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Thin Subwavelength Multilayer Cavity Resonator Using Metamaterials

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Abstract: **-** In this paper, theoretical analysis of a thin one-dimensional (1D) multilayer cavity resonator which was consisted of a combination of a conventional dielectric material and a negative permittivity and permeability material (metamaterial) is presented. In this analysis, the metamaterial slab can act as a phase compensator and, thus, by combining it with another conventional dielectric slab a 1D cavity resonator whose dispersion relation may not depend on the summation of thicknesses of the filling layers of this cavity, but instead it depends on the ratio of these thicknesses with the total thickness far less than the conventional $\lambda/2$, can be produced. As the reflecting walls of this cavity multilayer dielectric stack is applied for producing high reflectance. The stack consists of alternate quarter wave layers of high and low index materials. The formulation and relevant results are presented.

Key-Words: - Cavity resonator, phase compensator, metamaterials, negative index of refraction, high reflectance multilayer dielectric stack.

1 Introduction

Veselago predicted that lossless materials [1], which posses simultaneously negative permittivity, ε and negative permeability, μ would exhibit unusual properties such as negative index of refraction, $n=-\sqrt{\varepsilon\mu}$, antiparallel phase wave vector k and Poyinting vector S antiparllel phase, v_p and group velocities, v_g and time averaged energy flux, $S = uv_g$ opposite to the time averaged momentum density $P = u\,k/\omega$ where u is the time averaged energy density. Furthermore, if these materials are uniform, k, **E**, **H** form a left handed set of vectors. Therefore these materials are called left handed materials (LHM) or negative index of refraction materials (NIM). The quantities, S,u,P refer to the composite system consisting of EM field and material [2]. As a result of k and S being antiparallel, the refraction of an EM wave to the interface between a positive n and a negative n material would be at unusual side relative to the normal (negative refraction). Nowadays, this phenomenon has attracted a great deal of attention.

Negative refraction studies originate from left handed metamaterials [3-18] which are based on the Veselago's proposal, composed from periodically arranged negative permeability $\mu < 0$ and permittivity $\varepsilon < 0$ metal components, providing an effective medium with $n_{ref} = \sqrt{\varepsilon}\cdot\sqrt{\mu} < 0$.

In this paper the application of negative refraction phenomenon in thin layered cavities in conjunction with multilayer dielectric stack mirror.

2 Formulation of phase Compensation

The Veselago's approach of theoretical behavior of wave propagation is started from Maxwell's equations. From these equations one can clearly conclude that:

$$\mathbf{k}\times\mathbf{E} = \omega\mu\mathbf{H} \quad (1)$$

$$\mathbf{k}\times\mathbf{H} = -\omega\varepsilon\mathbf{E} \quad (2)$$

Assuming wave propagation in the form $e^{-jk\cdot r}$, where k is the propagation constant. From equation (1) and (2) we can see that the triad formed by **E**, **H,** and k can have different behavior, depending on the sign of ε and μ. If $\varepsilon < 0$ and $\mu < 0$, then the direction of $\mathbf{S} = \frac{1}{2}\mathbf{E}\times\mathbf{H}$ and k would be the same and hence the phase velocity and Poynting vector would point in the same direction. On the other hand, if $\varepsilon > 0$ and $\mu > 0$, they would have opposite sign. Such a medium would, among other things, have a negative index of refraction, accordingly to Veselago.

Now consider a two-layer structure in which the left layer is assumed to be a conventional lossless dielectric material with $\varepsilon_1 > 0$ and $\mu_1 > 0$ with thickness d_1 and the right layer is taken to be a lossless metamaterial with negative permittivity and permeability with thickness d_2. In the first layer, the direction of Poynting vector S is parallel with the direction of phase velocity or wave vector k, whereas in the second layer, these two directions are antiparallel. With proper choice of ratio of d_1 and d_2, one can have the phase of the wave at the left interface to be the same as the phase at the right interface, essentially with no constraint on the total thickness of the structure. The structure is shown in Fig. 1.

