Proceedings of the 5th WSEAS International Conference on Telecommunications and Informatics, Istanbul, Turkey, May 27-29, 2006 (pp249-253)

Using Nonlinear Filter for Adaptive Blind Channel Equalization

HAMIDREZA BAKHSHI
Dept. of Electrical and Computer Engineering
Shahed University
Qom Highway, Tehran,
IRAN

MOHAMMAD POOYAN
Dept. of Electrical and Computer Engineering
Shahed University
Qom Highway, Tehran,
IRAN

Abstract: InterSymbol Interference (ISI) of the radio propagation through channels is a major factor that limits the performance of mobile communication systems and can be compensated by equalization. One of the basic groups of equalizers is based on linear algorithms. In this paper, we propose the other method based on Hammerstein filter as a nonlinear memoryless blind equalizer. We have compared these two methods by simulation and have shown that the latter method results have better performance compared to the former ones; i.e. the Hammerstein filter introduces lower BER than linear algorithms, so makes it a viable alternative to former equalizers. For all methods studied in this paper, blind equalization is assumed. At last blind Hammerstein equalizer analyzed in order to study its convergence capability and unbiased property.

Keywords: Blind Equalization, Hammerstein Filter, Least Mean Square, Zero Forcing.

1 Introduction

Many systems suffer from InterSymbol Interference (ISI); so, the equalization methods should be employed to combat this effect. In the case of linear equalizers, two different criteria are used for determining the values of the equalizer coefficients. One criterion is based on the peak distortion at the output of the equalizer, namely the zero forcing algorithms [1], and the other one is based on the minimization of the mean square error at the output of the equalizer, named as Least Mean Square (LMS) [1]. These two algorithms can be performed without training sequence which results in blind equalization [2, 3].

In [4] Hammerstein filter is used as a memoryless filter for flat fading channel equalization but it can not be used in non flat fading channel equalization. In non flat fading channel (channel with memory grater than one) HDFE equalizer [4] is used for equalization.

We have proposed using nonlinear filters instead of linear ones for blind channel equalization. In particular, we have used Hammerstein filter for adaptive blind channel equalization and have shown that this filter can equalize channel with memory greater than one (non flat fading channel), blindly with the much lower complexity than HDFE. We have simulated three methods with different channels and have compared them with respect to bit error rate (BER). The simulation results confirm the faster convergence in blind adaptive case and also the better bit error rate performance of the proposed method over the traditional linear equalization techniques.

The paper is organized as follows; we propose the adaptive blind equalization technique based on Hammerstein filter in section 2. Analytical study of purposed method shown in section 3. In section 4, the simulation results are presented. finally in the last section, some concluding remarks are presented.

2 Blind Hammerstein Adaptive Equalization

The block diagram of the proposed equalizer is shown in Fig. 1, where *I(n)* is the *iid, BPSK* transmitted data, *f(n)* is the channel impulse response (modeled as taped delay lines [1]) and *x(n)* is the received signal available at the receiver. Noise at the input of the receiver is denoted as *v(n)* and assumed to be Additive White Gaussian Noise (AWGN).

2.1 Hammerstein Filter

Referring to Fig.1, the output of the Hammerstein filter $\hat{I}(n)$ has the form of [3, 4]:

$$\hat{I}(n) = g_0 + g_1 x(n) + g_2 x^2(n) + \Lambda + g_N x^N(n)$$

or

$$\hat{I}_k = \sum_{j=0}^{N} g_j x_k^j \, . \tag{1}$$

There are no delay terms, since we assume no memory for the filter. The weights g_i have to be determined such that this polynomial equalizes the channel.

2.2 The Hammerstein Filter as a Blind Equalizer

We used Hammerstein filter as a channel equalizer as shown in Fig. 1. The equation (1) can be written in matrix form as:

$$\begin{bmatrix} 1 & x(1) & x^2(1)\Lambda\ \Lambda\ \Lambda & x^N(1) \\ 1 & x(2) & x^2(2)\Lambda\ \Lambda\ \Lambda & x^N(2) \\ \Lambda\ \Lambda\ \Lambda\ \Lambda\ \Lambda\ \Lambda\ \Lambda\ \Lambda\ \Lambda\ \Lambda\ \Lambda & & & \mathrm{M} \\ 1 & x(m) & x^2(m)\Lambda\ \Lambda & x^N(m) \end{bmatrix} \begin{bmatrix} g_0 \\ g_1 \\ \mathrm{M} \\ g_N \end{bmatrix} = \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \\ \mathrm{M} \\ \hat{I}_N \end{bmatrix}$$

