$$\chi = \sum_{i=1}^{M} G_i \chi_i \tag{17}$$

where G_i is defined as the channel estimate of the *i-th* branch. In order to maximize the SNR, the channel estimate G_i is defined as the ratio of the desired signal amplitade to the variance of the noise and interferance components in the output, and is written as

$$G_i = \frac{E\{\chi_i \mid \alpha_{1,i}\}}{Var\{\chi_i \mid \alpha_{1,i}\}} \tag{18}$$

By combining (15) with (16), then the SNR, (S/N), at the output of the MRC, can be obtained as

$$\left(\frac{S}{N}\right) = \frac{E^2\{\chi_i \mid \alpha_1\}}{Var\{\chi_i \mid \alpha_1\}} \equiv N^2 E_c \gamma \tag{19}$$

where the reference user (1st user) is considered, and

$$\gamma = \sum_{i=1}^{M} \frac{(\alpha_{1,i})^2}{\sigma_i^2} \equiv \sum_{i=1}^{M} q_i \tag{20}$$

where the fading branch of the reference user $\{\alpha_{1,i}, i=1,...,M\}$ are modeled as correlated-Nakagami-*m* statistic. Therefore, it can be shown that γ has the Gamma pdf [13], and the pdf of γ is expressed in (12). By using of averaging conditional pdf of SNR as shown in (21) over the pdf of the correlated channel, the BER is approximately determined by

$$P_e^{case} = \int_0^{\infty} \phi(\sqrt{N^2 E_c \gamma_{case}}) f_\gamma(\gamma_{case}) d\gamma_{case} \tag{21}$$

For the differentiable reason of system performance analysis for different cases, include single user and multiple user cases, the γ_{case} in (21) is going to be replaced with the corresponding cases by means of the exact subscript. For example, γ_{mu-sc} represents the SNR of multi-user case with single carrier, while γ_{su-mc} indicates the SNR of single-user case with multi-carrier. Similarly, the means will be employed for the symbol, P_e^{case}, of average BER. The average BER of those cases will be illustrated in the next subsection, respectively. The $\phi(x)$ in (21) is the Gaussian Q-function and defined as

$$\phi(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-\frac{j^2}{2}} dt = \frac{2}{\sqrt{\pi}} e^{-x^2} \sum_{j=0}^{\infty} \frac{2^j x^{2j-1}}{(2j-1)!!} \tag{22}$$

3.1.Multi-user case

3.1.1. Multi-carrier

First, we consider a multi-user case with multi-carrier, and assume that the correlation between the spread codes is adjustable as $R_{I_i}(\ell M T_c) = \sum_{n'=\ell}^{N-1} C_{1,n'} C_{1,n-l} = 0$, such that the autocorrelation function, $R_{I_i}(0)$, which can be obtained from [4] and represented as

$$\begin{aligned} R_{I_i}(0) &= \int_{-\infty}^{\infty} S_{I_i}(f) df \\ &= \frac{(K-1)E_c}{2}\left(1-\frac{\mu}{4}\right) \end{aligned} \tag{23}$$

Thus, the conditional SNR, γ_{mu-mc}, of the multiple user case with multi-carrier at the output of the receiver can be determined from (19), which is written as

$$\begin{aligned} \gamma_{mu-mc} &= N^2 E_c \sum_{i=1}^{M} \frac{(\alpha_{1,i})^2}{\frac{(K-1)NE_c}{2}\left(1-\frac{\mu}{4}\right) + \frac{N\eta_0}{2}} \\ &= \left\{\frac{K-1}{2MN}\left(1-\frac{\mu}{4}\right) + \frac{\eta_0}{2MNE_c}\right\}^{-1} \frac{1}{M}\sum_{i=1}^{M}(\alpha_{1,i})^2 \end{aligned} \tag{24}$$

where N representes the chip number per symbol for the multi-carrier case, and $\eta_0/2$ is a double-sided PSD of the AWGN. For the purpose of calculating the system BER formula for this case, P_e^{mu-mc}, by substituting (24) into (21), and it can be obtained as (see the Appendix)

