Componentwise Program Construction:
The Equivalent Transformation Computation Model versus the Logic Programming Model

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Abstract: In the equivalent transformation (ET) computation model, a program is a set of procedural rewriting rules for answer-preserving transformation of problems with respect to given background knowledge. In this paper, we discuss an approach to program construction by creating and accumulating individually correct and efficient program components one by one, referred to as componentwise program construction. Basic requirements for componentwise program construction are identified, based on which we show that the ET model possesses several desirable properties for this program construction approach, in particular, compared with the logic programming model. In addition, we compare the expressive power of program components in the ET model and that of components in the logic programming model by viewing computation in the latter model as problem transformation using only one specific class of rewriting rules, i.e., single-head general unfolding-based rules, and then demonstrating that a larger class of rules is needed for effective computation.

Key–Words: Componentwise program construction, Equivalent transformation, Rule-based computation, Program synthesis, Rewriting rules, Computation paradigm

1 Introduction

Equivalent transformation (ET) is one of the most fundamental principles of computation, and it provides a simple and general basis for verification of computation correctness. Computation by ET was initially implemented in experimental natural language understanding systems at Hokkaido University during 1990–1992 [1], and the idea was further developed into a new computation model, called the ET model [2, 3]. A program in this model is a set of prioritized rewriting rules for answer-preserving transformation of problems, and a problem solving process consists in successive rule application. Besides extensive use in the domain of first-order terms, the model has been applied in several data domains, including RDF and XML (e.g. in [5]).

As opposed to declarative computation paradigms such as logic programming (LP) and functional programming (FP), programs are clearly separated from specifications in the ET model. A specification defines a set of problems of interest and provides background knowledge for declaratively determining the answer sets of the problems. From a specification a program consisting of rewriting rules is constructed. When applied in the domain of first-order terms and first-order atoms, questions often arise as to whether the ET model offers advantages over the LP model.

Advantages of the ET model are best seen from the viewpoint of program synthesis. In this paper, comparisons between the ET model and the LP model as regards componentwise program construction is discussed. Such a program construction approach demands a high level of program component independency—it requires that the correctness and the efficiency of a program component can be verified and evaluated, respectively, independently of other components—and, moreover, capabilities of a partial program to provide a meaningful clue to creation of new components towards completing the program. Limitations of the LP model in regard to these requirements are identified, and how the ET model complies with the requirements is described. Expressive power of program components in the two models are compared by viewing computation in the LP model as problem transformation using only one specific class of rewriting rules in the ET model, and then showing that a larger class of rules is necessary for enhancement of problem solving ability.
To start with, basic requirements for component-wise program construction are described in Section 2. Sections 3 and 4 analyze the two computation models with respect to these requirements. Section 5 illustrates the need for a wide variety of program components and reveals limitations of program components in the LP model as to the expressiveness aspect.

2 Basic Requirements

A program \( prg \) is partially correct with respect to a specification \( S \) iff for each problem \( prb \) defined by \( S \), \( prg \) yields the correct answer set of \( prb \) provided that it terminates when executing \( prb \). It is correct with respect to \( S \) iff it is partially correct with respect to \( S \) and it terminates when executing each problem defined by \( S \). Componentwise program construction is a process of generating a correct and sufficiently efficient program by creating and accumulating individually correct and efficient program components one by one on demand. This program construction approach imposes the following requirements:

(Rq1) The correctness (respectively, the efficiency) of a program component can be defined (respectively, measured).

(Rq2) The correctness (respectively, the efficiency) of a component can be verified (respectively, evaluated) independently of other components.

(Rq3) Accumulation of individually correct (respectively, individually efficient) components yields a partially correct (respectively, an efficient) program.

(Rq4) There is a wide choice of components that may be added towards completing a program.

(Rq5) Appropriate components to be added towards completing a program can be selected and created at low cost.

The first three requirements enable decomposition of program partial correctness and program efficiency into component correctness and component efficiency, respectively. The fourth requirement is concerned with the possibility of obtaining high-quality programs, especially in regard to program efficiency and program termination. It is complemented by the fifth requirement, which is concerned with the cost of a component accumulation process. These requirements altogether form a basis for analysis of the componentwise program construction in the LP model and in the ET model in the following two sections.

let \( D = \emptyset \) and \( G' = G \)
let \( Pred \) be the set of all predicates occurring in \( G \)
repeat
(a) select and remove a predicate \( p \) from \( Pred \)
(b) \( G'' := G' \setminus \{p\} \)
(c) find a predicate definition \( D_p \) of \( p \) such that
   (i) \( G'' = M(D_p \cup \text{unit}(G'')) \)
   (ii) \( D_p \) is efficient
(d) \( D := D \cup D_p \)
(e) \( G' := G'' \)
until \( Pred = \emptyset \)

