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DOUBLE CORE PHOTONIC CRYSTAL FIBER

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Abstract: - In this study, coupling between two cores in a specific photonic crystal fiber (PCF) structure is investigated via Localized Basis Function (LBF) method. The PCF structure considered here consists of an unbounded periodic cladding and two high index core regions. In the LBF method, a guided mode localized in the core regions and periodic variation of the refractive index in the transverse domain are decomposed by using Hermite-Gaussian type Localized Basis Functions and Fourier type expansions. Simulation results are presented which demonstrate that the coupling between two cores in the PCF structure can be obtained in accurate and numerically efficient manner utilizing the LBF approach proposed in this work.

Key-Words: - periodic structures, photonic crystal fiber, eigenvalues, coupling.

1 Introduction

In recent years, there has been an increasing interest in photonic crystal fibers (PCF's). PCF's are single material optical fibers with a periodic array of air holes made in their cross-sections running along the entire length of the fiber. The large and controllable periodic variations of the refractive index in the transverse domain offered by these fibers opens up exciting new opportunities for the control and guidance of light [1, 2, 3].

In a PCF, light can be guided using either one of two quite different mechanisms: Total Internal Reflection (TIR) mechanism and Photonic Band Gap (PBG) effect [1]. TIR occurs when the refractive index of the core is higher than that of the cladding surrounding it. TIR is a well-known mechanism and has widely been used in describing propagation in optical waveguides. Within the last decade there has been an increasing interest in a physical mechanism known as PBG, which provides some new opportunities in confining and controlling light in fibers. PBG can be obtained by introducing a periodic perturbation into the cross-section of the fiber. The main property characterizing the PBG structures is the occurrence of pass and stop bands in the frequency spectrum.

In this study, coupling between the cores in a specific photonic crystal fiber (PCF) structure depicted in Fig.1 is investigated via Localized Basis Function (LBF) method. The PCF structure considered here, consists of an unbounded cladding region formed by introducing circular air perforations in a lossless dielectric conforming to a periodic pattern of hexagonal symmetry except for "defect" regions obtained by removing two of the air holes, as seen from the Fig.1. Guided mode energy is concentrated in the vicinity of these defects, which act as the (high-index) core regions for the PCF. Hermite-Gaussian type Localized Basis Functions (LBF) [4, 5, 6] are utilized together with Fourier type expansions in representing the localized guided modal fields and the periodic variation of the refractive index in the transverse domain. Because the refractive indices of the defects are higher than the "effective index" of the cladding [7], the guidance of the electromagnetic field in the PCF structure can also be thought to result due to TIR mechanism. Numerical results are given which demonstrate the applicability of the presented method.

2 Method

In the LBF method considered in this paper, expansions based on Hermite-Gaussian (HG) functions are used for representing the fields within the core (defect) regions whereas a mixed representation utilizing HG functions and 2-D Fourier series are used to model the periodic refractive index variation together with the defect (core) regions. These expansions reduce the problem of determining propagation characteristics of the PCF to a matrix eigenvalue problem.

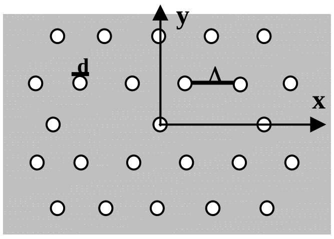

Fig.1 The periodic cladding and two core regions.

In our work, scalar wave approximation is utilized. This approximation is valid when the ratio of the air hole diameter (d) to the hole separation (Λ) is small ($d/\Lambda<0.4$) [5], as considered in this work. A modal electric field component of the coupled structure is expanded as

$$E(x,y)=\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\left(\Psi_i(x+\Lambda)+\Psi_i(x-\Lambda)\right)\Psi_j(y) \qquad (1)$$

where N is the number of terms retained in the expansion and ψ_i stands for orthonormal basis of Hermite-Gaussian functions:

$$\Psi_i(x)=\frac{2^{-(i-1)}\pi^{-1/4}}{\sqrt{(2(i-1))!w}}\exp\left(\frac{-x^2}{2w^2}\right)H_{2i}\left(\frac{x}{w}\right) \qquad (2)$$

Here H_i is the ith-order Hermite polynomial. w in (2) which represents the characteristic width of the basis set is taken to be $\Lambda/2$ where Λ is the separation between the holes.