or

$$X_m G = \hat{I}_m \, . \tag{2}$$

The output of the channel is:

$$x_k = \sum_{j=1}^{L} f_j I_{k-j} + v_k + f_0 I_k \, . \tag{3}$$

The equation (3) has three parts; the first part indicates ISI, the second part is noise and the third part is desired signal. In order to eliminate ISI and noise, we should minimize MSE by statistical Gradient method. The MSE cost function is:

$$J_k = E\left\{(I_k - \hat{I}_k)^2\right\} \tag{4}$$

Using equation (1), we have:

$$J_k = E\{I_k^2\} + E\{(\sum_{j=0}^{N} g_j x_k^j)^2\} - 2E\{I_k \sum_{j=0}^{N} g_j x_k^j\} \, . \tag{5}$$

In order to minimize MSE, the Gradient vector should be set to zero:

$$\nabla_k = \frac{\partial J_k}{\partial g_j} = 2E\{x_k^j (\sum_{j=0}^{N} g_j x_k^j - I_k)\} = 0 \tag{6}$$

Using equation (6), we can determine $\{g_i\}$ coefficients by the following iterative equation:

$$g_j^{k+1} = g_j^k + \Delta \varepsilon_k x_k^j \tag{7}$$

where, $\varepsilon_k = I_k - \sum_{j=0}^{N} g_j^k x_k^j$ indicates the error vector and Δ is the step size.

In non-blind estimations, we can use training sequence instead of I_k. But in blind estimations, we can estimate I_k by:

$$I_k = \operatorname{sgn}(\sum_{j=0}^{N} g_j^k x_k^j) \, . \tag{8}$$

3 Analytical Study of Blind Hammerstein Equalizer

We can extend equation (7) to represent the coefficient respect to initial value ($g_i^{(0)}$), for this manner this equation writes as:

$$g_i^{(1)} = g_i^{(0)} + \Delta \varepsilon_0 x_1^i \tag{9}$$

$$g_i^{(2)} = g_i^{(1)} + \Delta \varepsilon_1 x_2^i = g_i^{(0)} + \Delta \varepsilon_0 x_0^i + \Delta \varepsilon_1 x_1^i$$

$$\mathrm{N}$$

$$g_i^{(k+1)} = g_i^{(0)} + \Delta \sum_{k=0}^{k} \varepsilon_k x_k^i$$

If the initial value is chosen zero in equation (9), then:

$$g_i^{(k+1)} = \Delta \sum_{k=1}^{k} \varepsilon_k x_k^i = \Delta \sum_{k=0}^{k} \left(I_k - \hat{I}_k\right) x_k^i = \Delta \sum_{k=0}^{k} I_k x_k^i - \Delta \sum_{k=0}^{k} \hat{I}_k x_k^i \tag{10}$$

For large amount of k, sigma can be approximated by expectation and subsisted x_k by (3), $\hat{I}$ by (1) so the (10) rewrite as:

$$g_i^{(k+1)} = \Delta k \, E\left\{ I_k \left(\sum_{k=0}^{L} f_e I_{k-e} + n_k \right)^i \right\} - \Delta k \, E\left\{ \left(\sum_{j=0}^{N} g_j^{(k)} x_k^j \right) x_k^i \right\} \tag{11}$$

Input data $\{I_k\}$ assumed to be iid with zero mean and data are in depended to noise so the first term in (11) writes as (12).

$$E\left\{ I_k . \left(\sum_{k=0}^{L} f_e I_{k-e} + n_k \right)^i \right\} = E\left\{ f_0^i I_k^{i+1} \right\} = f_0^i \, E\left\{ I_k^{i+1} \right\} . \tag{12}$$

If we suppose strict stationary of (i+1) order so $E\left\{I_k^{i+1}\right\}$ is equal to $(i+1)_{th}$ moment of I and (12) Lead to:

$$E\left\{ I_k \left(\sum_{k=0}^{L} f_e I_{k-e} + n_k \right)^i \right\} = f_0^i m_I (i+1) \tag{13}$$

The second part of (11) can be rewrite as:

$$E\left\{\left(\sum_{j=0}^{N} g_j^{(k)} x_k^i\right) x_k^i\right\} = E\left\{\sum_{j=0}^{N} g_j^{(k)} x_k^{i+j}\right\} \quad (14)$$