Figure 1: Componentwise program construction in the LP model

<table>
<thead>
<tr>
<th>Set</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G )</td>
<td>( G_{pal} \cup G_{rv} \cup G_{ap} \cup G_{palpal} )</td>
</tr>
<tr>
<td>( G_{pal} )</td>
<td>{ ( \text{pal}(t) \mid t ) is a ground palindrome list }</td>
</tr>
<tr>
<td>( G_{rv} )</td>
<td>{ ( \text{rv}(t_1, t_2) \mid t_1, t_2 ) are ground lists, and ( t_1 ) is the reverse of ( t_2 ) }</td>
</tr>
<tr>
<td>( G_{ap} )</td>
<td>{ ( \text{ap}(t_1, t_2, t_3) \mid t_1, t_2, t_3 ) are ground lists, and appending ( t_2 ) to ( t_1 ) yields ( t_3 ) }</td>
</tr>
<tr>
<td>( G_{palpal} )</td>
<td>{ ( \text{palpal}(t) \mid t ) is a ground list, and ( \text{pal}({1}[t]) ) and ( \text{pal}({2}[t]) ) are palindromes }</td>
</tr>
</tbody>
</table>

Table 1: Background knowledge

3 A Solution in the LP Model

3.1 Componentwise Program Construction

In the LP paradigm, a set of definite clauses is regarded as a program, and a specification is a pair \( \langle G, Q \rangle \), where \( G \) is a set of all true ground atoms in the problem domain, which is assumed in this paper to be known beforehand and is regarded as background knowledge, and \( Q \) is a set of atoms, each of which represents a query of interest. A program is executed using some proof engine, e.g., a Prolog-like SLD-resolution-based inference engine.

For any atom \( a \), let \( \text{rep}(a) \) denote the set of all ground instances of \( a \). Given a set \( Q \) of atoms, let \( \text{rep}(Q) = \bigcup \{ \text{rep}(q) \mid q \in Q \} \). Given a set \( D \) of definite clauses, let \( M(D) \) denote the minimal model of \( D \). A program construction problem in the LP model can be formalized by:

Given a specification \( \langle G, Q \rangle \), find a set \( D \) of definite clauses such that \( M(D) \subseteq G \) and \( \text{rep}(Q) \cap G \subseteq M(D) \), and \( D \) is efficient with respect to \( Q \).

Predicate definitions, each of which is a set of definite clauses with the same head predicate, are often regarded as program components. Following the componentwise program construction approach, a logic program is constructed from a given specification \( \langle G, Q \rangle \) by using the algorithm in Fig. 1. To illus-
3.2 Analysis

Taking predicate definitions as program components, we now analyze componentwise program construction in the LP model. Referring to Fig. 1, Condition (c) provides the notion of correctness of a predicate definition $D_p$ with respect to $G'$ and $G''$. This notion is independent of any other predicate definition existing in $D$. Moreover, a set of individually correct predicate definitions is always partially correct when it is regarded as a logic program. Requirements (Rq1), (Rq2), and (Rq3) are fully fulfilled in regard to the correctness aspect.

As to the efficiency aspect, when a node in an SLD-tree is expanded using one predicate definition, each clause in the definition possibly yields one child node, and, therefore, the number of resulting child nodes is bounded above by the number of clauses in the definition. As such, the number of clauses in the definition serves as a measure of its efficiency—the smaller the number is, the more efficient the definition tends to be (when it is regarded as a program component). The number of clauses, however, provides only a rough measure—the exact number of resulting child nodes may be far less than the number of clauses in the definition since the unification of a goal atom and the heads of some clauses in the definition may fail.

In general, as a predicate definition contains fewer definite clauses, the number of clauses in the definition provides a more accurate efficiency measure (i.e., a lower upper bound for the exact number of child nodes resulting from node expansion). Since a logic program typically involves several predicate definitions with multiple definite clauses, approximation of program efficiency based on the number of clauses in predicate definitions tends to be inaccurate. Requirements (Rq1), (Rq2), and (Rq3) are thus only poorly satisfied in regard to the efficiency aspect.

As will be discussed in Section 5, from the viewpoint of computation, predicate definitions are regarded as components of only one specific kind in the ET model, i.e., single-head general unfolding-based rules, and Requirement (Rq4) is not satisfied accordingly. (It will also be shown in Section 5 that employment of such a restricted class of components alone makes it impossible to construct correct logic programs for dealing with some class of queries.) As to Requirement (Rq5), although any arbitrary predicate occurring in a given background knowledge can be selected at Step (a) of Fig. 1, construction of a predicate definition often involves creation of a number of definite clauses and the overall construction cost can be considerable. The first row of Table 2 concludes this analysis.