The squared expression of the refractive index in the transverse domain is separated into two parts: corresponding to the periodic lattice of holes which may be described using a Fourier series, and that corresponding to the defects (i.e., the cores) which will again be described using localized orthonormal Hermite-Gaussian functions. We thus write,

$$n^2(x,y)=n_{defects}(x,y)+n_{periodic}(x,y)$$

$$n^2(x,y)=\sum_{i=1}^{N_1}\sum_{j=1}^{N_1}C_{ij}(\Psi_i(x+\Lambda)+\Psi_i(x-\Lambda))\Psi_j(y)$$

$$+\sum_{i=1}^{N_2}\sum_{j=1}^{N_2}D_{ij}\cos\left(\frac{2\pi(i-1)x}{lx}\right)\cos\left(\frac{2\pi(j-1)y}{ly}\right) \qquad (3)$$

where N_1 and N_2 terms are retained in expansions for the defect regions and the periodic cladding region holes, respectively. lx and ly are the periods along the x and the y axis, which for the structure shown in Fig.2 are determined as $lx=\Lambda$ and $ly=1.732\Lambda$, respectively.

ψ_i in (3) has the same functional form as given by (2) however, w is now replaced by $w=0.26d$ (d: the air hole diameter), i.e. the basis sets used in representing modal fields and $n^2(x, y)$ within the defect regions are defined with different characteristic widths.

Expansion coefficients D_{ij} are evaluated via inner products defined over the unit cell (UC),

$$D_{ij}=\iint_{UC}n^2(x,y)\cos\left(\frac{2(i-1)\pi x}{lx}\right)\cos\left(\frac{2(j-1)\pi y}{ly}\right)dxdy \qquad (4)$$

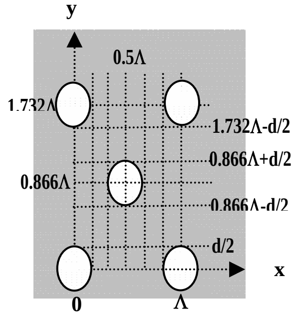

Fig.2 A periodic unit cell of the unbounded periodic cladding structure.

On the other hand C_{ij} coefficients in (3) are evaluated via inner products defined over one of the defect regions (DR),

$$C_{ij} = \iint_{DR} n^2(x,y)\Psi_i(x)\Psi_j(y)dxdy \tag{5}$$

The representations for the scalar electric field component and the squared expression of the refractive index in the transverse domain given in (1) and (3), respectively, are substituted into the scalar wave equation.

$$\frac{1}{k_0^2}\nabla^2 E + (n^2 - \bar{n}^2)E = 0 \tag{6}$$

where $\bar{n} = \beta/k_0$ denotes the modal index.

Next, each term of the resulting expression is multiplied by $\psi_k(x)\psi_l(y)$ and integrated over the entire transverse (x, y) plane. Making use of the orthonormality of the Hermite-Gaussian basis functions, the problem is then reduced to matrix form:

$$MV = \bar{n}^2 M_1 V \tag{7}$$

Here V denotes the vector ($N^2\times1$) of expansion coefficients a_{ij}, and the elements M_{klij} and M_{1klij} of $N^2\times N^2$ matrixes M and M_1 are obtained respectively as

$$M_{klij} = \frac{1}{k_0^2} I^{(1)}_{klij} + I^{(2)}_{klij} \tag{8}$$

$$I^{(1)}_{klij} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Psi_k(x)\Psi_l(y)\nabla^2\left[(\Psi_i(x+\Lambda) + \Psi_i(x-\Lambda))\Psi_j(y)\right]dxdy \tag{9}$$

$$I^{(2)}_{klij} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} n^2\Psi_k(x)\Psi_l(y)(\Psi_i(x+\Lambda) + \Psi_i(x-\Lambda))\Psi_j(y)dxdy \tag{10}$$

$$M_{1klij} = I^{(3)}_{klij} \tag{11}$$

$$I^{(3)}_{klij} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Psi_k(x)\Psi_l(y)(\Psi_i(x+\Lambda) + \Psi_i(x-\Lambda))\Psi_j(y)dxdy \tag{12}$$

3 Numerical Results

The specific PCF structure of hexagonal symmetry considered in this work is shown in Fig.1. In all numerical calculations presented here we assumed $d/\Lambda=0.4$ and $n_a=1$, $n_s=1.45$, corresponding to the refractive indices of air and silica substrate respectively.

Fig.3 demonstrates that the convergence properties of the representation given in (3) are fairly good and the refractive index profile can be represented quite accurately by using reasonable number of terms in the expansions.

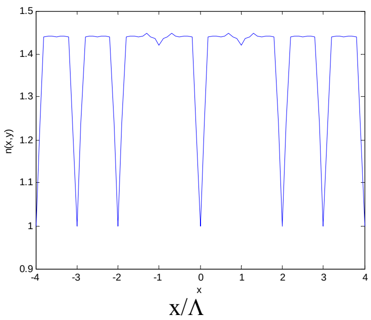

Fig.3 Variation of index profile along x profile obtained from (3) using $N_1=10$, $N_2=100$.