Near the convergence, the amounts of $g_j^{(k)}$ are approximately constant and also $g_j^{(k)}$ known in step k+1 and by strict stationary (14) can be written as

$$E\left\{\sum_{j=0}^{N} g_j^{(k)} X_k^{i+j}\right\} = \sum_{j=0}^{N} g_j^{(k)} m_x(i+j) \quad (15)$$

If (15) is combined with (13) the updating rule for Hammerstein Coefficient is calculated by:

$$g_j^{(k+1)} = \Delta k\ f_0^{\,j} m_I(j+1) - \Delta k \sum_{i=0}^{N-1} g_i^{(k)} m_x(i+j) \quad (16)$$

It is useful to write (16) as a matrix form.

$$\frac{1}{k\Delta}\begin{bmatrix} g_0^{(k+1)} \\ g_1^{(k+1)} \\ \mathrm{M} \\ g_{N-1}^{(k+1)} \end{bmatrix} = -\begin{bmatrix} m_x(0) & m_x(1) & \Lambda & m_x(N-1) \\ m_x(1) & m_x(2) & \Lambda & m_x(N) \\ \mathrm{M} & & & \\ m_x(N-1) & m_x(N) & \Lambda & m_x(2N-2) \end{bmatrix}\begin{bmatrix} g_0^{(k)} \\ g_1^{(k)} \\ \mathrm{M} \\ g_{N-1}^{(k)} \end{bmatrix} + \begin{bmatrix} f_0 m_I(1) \\ f_0^1 m_I(2) \\ \mathrm{M} \\ f_0^{N-1} m_I(N) \end{bmatrix}$$

$$g^{(k+1)} = Mg^{(k)} + U \quad (17)$$

This equation is vary important because describe a discrete state-space equation for Hammerstein Coefficient updating. So convergences of adaptive system lead to stability discussion of state-space system. This state space system is stable if and only if all he singular value of matrix m is not exceeding one.

$$M = V\Lambda V^H \quad , \quad \Lambda = diag\left[\lambda_0, \Lambda\ \lambda_{N-1}\right]$$

$$\lambda_{\max} = \arg\max(\lambda)\ ,\ \lambda_{\max} < 1 \quad (18)$$

In other to discuss about step size in this problem if the max singular of M defined as $\lambda_{\max}$ then the step size must be lower than $\dfrac{1}{k\lambda_{\max}}$. For finding a useful bound for $\lambda_{\max}$ considers that $\lambda_{\max}$ is lower than the sum of the singular value of M or trace of M so (18) rewrite as:

$$\lambda_{\max} < \sum_i \lambda_i = tr(M) = \sum_{i=0}^{N} m_x(2i) =$$
$$\sum_{i=0}^{N} \sum_{j=0}^{i/2} \left\{\binom{2i}{2j}\left(\sum_{l=0}^{L} f_l^{\ 2j}\right)\left(\frac{(2j)!}{2^j (j!)}\delta_n^{2j}\right)\right\}. \quad (19)$$

After successful convergence by initial value close to zero the even coefficients become zeros and Hammerstein inform as odd polynomial. After convergence the mean of estimation error is calculated as:

$$E\{e\} = E\left\{\hat{I}_k - I_k\right\} = E\left\{\hat{I}_k\right\}$$
$$= E\left\{\sum_{i=1}^{\frac{N}{2}} g_{2i+1} x_k^{2i+1}\right\} = \sum_{i=1}^{\frac{N}{2}} g_{2i+1} E\left\{x_k^{2i+1}\right\}$$
$$= \sum_{i=1}^{\frac{N}{2}} g_{2i+1}\left(\sum_{j=1}^{2i+1}\binom{2i+1}{j} E\{n_k^{2i=1-j}\} E\{(\sum_{l=0}^{L} f_l I_{k-l})^j\}\right) \quad (20)$$
$$E\{n_k^{2i=1-j}\} = \begin{cases} 0 & j = 2m \\ \dfrac{(2(i-m))!}{2^m((i-m)!)}\delta_n^{2(i-m)} & j = 2m+1 \end{cases}$$
$$,\quad E\{(\sum_{l=0}^{L} f_l I_{k-l})^j\} = \begin{cases} 0 & j = 2m+1 \\ \left(\sum_{l=0}^{l=L} f_l^{\ 2j}\right) & j = 2m \end{cases}$$
$$E\{e\} = 0$$