4 A Solution in the ET Model

4.1 Computation by ET: An Introductory Example

Assume again as background knowledge the set of true ground atoms defined by Table 1. Consider the problem “find all ground terms $t$ such that $palpal(t)$ is true.” This problem is represented in the ET model as a set consisting of a single definite clause

$$\text{ans}(X) \leftarrow \text{palpal}(X),$$

where $\text{ans}$ stands for “answer,” and this definite clause is intended to mean “$X$ is an answer if $\text{palpal}(X)$ is true.” The rewriting rules in Fig. 3 are devised for solving this problem. Table 3 illustrates a sequence of problem transformation steps by successive application of these rules, where atoms to which the rules are applied are underlined and the rule applied in each step is given in the last column. The transformation sequence changes the initial problem “find all ground terms $t$ such that $palpal(t)$ is true” into the singleton set $\{\text{ans}([]) \leftarrow \}$, which means “the empty list is an answer (unconditionally) to the problem and there exists no other answer.” The correctness of this computation can be verified by proving

![Table 2: Comparisons between the LP model and the ET model](image-url)
that each rule in Fig. 3 is an answer-preserving rule with respect to the background knowledge in Table 1.

**4.2 Componentwise Program Construction**

In the ET model, a problem is represented as a set of definite clauses. To find its answer set, a given problem is successively transformed by using rewriting rules into a simpler but equivalent problem from which the answer set can be readily obtained. A program in this model is a set of prioritized rewriting rules, and a specification is a pair \( \langle G, Q \rangle \), where \( G \) is a set of all true ground atoms in the problem domain, i.e., background knowledge, and \( Q \) is a set of problems of interest. It is required that the head predicate of each clause in a problem in \( Q \) does not occur in \( G \). The answer set of a problem \( prb \) with respect to a specification \( \langle G, Q \rangle \) is defined as the set

\[
\{ a \mid (a \leftarrow b_1, \ldots, b_n) \text{ is a ground instance of some definite clause in } prb \text{ and } \{b_1, \ldots, b_n\} \subseteq G \},
\]

which is referred to as \( T_{prb}(G) \).

In order to assure correct computation results, only answer-preserving rewriting rules—correct rewriting rules—are used. A rewriting rule is said to be correct with respect to a set \( G \) of ground atoms iff for any problems \( prb \) and \( prb' \), if the rule transforms \( prb \) into \( prb' \), then \( T_{prb}(G) = T_{prb'}(G) \). Obviously, the correctness of one rewriting rule does not depend on any other rewriting rule, and can be checked individually. Requirements (Rq1) and (Rq2) are thus fulfilled as to the correctness aspect. A correct rewriting rule with respect to \( G \) is called an \textit{ET rule} with respect to \( G \).

A program construction problem in this model is formalized as follows:

Given a specification \( \langle G, Q \rangle \), find a set \( P \) of prioritized ET rules such that \( P \) can compute the answer set of each problem in \( Q \) successfully and \( P \) is efficient with respect to \( Q \).

An algorithm for componentwise program construction in the ET model is shown in Fig. 4.

**4.3 Analysis**

ET rules are program components in the ET model. How ET rules fulfill Requirements (Rq1) and (Rq2) in regard to the correctness aspect has been discussed in Subsection 4.2. Since a set of unit clauses obtained from a sequence of problem transformation steps using ET rules always yields a correct answer set, accumulation of ET rules always results in a partially correct program. Requirement (Rq3) as to the correctness side is also fully satisfied.

As to the efficiency side, a rule with fewer bodies is considered as a more efficient program component inasmuch as it narrows down a search space. Obviously, the number of rule bodies is an individual property of a rule. In the LP model, a program often
5 Necessity for a Large Variety of Rewriting Rules

Referring to Fig. 3, a classification of rules will be introduced. The need for a large variety of rule classes will be explained by pointing out that severe restriction on the choice of program components in the LP model is the root cause of the failure of logic programs in finding the answer set of the query illustrated in Subsection 4.1.

The rule \( r_{rv2} \) in Fig. 3 makes replacement of two atoms simultaneously (see, e.g., its application to the sixth problem in Table 3), and is called a multi-head rule. Every other rule in the figure replaces a single atom at a time, and is called a single-head rule. Each single-head rule in the figure operates as an unfolding rule using some definite clauses, and is called an unfolding-based rule. Note that the multi-head rule \( r_{rv2} \) is devised based on the functionality of the “reverse” relation, and its operation is completely different from unfolding. The rules \( r_{pal} \) and \( r_{pal} \) are applicable to any \( pal \)-atom and \( pal \)-atom, respectively, containing any arbitrary term, and are called general rules. All other rules in the figure are applicable to atoms having certain specific patterns, and are called specialized rules. Employment of specialized rules allows content-based control of computation [3]—an appropriate transformation step can be decided based on the run-time contents of clauses occurring in a computation state.