$$\begin{aligned} P_e^{mu-mc} = \sqrt{\frac{2}{\pi^{1/2}}} \sum_{j=0}^{\infty} & \frac{2^{j/2}\left(\left\{\frac{K-1}{2MN}\left(1-\frac{\mu}{4}\right)+\frac{\eta_0}{2MNE_C}\right\}^{-1}\frac{1}{M}\right)^{j-1/2}}{\sqrt{(2j-1)!!}} \\ & \times \prod_{i=1}^{L}\left(\frac{\lambda_1}{\lambda_i}\right)^m \sum_{k=0}^{\infty} \frac{\delta_k}{\lambda_1^{Lm+k}\Gamma(Lm+k)} U(\gamma) \\ & \times \frac{\Gamma(j-1/2+Lm+k)}{\left(1+\left\{\frac{K-1}{2MN}\left(1-\frac{\mu}{4}\right)+\frac{\eta_0}{2MNE_C}\right\}^{-1}\frac{1}{\lambda_1 M}\right)^{(j-1/2+Lm+k)}} \end{aligned} \tag{25}$$

where δ_k is defined in (14), $U(\gamma)$ is the unit step function, and $MNE_c = N_1 E_{c1} = E_b$, where N_1

and E_{c1} are length and energy of the spread code, respectively, E_b denotes the bit energy, and $(2n+1)!! = 1\cdot 3\ldots(2n+1)$.

3.1.2. Single-carrier

Similarly, the conditional SNR, γ_{mu-sc} , of a single-carrier RAKE receiver, can be determined as

$$\gamma_{mu-sc} = \left\{\frac{K-1}{2N_1}\left(1-\frac{\mu}{4}\right)+\frac{\eta_0}{2N_1E_{c1}}\right\}^{-1}\sum_{i=1}^{M}\left(\alpha_i^{(1)}\right)^2 \tag{26}$$

where the symbol of the length and the chip energy of the spreading sequence are replaced with the symbols N_1 , and E_{c1}, respectively. The system BER, P_e^{mu-sc} , of this case can be obtained as

$$P_e^{mu-mc} = \frac{\sqrt{2}}{\pi}\sum_{j=0}^{\infty}\frac{2^{j/2}\left(\left\{\frac{K-1}{2N_1}\left(1-\frac{\mu}{4}\right)+\frac{\eta_0}{2N_1E_{c1}}\right\}^{-1}\right)^{j-1/2}}{\sqrt{(2j-1)!!}}$$
$$\times\prod_{i=1}^{L}\left(\frac{\lambda_1}{\lambda_i}\right)^m\sum_{k=0}^{\infty}\frac{\delta_k}{\lambda_1^{Lm+k}\Gamma(Lm+k)}U(\gamma) \tag{27}$$
$$\times\frac{\Gamma(j-1/2+Lm+k)}{\left(1+\left\{\lambda_1\left[\frac{K-1}{2N_1}\left(1-\frac{\mu}{4}\right)+\frac{\eta_0}{2N_1E_{c1}}\right]\right\}^{-1}\right)^{(j-1/2+Lm+k)}}$$

3.1.3. Multi-carrier and PBI

The conditional SNR, $\gamma_{mu-mc-PBI}$, of a multi-carrier with PBI can be determined from (19) and expressed as

$$\gamma_{mu-mc-PBI} = \left\{\frac{K-1}{2MN}\left(1-\frac{\mu}{4}\right)+\frac{\eta_0}{2MNE_c}+\frac{\eta_J}{2MNE_c}\right\}^{-1}\frac{1}{M}\sum_{i=1}^{M}\left(\alpha_{1,i}\right)^2 \tag{28}$$

where η_J represents the JSR defined in (4). By using of the same steps as that of the derived results shown in (31), and the system BER under this assumption, $P_e^{mu-mc-PBI}$, can be determined as