So the estimator is asymptotically unbiased. The error variance also can be calculated iteratively as follow:

$$\delta_e^k = E\{(I_k - \hat{I}_k)^2\} = \delta_I^2 - 2E\{I_k \sum_{i=0}^{N} g_i^k x_k^i\}$$
$$+ E\{\sum_{i=0}^{N}\sum_{j=0}^{N} g_i^k g_j^k x_k^{i+j}\} \quad (21)$$

If the first tap of the channel impulse response is known, then (21) become simpler by using $E\{I_k x_k^i\} = \begin{cases} 0 & i = 2m \\ f_0^{\,i}\, m_I(i+1) & i = 2m+1 \end{cases}$, $m_I(i) = \begin{cases} 0 & i = 2m+1 \\ 1 & i = 2m \end{cases}$ in it.

$$\delta_e^k = (1 - 2\sum_{l=0}^{L} f_0^{2i+1} g_{2i+1}^k) + \sum_{i=0}^{N}\sum_{j=0}^{N} g_i^k g_j^k m_x(i+j)\}$$
$$= (1 - 2\sum_{l=0}^{L} f_0^{2i+1}(g_{2i+1}^{k-1} + \Delta e_{k-1} x_{k-1}^{2i+1})) +$$
$$\sum_{i=0}^{N}\sum_{j=0}^{N}(g_{2i+1}^{k-1} + \Delta e_{k-1} x_{k-1}^{2i+1}) g_{2i+1}^{k-1} + \Delta e_{k-1} x_{k-1}^{2i+1}) m_x(i+j)\}$$
$$= \left[(1 - 2\sum_{l=0}^{L} f_0^{2i+1} g_{2i+1}^{k-1}) + \sum_{i=0}^{N}\sum_{j=0}^{N} g_i^{k-1} g_j^{k-1} m_x(i+j)\}\right] +$$
$$\Delta e_{k-1}\left[\sum_{i=0}^{N}\sum_{j=0}^{N}(x_{k-1}^j g_i^{k-1} + x_{k-1}^i g_j^k) m_x(i+j) - 2\sum_{i=1}^{N/2} f_0^{2i+1} x_{k-1}^{2i+1}\right]$$
$$\delta_e^k = \delta_e^{k-1} + \Delta e_{k-1}\left[\sum_{i=1}^{N}\sum_{j=0}^{N}(x_{k-1}^j g_i^{k-1} + x_{k-1}^i g_j^k) m_x(i+j) - 2\sum_{i=1}^{N/2} f_0^{2i+1} x_{k-1}^{2i+1}\right]. \quad (22)$$

4 Simulation Results

For evaluation of the BER performance of the receiver discussed in this paper, we simulated the equivalent base band system shown in Fig. 1. As a purpose of comparison, we have done the simulation process for the three blind equalizations studied in this paper and have compared the results for different channels. For the simulation, we generate bit stream of *i(n)* uniformly from {-1,1}.

Fig. 2 shows that the Hammerstein equalizer have better BER performance compared to two other linear methods in the practical region of E_b/N_0.

We simulated different orders of Hammerstein filter and observed that the Hammerstein filter of order 10 has the best performance. Fig. 3 shows the convergence property of Hammerstein equalizer and MMSE equalizer versus number of iterations. Fig. 4 shows BER of three methods versus E_b/N_0 for another channel with deep nulls (its impulse response is shows in Fig. 5). The simulation results show that the two linear methods can not equalize the channel appropriately; however, the Hammerstein equalizer has done it with an outstanding performance.

5 Conclusion

In this paper we have introduced a new approach for blind channel equalization, using Hammerstein filter as equalizer at the receiver. Regarding BER performance of the three studied methods, we observe that Hammerstein filter has the best performance compared to other studied linear equalization methods.

We can conclude that Hammerstein filter as a nonlinear filter can equalize non flat channels more properly than linear equalizers due to the nonlinear nature of the optimum estimation. It has also been observed that by using the proposed method, the deficiency of Hammerstein equalizers in not having memory can be solved with the much lower complexity than HDFE [4].