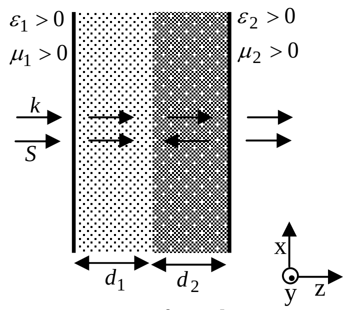

Fig. 1, parameters of two layer structure

At first slab, by passing the wave through it, the phase at the end of slab is obviously different from the phase at the beginning of the slab by amount $n_1 k_0 d_1$, where $k_0 = \omega\sqrt{\varepsilon_0 \mu_0}$. In metamaterial slab the wave at the end of the slab is different from the phase at the beginning of it by the amount $-n_2 k_0 d_2$. (The absolute value of n_2 is positive) So the total phase difference between the front and back faces of this two layer structure is $k_0(n_1 d_1 - n_2 d_2)$. Therefore, phase difference is developed by traversing the first slab, can be decreased and even cancelled by traversing the second slab. If the ratio of d_1 and d_2 is chosen to be $\frac{d_1}{d_2} = \frac{n_2}{n_1}$, then the total phase difference between the front and back faces of this two-layer structure becomes zero and not even multiples of π [19]. So the metamaterial slab with negative permittivity and permeability and at given frequencies can act as the phase compensator in this structure. Phase cancellation in this geometry does not depend on the sum of the thicknesses of layers; rather it depends on the ratio of them.

The fields inside the ordinary dielectric slab $0 \le z \le d_1$ can simply be written as

$$E_1 = a_x E_{01} \sin(n_1 k_0 z) \tag{3}$$

$$H_1 = a_y \frac{n_1 k_0}{j\omega\mu_1} E_{01} \cos(n_1 k_0 z) \tag{4}$$

And in metamaterial slab $d_1 \le z \le d_1 + d_2$ the fields are written as

$$E_2 = a_x E_{01} \sin(n_2 k_0 (d_1 + d_2 - z)) \tag{5}$$

$$H_2 = -a_x \frac{n_2 k_0}{j\omega\mu_2} E_{02} \cos(n_2 k_0 (d_1 + d_2 - z)) \tag{6}$$

The continuity of electric and magnetic fields at the interface between two layers would lead to

$$\begin{aligned} &E_{01} \sin(n_1 k_0 d_1) - E_{02} \sin(n_2 k_0 d_2) = 0 \\ &\frac{n_1}{\mu_1} E_{01} \cos(n_1 k_0 d_1) + \frac{n_2}{\mu_2} E_{02} \cos(n_2 k_0 d_2) = 0 \end{aligned} \tag{7}$$

In order to have a nontrivial solution $(E_{01} \ne 0)$ and $(E_{02} \ne 0)$ the determinant in (7) must be zero. That is

$$\frac{n_2}{\mu_2} \tan(n_1 k_0 d_1) + \frac{n_1}{\mu_1} \tan(n_2 k_0 d_2) = 0 \tag{8}$$

From this equation we can have

$$\frac{\tan(n_1 k_0 d_1)}{\tan(n_2 k_0 d_2)} = \frac{n_1 |\mu_2|}{n_2 |\mu_1|} \Rightarrow \frac{d_1}{d_2} \cong \frac{|\mu_2|}{|\mu_1|} \tag{9}$$

This relation shows how the thickness of layers is related in order to have nontrivial solution.

The electric and magnetic fields in cavity are given as

$$E_1 = a_x E_0 \sin(n_2 k_0 d_2) \sin(n_1 k_0 z) \tag{10}$$

$$H_1 = a_y \frac{n_1 k_0}{j\omega\mu_1} E_0 \sin(n_2 k_0 d_2) \cos(n_1 k_0 z) \tag{11}$$

$$E_2 = a_x E_0 \sin(n_1 k_0 d_1) \sin(n_2 k_0 (d_1 + d_2 - z)) \tag{12}$$

$$H_2 = -a_y \frac{n_2 k_0}{j\omega\mu_2} E_0 \sin(n_1 k_0 d_1) \cos(n_2 k_0 (d_1 + d_2 - z)) \tag{13}$$

Applying this idea structure can lead to several applications in design of some devices and components such as cavities.

3 Multilayer stack mirror in 1D metamaterial Cavity

The described idea has the potential of application in design a compact 1D cavity resonator. To realize this cavity two perfect reflectors should be put at the two open surfaces of it. Multilayer dielectric stack can be used instead of these reflectors. Multilayer films are widely used in science and industry for control o light. Optical surfaces having virtually any desired reflectance and transmittance characteristics may be produced by means of thin film coating.

These films are usually deposited on glass or metal substrates by high vacuum evaporation. These structures have so many applications include such things like antireflectance surfaces, heat reflecting and heat transmitting mirrors, one way mirrors, optical filters and so on. To model these structures first consider the case of single layer of dielectric of index n_1 and thickness d between two infinite media of indices n_0 and n. The structure is shown in Fig. 2. For simplicity we develop the theory for normally incident light. The modifications for the general case of oblique incidence are easily made. The amplitude of the of electric vector of the incident beam is E_0

that of the reflected beam is E_0', and that of the transmitted beam is E_t. The electric field amplitudes in the film are E_1 and E_1' for the forward and backward traveling waves respectively.