$$P_e^{mu-mc-PBI} = \frac{\sqrt{2}}{\pi}\sum_{j=0}^{\infty}\frac{2^{j/2}\left(\left\{\frac{K-1}{2MN}\left(1-\frac{\mu}{4}\right)+\frac{\eta_0}{2MNE_C}+\frac{\eta_J}{2MNE_c}\right\}^{-1}\frac{1}{M}\right)^{j-1/2}}{\sqrt{(2j-1)!!}}$$
$$\times\prod_{i=1}^{L}\left(\frac{\lambda_1}{\lambda_i}\right)^m\sum_{k=0}^{\infty}\frac{\delta_k}{\lambda_1^{Lm+k}\Gamma(Lm+k)}U(\gamma)$$
$$\times\frac{\Gamma(j-1/2+Lm+k)}{\left(1+\left\{\frac{K-1}{2MN}\left(1-\frac{\mu}{4}\right)+\frac{\eta_0}{2MNE_C}+\frac{\eta_J}{2MNE_c}\right\}^{-1}\frac{1}{\lambda_1 M}\right)^{(j-1/2+Lm+k)}}$$

(29)

3.2.Single-user case

3.2.1. Multi-carrier

Next, the conditional SNR of single-user and multiple-carrier case, γ_{su-mc} , at the output of the receiver can be calculated with the same way as that adopted in the case of multiple user's, and the values of the user number will be substituted by *K=1* into (26). The conditional SNR, γ_{su-mc} , becomes as

$$\gamma_{su-mc} = N^2E_c\sum_{i=1}^{M}\frac{(\alpha_{1,i})^2}{\sigma_i^2} = \frac{2NE_c}{\eta_0}\sum_{i=1}^{M}(\alpha_{1,i})^2 \tag{30}$$

By following the same steps as shown in (25), we can obtain the system BER, P_e^{su-mc} , for single-user and multi-carrier case as

$$P_e^{su-mc} = \frac{\sqrt{2}}{\pi}\sum_{j=0}^{\infty}\frac{2^{j/2}\left(1+\frac{2NE_c}{\lambda_1\eta_0}\right)^{j-1/2}}{\sqrt{(2j-1)!!}}$$
$$\times\prod_{i=1}^{L}\left(\frac{\lambda_1}{\lambda_i}\right)^m\sum_{k=0}^{\infty}\frac{\delta_kU(\gamma)}{\lambda_1^{Lm+k}\Gamma(Lm+k)}\frac{\Gamma(j-1/2+Lm+k)}{\left(1+\frac{2NE_c}{\lambda_1\eta_0}\right)^{(j-1/2+Lm+k)}} \tag{31}$$

3.2.2. Single- carrier

Similarly, the conditional SNR of a single-carrier RAKE receiver, γ_{su-sc} , is given as

$$\gamma_{su-sc} = \frac{2N_1^2E_{c1}}{\eta_0}\sum_{i=i}^{L}(\hat{\alpha}_{1,i})^2 \tag{32}$$

where L is the number of resolvable paths of the channels. Note that the parameter has been set as (27) $MNE_c = N_1E_{c1} = E_b$ in the last equation. Thus the average BER, P_e^{su-sc} , of single-user and single-carrier can be determined as

$$P_e^{su-sc} = \sqrt{\frac{2}{\pi^{1/2}}}\sum_{j=0}^{\infty}\frac{2^{j/2}\left(\frac{2N_1^2E_{c1}}{\eta_0}\right)^{j-1/2}}{\sqrt{(2j-1)!!}}$$
$$\times\prod_{i=1}^{L}\left(\frac{\lambda_1}{\lambda_i}\right)^m\sum_{k=0}^{\infty}\frac{\delta_kU(\gamma)}{\lambda_1^{Lm+k}\Gamma(Lm+k)}\frac{\Gamma(j-1/2+Lm+k)}{\left(1+\frac{2NE_c}{\lambda_1\eta_0}\right)^{(j-1/2+Lm+k)}} \tag{33}$$

3.2.3. Multi-carrier with PBI

When the effect of PBI is considered, the conditional SNR, $\gamma_{su-mc-PBI}$, of multi-carrier and single- user case can be written as

$$\gamma_{su-mc-PBI} = \left(\frac{2MNE_c}{\eta_0}+\frac{2MNE_c}{\eta_J}\right)\frac{1}{M}\sum_{i=1}^{M}\left(\alpha_{1,i}\right)^2 \tag{34}$$