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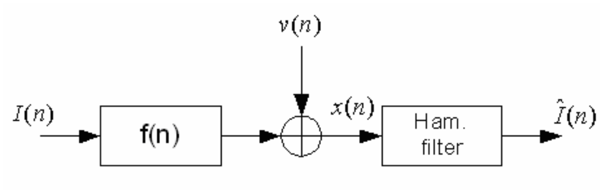

Fig. 1: Block diagram of the system with Hammerstein type equalizer

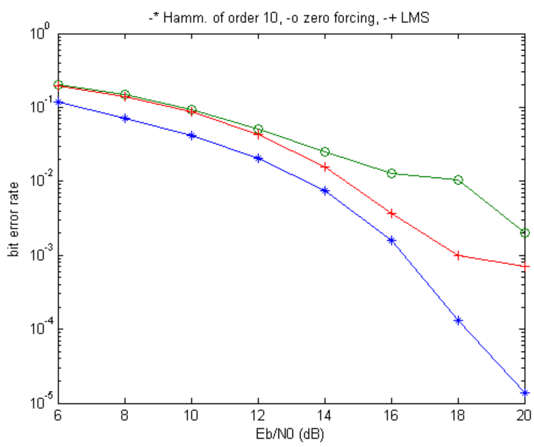

Fig. 2: Bit error rate versus E_b/N_0 (dB) for three methods of Hammerstein filter of order 10, Zero forcing and LMS.
Channel: $f(z) = .89 + .0133z^{-1} + .01z^{-2} + .4556z^{-3}$

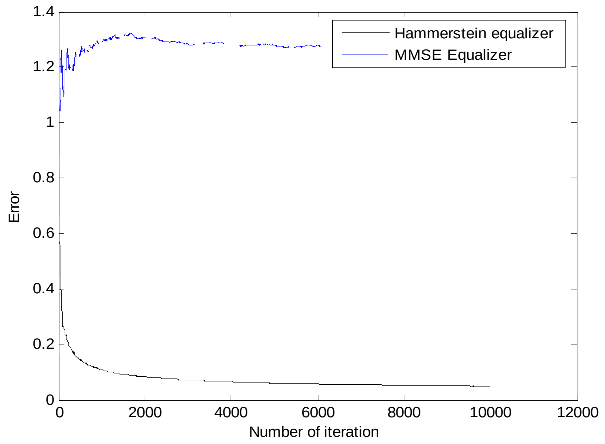

Fig. 3: Convergence property
Channel: $f(z) = .7208 + .0108z^{-1} + .0081z^{-2} + .693z^{-3}$

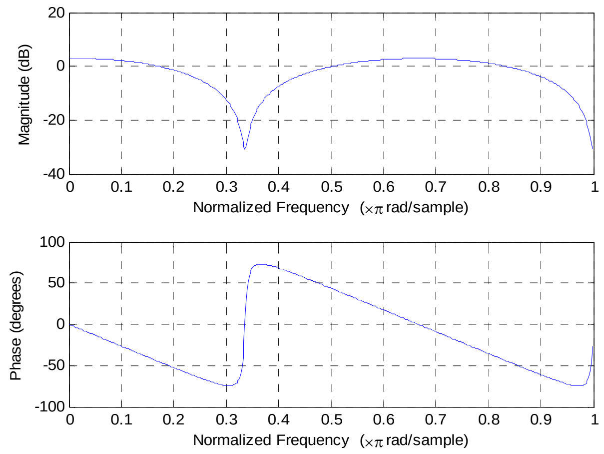

Fig. 5: Magnitude and Phase of the channel with impulse response:
$f(z) = .7208 + .0108z^{-1} + .0081z^{-2} + .693z^{-3}$

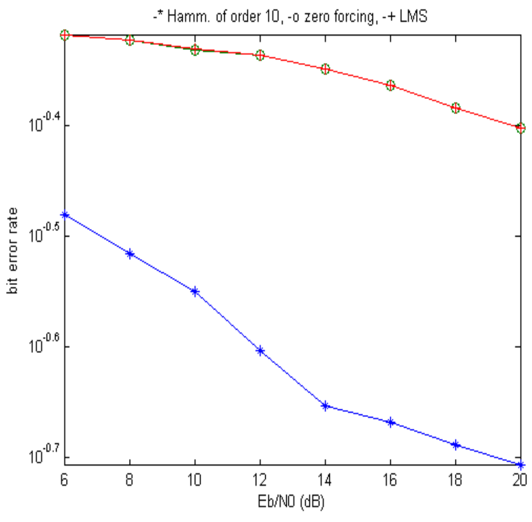

Fig. 4: Bit error rate versus E_b/N_0 (dB) for three methods of Hammerstein filter of order 10, Zero forcing and LMS.
Channel: $f(z) = .7208 + .0108z^{-1} + .0081z^{-2} + .693z^{-3}$