Utilizing same values of d/Λ, n_a, n_s in all computations, we have investigated the dependence on normalized wavelength Λ/λ of the coupling between the two defect regions resulting from the removal of air holes centered at $x=\pm\Lambda$ and $y=0$ (see Fig.1).

The variation with x/w of the normalized amplitude of the transverse electric field of the dominant even symmetric mode of the coupled structure is depicted in Fig.4 for $\Lambda/\lambda=5.11$. This value of Λ/λ corresponds to a separation of about 10λ between the centers of the two defect regions. Hence, it is expected that at this rather weak coupling condition the field variation of the composite system will be almost identical to the superposition of the fields of individual PC fibers.

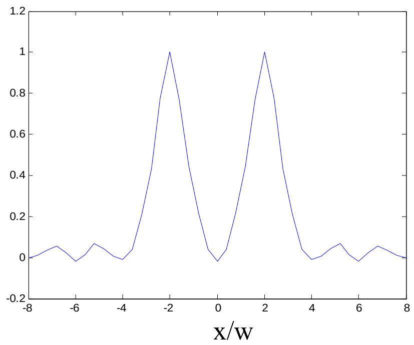

Fig.4 The variation of the normalized modal field amplitude with x/w. ∧/λ=5.11, $\bar{n}_e = 1.4479$

Indeed the obtained mode index completely agrees with that obtained for the single defect case. It should however be kept in mind that the coupled system does, of course, support also an odd symmetric mode, which in this case corresponds to the same mode index and its variation can be obtained via simply inverting the field amplitudes shown in Fig.4 in regions ± x.

On the other hand, as ∧/λ decreases the electrical distance between the two defect regions becomes smaller and the modal indices of the even $\bar{n}_e$ and odd $\bar{n}_o$ symmetric modes of the structure begin to differ from the modal index $\bar{n}$ obtained for the single defect case. In Fig.5 we have plotted the variation with x/w of the even symmetric dominant mode for ∧/λ=0.9 which clearly indicates the overlap between the fields supported by individual defect regions.

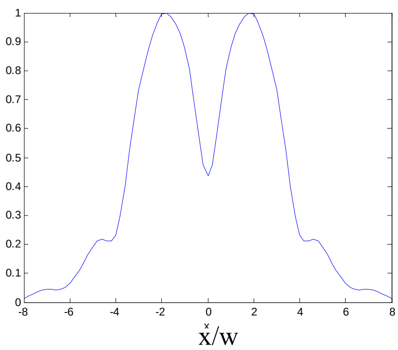

Fig.5 The variation of the normalized modal field amplitude with x/w. ∧/λ=0.92, $\bar{n}_e = 1.4009$

In Fig.6 the variation of modal indices of the dominant even symmetric and odd symmetric modes of the two core photonic crystal fiber with ∧/λ is given together with the modal index of a single core (defect) fiber. It is well known that in the weak coupling limit standard Coupled Mode Theory (CMT) yields a good approximation for estimating the splitting between $\bar{n}_e$ and $\bar{n}_o$. In case when two waveguides are identical, CMT predicts that $\bar{n}_e, \bar{n}_o$ and $\bar{n}$ are related as [8]

$$\bar{n} = \frac{\bar{n}_e + \bar{n}_o}{2} \tag{13}$$

Our numerical results indicate that the calculated values of modal indices are in good agreement with (13) and hence provides a validation for the applicability of the scalar wave approximation in the parameter regime consider in this treatment.

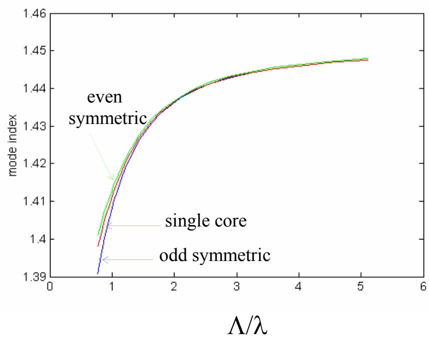

Fig.6 The variation of indices of the dominant modes in a double core PCF and modal index of a single core PCF with ∧/λ.

4 Conclusion

In this study, the coupling between the cores in the specific photonic crystal fiber (PCF) structure is investigated via Localized Basis Function (LBF) method. Hermite-Gaussian type Localized Basis Functions (LBF's) [4, 5, 6] are utilized together with Fourier type expansions in representing the localized guided modal field and the periodic variation of the refractive index in the transverse domain. We have presented numerical results to illustrate the effectiveness of the presented approach in addressing the coupling between the two defect regions (double core) in a PCF. It is also shown that scalar wave approximation provides sufficient accuracy for treating coupling in the two core PCF in the parameter regime considered. However as noted in the literature this approximation breaks down when d/∧ is increased, corresponding to larger air holes. In this case the presented approach can also be used with little modification [6] to obtain full wave solutions for the vector fields of the dominant mode in PCF.

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