The system BER , $P_e^{su-mc-PBI}$, is also can be determined by the same procedures of the last case, and obtained as

$$P_e^{su-mc-PBI} = \sqrt{\frac{2}{\pi^{1/2}}} \sum_{j=0}^{\infty} \frac{2^{j/2}\left(\left(\frac{2MNE_c}{\eta_0}+\frac{2MNE_c}{\eta_J}\right)\frac{1}{M}\right)^{j-1/2}}{\sqrt{(2j-1)!!}} \times \prod_{i=1}^{L}\left(\frac{\lambda_1}{\lambda_i}\right)^{m} \sum_{k=0}^{\infty} \frac{\delta_k U(\gamma)}{\lambda_1^{Lm+k}\Gamma(Lm+k)} \times \frac{\Gamma(j-1/2+Lm+k)}{\left(1+\left(\frac{2MNE_c}{\eta_0}+\frac{2MNE_c}{\eta_J}\right)^{-1}\frac{1}{\lambda_1 M}\right)^{(j-1/2+Lm+k)}} \quad (35)$$

Since the reason of editorial size limitation, except the detail procedures of the derivative for multi-user and multi-carrier case is shown in the Appendix, the BER performance for the other cases will not be presented in the paper.

4. Performance Evaluation

The performance with average BER of a MC-DS-CDMA system work over correlated Nakagami-*m* fading channels is numerically analyzed in this section. The Matlab package language is applied to numerically analyze for the every kinds of derived equations. The results of E_b / N_0 versus BER for multi-user and multi-carrier case were considered in Fig. 3, in which the parameter of user number is assigned to $K=50$ and $K=120$, and assuming that the different values of correlation coefficients $\rho=0.16$, $\rho=0.25$, $\rho=0.36$ and $\rho=0.49$, which were suggested in the research paper [12]. It is known that the system performance will become much better when the values of the correlation coefficient between subcarriers are decrease. By setting the length of the spreading sequence is *N=128*, and the same user number are that utilized in Fig. 3, the plots of SNR versus BER with different fading figure, *m=2*, and *5*, illustrated in Fig. 4. It is reasonable to say the fact that the much more of the fading figure, *m* value, in Nakagami-*m* statistic distribution, the better of the system performance from Fig. 4. The results from different effect of PBI (Jimming- to-signal ratio, JSR) are presented in Fig. 5 to Fig. 8. In Fig. 5 the BER versus E_b / N_0 curves for multi-carrier case with different JSR values are presented. It is shown that the performance will become inferior when the JSR is increase gradually, that is, the best one of the performance is the curve $JSR = 0dB$ appeared in Fig. 5. The results from the same conditions considered in Fig. 5 is also adopted in Fig. 6 just with different number of subcarrier, $N=128$. The affect of the different subcarrier number can be understood from Fig. 5 and Fig. 6. By the way, the average BER for different JSR with single and multi-carrier cases is also shown in Fig. 7. The curves illustrated in Fig. 7 say that average BER will become degradation when the JSR increases. The impact of JSR on both single carrier and multi-carrier DS-CDMA systems are compared and shown in Fig. 7. The PBI is caused from the distinct carrier can be clearly known in this comparison, in which the performance for single-carrier system is always better than that of multi-carrier system. It is worth to note that this phenomenon appears in Fig. 7 will become less significantly as the values of PBI are increase. The results of BER versus SNR with different correlation coefficients and with no PBI for multipath are shown in Fig. 8. It is obviously that the correlation coefficient definitely affects the performance whether the PBI exists or not. The results from some of the figures have significantly reminded that the correlation coefficient is not negligible, it is necessary to pay much attention in designing related to the CDMA systems.