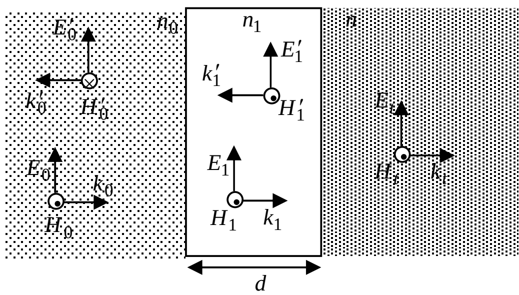

Fig. 2, Wave vectors and their associated electric and magnetic fields for the case of normal incidence on a single dielectric layer

By applying the continuity of electric and magnetic fields at boundaries and considering the phase factors of traveling waves after some simple algebraic calculations we obtain the following matrix form

$$\begin{bmatrix} 1 \\ n_0 \end{bmatrix} + \begin{bmatrix} 1 \\ -n_0 \end{bmatrix} r = M \begin{bmatrix} 1 \\ n \end{bmatrix} t \qquad (14)$$

Where the parameters of above equation are as below:

$$r = \frac{E_0'}{E_0} \qquad (15)$$

$$t = \frac{E_t}{E_0} \qquad (16)$$

$$M = \begin{bmatrix} \cos kd & -\frac{j}{n_1} \sin kd \\ -jn_1 \sin kd & \cos kd \end{bmatrix} \qquad (17)$$

where r, t and M are reflection coefficient, transmission coefficient and transfer matrix of the film.

Now suppose that we have N layers having indices of refraction $n_1, n_2, n_3, \ldots n_N$ and thicknesses $d_1, d_2, d_3, \ldots d_N$ respectively. In the same way that the equation (14) was derived, the reflection and transmission coefficients of the multilayer are related by a similar matrix equation:

$$\begin{bmatrix} 1 \\ n_0 \end{bmatrix} + \begin{bmatrix} 1 \\ -n_0 \end{bmatrix} r = M_1 M_2 \ldots M_N \begin{bmatrix} 1 \\ n \end{bmatrix} t = M \begin{bmatrix} 1 \\ n \end{bmatrix} t = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 1 \\ n \end{bmatrix} t \qquad (18)$$

where

$$r = \frac{An_0 + Bnn_0 - C - Dn}{An_0 + Bnn_0 + C + Dn} \qquad (19)$$

$$t = \frac{2n_0}{An_0 + Bnn_0 + C + Dn} \qquad (20)$$

The reflectance R and transmittance T are then given by $R = |r|^2$ and $T = |t|^2$, respectively. In order to obtain high value of reflectance in a multilayer film, a stack of alternative layers of high index, n_h and low index, n_l materials is used. The thickness of each layer being quarter wavelength and the product of two adjacent ones is:

$$\begin{bmatrix} 0 & -\frac{j}{n_h} \\ -jn_h & 0 \end{bmatrix} \begin{bmatrix} 0 & -\frac{j}{n_l} \\ -jn_l & 0 \end{bmatrix} = \begin{bmatrix} -\frac{n_l}{n_h} & 0 \\ 0 & -\frac{n_l}{n_h} \end{bmatrix} \qquad (21)$$

If the stack consists of 2N layers, the transfer matrix of the complete multilayer stack is

$$M = \begin{bmatrix} (-\frac{n_h}{n_l})^N & 0 \\ 0 & (-\frac{n_l}{n_h})^N \end{bmatrix} \qquad (22)$$

Fro simplicity both n and n_0 are assumed to be unity, so the reflectance of multilayer stack can be calculated as follows:

$$R = \left[\frac{(n_h/n_l)^{2N} - 1}{(n_h/n_l)^{2N} + 1} \right]^2 \qquad (23)$$

The reflectance thus approaches unity for large N.

3 Results and Conclusion

In this paper a 1D cavity resonators utilizing the concept of lossless metamaterials in which both permittivity and permeability possess negative real values at given frequencies with multilayer dielectric mirror, is introduced. The slab of metamaterial having negative permittivity and permeability functioned as a phase compensator. As it is shown, when the cavity is filled with two layers of materials; the first layer assumed to be a lossless conventional material and the second layer is taken to be the metamaterial with negative permittivity and permittivity, the nontrivial 1D solutions for such a cavity, in principle, depend on the ratio of thicknesses of the two layers, not the sum of thicknesses. In other words, the cavity can conceptually be thin and can still be resonant, as long as the ratio of thicknesses is satisfied in the special dispersion relation. This arrangement provides possibility for having subwavelength thin compact cavity resonators and other components.

Regarding the multilayer dielectric mirror, for example, a 20layer stack of zinc sulfide $(n_h = 2.3)$ and magnesium fluoride $(n_l = 1.35)$ gives a reflectance of more than 0.999. This maximum reflectance, of course occurs only at one wavelength and make this structure a good candidate for applying instead of mirrors.

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