When computation by SLD resolution is viewed in the ET framework, expansion of a node (generation of its children) in a search tree for finding SLD-refutations corresponds to an unfolding transformation step. Accordingly, computation in logic programming can be seen as computation using only one specific class of rewriting rules, i.e., single-head general unfolding-based rules. By employment of such a restricted class of rules alone, it is often difficult to achieve effective computation control, in particular, for preventing infinite computation or for improving computation efficiency.

As an example, consider a logic program consisting of the definite clauses in Fig. 2, which is constructed from the set \( G \) of true ground atoms in Table 1, and the query “find all ground terms \( t \) such that \( palpal(t) \) is true” illustrated in Subsection 4.1, which is represented in LP as the single goal atom \( palpal(X) \). When executing this query, this logic program enters infinite computation after giving \( X = [] \), i.e., it fails to infer that the empty list is the “only” possible ground instance of \( X \) that satisfies the query, and thus, does not yield the correct answer set. Indeed, as will shown in this section, any logic program for checking palindromes fails to terminate when ex-
executing this query.

This difficulty is overcome in the ET model by content-based control of computation and the possibility of employing several types of rewriting rules, including specialized rules and multi-head rules. To illustrate, attention will now be drawn to the role of the multi-head rule \( r_{rv2} \) in successful termination of the transformation sequence in Table 3. First, consider the two atoms \( \text{pal}(1[X]) \) and \( \text{pal}(2[X]) \) in the second problem in the table, and the case when \( X \) is instantiated into a nonempty ground list, say \( l_X \). By the definition of \( G_{pal} \) in Table 1, the first \( \text{pal} \)-atom restricts the last element of \( l_X \) to 1, whereas the second one restricts it to 2. This contradiction proves nonexistence of any answer other than \( X = [] \). In terms of computation, finding this contradiction involves exchange of information about the restrictions on \( l_X \) between descendants of the first \( \text{pal} \)-atom and those of the second \( \text{pal} \)-atom in a computation process.

Now consider the transformation step using the multi-head rule \( r_{rv2} \) in Table 3. Prior to this transformation step, \( X \) is the only information connection between the two groups of descendants. Via this connection, however, the constraint on the last element of \( l_X \) cannot be exchanged in a finite form. For example, passing the restriction “the last element of \( l_X \) must be 1” through \( X \) to the descendants of the \( \text{pal}(2[X]) \) entails passing infinitely many patterns such as \( X = [1] \), \( X = [v_1, 1] \), \( X = [v_1, v_2, 1] \), \( X = [v_1, v_2, v_3, 1] \), and so forth, where the \( v_i \) are newly introduced variables. With this information connection alone, a contradiction must be found for each such pattern, one at a time, leading to infinite computation.

The multi-head rule \( r_{rv2} \) provides a simple remedy. It creates an additional information connection, i.e., \( A_1 \), through which the constraint on the last element of \( l_X \) can be passed to \( ap(A_1, [2], [2[X]]) \) as a single finite pattern, i.e., \( A_1 = [1[v]] \), where \( v \) is some new variable. Consequently, the all-embracing contradiction can be found using a finite number of transformation steps. Such an additional information connection (common variable) cannot be created by single-head rules. Since computation in logic programming corresponds to computation using only single-head general unfolding-based rules, any logic program for checking palindromes has no information connection other than \( X \) when executing the query \( \leftarrow \text{pal}(1[X]), \text{pal}(2[X]) \), and therefore fails to terminate.

6 Concluding Remarks

Theoretically, program synthesis can be viewed as a search for a sufficiently efficient program in a certain space of correct programs with respect to a given specification. The chance that such a program can be found increases as the program space is larger. Clear-cut separation between specifications and programs in the ET model opens up the possibility of extending a correct-program space. A specification in this model provides background knowledge for declaratively defining the answers to problems, whereas a program consists of procedural rewriting rules for computing the answers by program transformation. A very large class of rules can be employed—any rule whose application always results in answer-preserving transformation with respect to given background knowledge can serve as an ET rule. As a result, various classes of rules, with varying expressive power, can be introduced. For example, in the Equivalent Transformation Interpreter (ETI) system² developed at Hokkaido University, rules with guard conditions (possibly involving extra-logical predicates), rules with execution parts, and multi-head rules are provided.

The algorithm for componentwise program construction discussed in Section 4 provides a basis for rule generation in the second phase. Based on this framework, a program synthesis system has been implemented and used for constructing many nontrivial programs.

References:


²ETI is an interpreter system that supports ET-based problem solving. It is available at http://assam.cims.hokudai.ac.jp/etpro.