5. Conclusions

In this paper the approximate expression of system performance for MC-DS-CDMA system with correlation coefficient between fading branch were evaluated. The pdf of SNR at the output or Rake (MRC) receiver for different cases combination with multi-user and single-user cases were determined, meanwhile, the sum of Gamma variates is applied in the procedures to calculate the system BER performance. Besides, the influence of JSR is also assumed for one of the system parameter. By comparison the system BER with the different correlation coefficients, branch number, and number of subcarrier were considered to analysis for the validation. The derived results prove that the correlation coefficients will degrade the performance of MC-DS-CDMA system definitely.

Appendix：

The average BER of MC-DS-CDMA system with the case assumed that is under multi-user with multi-carrier was derived in this appendix. The formula of determining average BER is given in (21), where $\phi(\cdot)$ represents the conditional pdf of SNR is able to be written as [14]

$$\phi(\mathrm{x}) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{j^2}{2}} dt = \frac{2}{\sqrt{\pi}} e^{-x^2} \sum_{j=0}^{\infty} \frac{2^j x^{2j-1}}{(2j-1)!!} \qquad \text{(A-1)}$$

where $(2n+1)!! = 1 \cdot 3 \ldots (2n+1)$, and the correlated-Nakagami-*m* distribution. By substituting (6), (12) and (24) into (21), then the average BER becomes as

$$\begin{aligned}
P_e^{mu-mc} &= \int_0^\infty \frac{\sqrt{2}}{\pi} e^{-\left(\left\{\frac{K-1}{2MN}\left(1-\frac{\mu}{4}\right)+\frac{\eta_0}{2MNE_C}\right\}^{-1}\frac{1}{M}\gamma\right)} \\
&\quad \times \sum_{j=0}^{\infty} \frac{2^{j/2}\left(\left\{\frac{K-1}{2MN}\left(1-\frac{\mu}{4}\right)+\frac{\eta_0}{2MNE_C}\right\}^{-1}\frac{1}{M}\gamma\right)^{j-1/2}}{\sqrt{(2j-1)!!}} \\
&\quad \times \prod_{i=1}^{L}\left(\frac{\lambda_1}{\lambda_i}\right)^m \sum_{k=0}^{\infty} \frac{\delta_k \gamma^{Lm+k-1} e^{-\gamma/\lambda_1}}{\lambda_1^{Lm+k}\Gamma(Lm+k)} U(\gamma) d\gamma \\
&= \frac{\sqrt{2}}{\pi} \sum_{j=0}^{\infty} \frac{2^{j/2}\left(\left\{\frac{K-1}{2MN}\left(1-\frac{\mu}{4}\right)+\frac{\eta_0}{2MNE_C}\right\}^{-1}\frac{1}{M}\right)^{j-1/2}}{\sqrt{(2j-1)!!}} \\
&\quad \times \prod_{i=1}^{L}\left(\frac{\lambda_1}{\lambda_i}\right)^m \sum_{k=0}^{\infty} \frac{\delta_k}{\lambda_1^{Lm+k}\Gamma(Lm+k)} U(\gamma) \\
&\quad \times \int_0^\infty e^{-\left(\left\{\frac{K-1}{2MN}\left(1-\frac{\mu}{4}\right)+\frac{\eta_0}{2MNE_C}\right\}^{-1}\frac{1}{M}\gamma\right)} \gamma^{j-1/2} \gamma^{Lm+k-1} e^{-\gamma/\lambda_1} dr \\
&= \sqrt{\frac{2}{\pi^{1/2}}} \sum_{j=0}^{\infty} \frac{2^{j/2}\left(\left\{\frac{K-1}{2MN}\left(1-\frac{\mu}{4}\right)+\frac{\eta_0}{2MNE_C}\right\}^{-1}\frac{1}{M}\right)^{j-1/2}}{\sqrt{(2j-1)!!}} \\
&\quad \times \prod_{i=1}^{L}\left(\frac{\lambda_1}{\lambda_i}\right)^m \sum_{k=0}^{\infty} \frac{\delta_k}{\lambda_1^{Lm+k}\Gamma(Lm+k)} U(\gamma) \qquad \text{(A-2)} \\
&\quad \times \int_0^\infty \gamma^{j-1/2+Lm+k-1} e^{-\gamma\left(1+\left\{\frac{K-1}{2MN}\left(1-\frac{\mu}{4}\right)+\frac{\eta_0}{2MNE_C}\right\}^{-1}\frac{1}{\lambda_1 M}\right)} dr
\end{aligned}$$

By using of the given integral formulas as [12]

$$\int_0^\infty \mathrm{x}^{\nu-1} e^{-\Lambda x} dx = \frac{1}{\Lambda^\nu}\Gamma(\nu) \qquad [\mathrm{Re}\ \ \Lambda > 0, \quad \mathrm{Re}\ \ \nu > 0]$$

and the variables are changed as $\nu = j - 1/2 + Lm + k$ and

$$\Lambda = 1 + \left\{\frac{K-1}{2MN}\left(1-\frac{\mu}{4}\right)+\frac{\eta_0}{2MNE_C}\right\}^{-1}\frac{1}{\lambda_1 M} \qquad \text{(A-3)}$$

then the equation of (A-2) becomes as

$$\begin{aligned}
P_e^{mu-mc} &= \sqrt{\frac{2}{\pi^{1/2}}} \sum_{j=0}^{\infty} \frac{2^{j/2}\left(\left\{\frac{K-1}{2MN}\left(1-\frac{\mu}{4}\right)+\frac{\eta_0}{2MNE_C}\right\}^{-1}\frac{1}{M}\right)^{j-1/2}}{\sqrt{(2j-1)!!}} \qquad \text{(A-4)} \\
&\quad \times \frac{\Gamma(j-1/2+Lm+k)}{\Lambda^{(j-1/2+Lm+k)}} \prod_{i=1}^{L}\left(\frac{\lambda_1}{\lambda_i}\right)^m \sum_{k=0}^{\infty} \frac{\delta_k}{\lambda_1^{Lm+k}\Gamma(Lm+k)} U(\gamma)
\end{aligned}$$

where δ_k is defined in (14), $U(\gamma)$ denotes the unit step function, Λ is defined in (A-3), and $SNR = MNE_c / \eta_0$.

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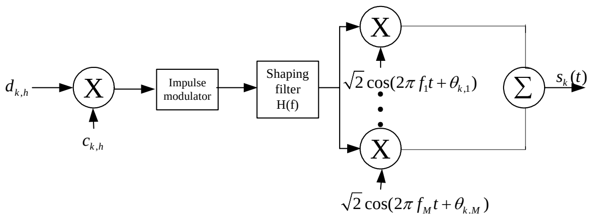

Fig 1. The transmitter block diagram of an MC-DS-CDMA system

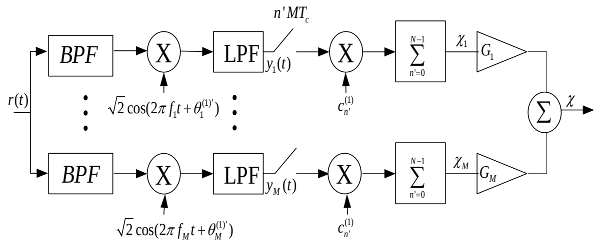

Fig 2. The receiver block diagram of a reference user

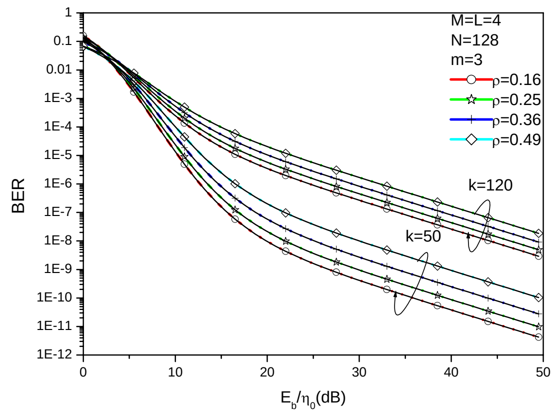

Fig. 3. BER versus SNR with different user numbers

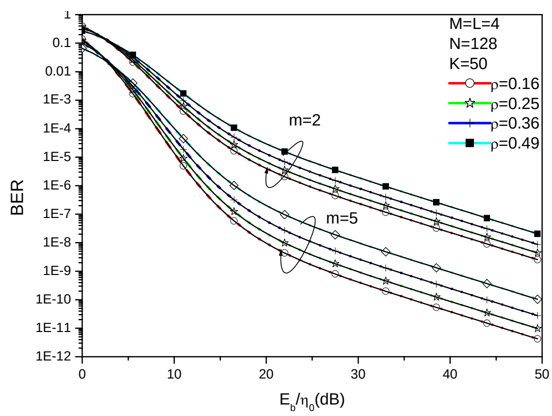

Fig. 4. BER versus SNR with different fading parameters

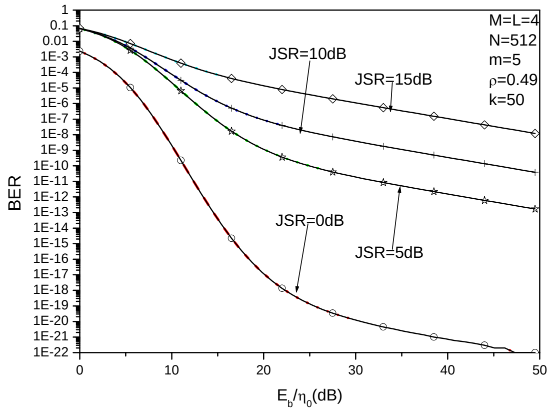

Fig. 5. BER versus SNR with different JSR values

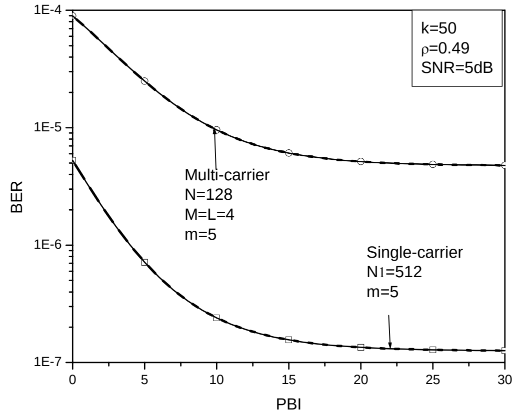

Fig. 7. BER versus PBI for multi-carrier and single-carrier case

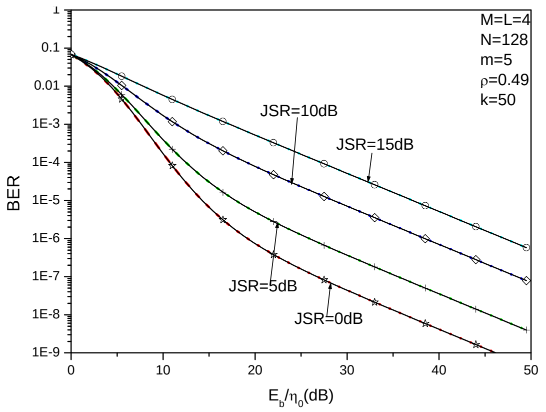

Fig. 6. BER versus SNR with different JSR values

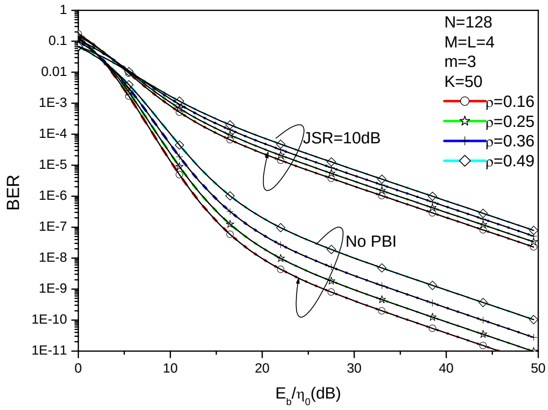

Fig. 8. BER versus SNR with JSR=10dB and no PBI