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Stability Analysis of a Sliding-Mode Speed Observer during Transient State

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Abstract: - The recent adaptive sliding-mode speed observer [1] is stable in the Lyapunov sense under a constant speed condition. During transient state, this observer may become momentarily unstable because mechanical dynamics could prominently appear in the time derivative of Lyapunov function ($\dot{V}$). In effect, $\dot{V}$ may be either positive or negative. A feasible analysis of the transient stability of this observer is to determine two important solutions according to the quadratic inequality concerning angular acceleration of the rotor. As a consequence, the theorem of Lasalle's invariant set was employed to explain stability scenario since prior ending of transient state up to steady state. Some simulation results are shown to indicate whether this observer is stable during the transient state.

Key-Words: - Induction motor; Speed observer; Transient state; Quadratic inequality; Invariant set; Lyapunov function

Nomenclature

B_t	Viscous friction coefficient (N·m·s/rad)
f_s	Stator or supply frequency (Hz)
i_{eq}	Equivalent current in proportion to electromagnetic torque
i_s	Stator current vector
$i_{s\alpha}$, $i_{s\beta}$	α and β components of stator current (A)
J_t	Moment of inertia (kg·m^2)
K	2×2 surface gain matrix
k_{si} , k_{ri} , k_{mi} , $k_{\omega i}$	Integral gains
k_{sp} , k_{rp} , k_{mp} , $k_{\omega p}$	Proportional gains
K_T	Torque constant
L_r	Rotor self-inductance (H)
L_s	Stator self-inductance (H)
M	Mutual inductance (H)
p	Number of poles
R_m	Core loss resistance (Ω)
R_r	Rotor resistance (Ω)
R_s	Stator resistance (Ω)
s	Slip
S_i	Surface vector
T_e	Electromagnetic torque (N·m)
T_L	Load torque (N·m)
U_o	Correction vector
V	Lyapunov or scalar function
v_s	Stator voltage vector
α , β	Components in fixed stator coordinates
σ	Total leakage factor
ψ_r	Rotor flux linkage vector
$\psi_{r\alpha}$, $\psi_{r\beta}$	α and β components of rotor flux (Wb)
ω_s	Stator angular velocity (rad/sec)
ω_m	Mechanical shaft speed (rad/sec or rpm)
ω_r	Electrical rotor (angular) speed (rad/sec)
$\dot{\omega}_r$	Rotor (angular) acceleration or deceleration (rad/sec^2)
ω_{sl}	Angular velocity of slip (rad/sec)

1 Introduction

The recent development of an adaptive sliding-mode speed observer [1] provides a practical observer that is stable in the Lyapunov sense under almost constant speed of motor revolution. During transient state, the rotor acceleration $(\dot{\omega}_r)$ is not zero, and stability has not yet been guaranteed. The observer is thus suitable for steady-state operation such as paper mill, rubber extrusion, etc. This limitation could be overcome if the observer stability in transient state were guaranteed. Without this limit, the application of the observer for servo control would be possible.

This article investigates transient stability of the adaptive sliding-mode observer via LaSalle's Theorem. Firstly, description of selecting a Lyapunov function for the PI adaptive laws is imparted. Secondly, detailed investigations and discussions on transient stability of the observer via an invariant set are presented. Thirdly, simulation results are presented to verify the claim.

2 Lyapunov Function for PI Adaptive Laws

In the earlier work [1], an improvement of parameter estimations inside the adaptive sliding-mode speed observer was accomplished by four proportional plus integral (PI) adaptive laws instead of its solitary integral elements. In order to ensure its stability, a Lyapunov function candidate will be chosen and examined. Our speed observer for a three phase induction motor (IM) taking core-loss into account is written as follows:

$$\left(d\hat{i}_s/dt\right) = \hat{A}_{11}\hat{i}_s + \hat{A}_{12}\hat{\psi}_r + B_1 v_s + \hat{D}_1\hat{\psi}_r + U_o\,, \tag{1}$$

$$\left(d\hat{\psi}_r/dt\right) = \hat{A}_{21}\hat{i}_s + \hat{A}_{22}\hat{\psi}_r + \hat{D}_2\hat{\psi}_r\,, \tag{2}$$

where the meaning of each term is given in Appendix. Four PI adaptive laws running parallel to the observer concurrently estimate stator, rotor, and core-loss resistances and rotor angular speed of the induction motor. These PI laws are denoted compactly as

$$\dot{\hat{R}}_s = -k_{sp}\dot{\Theta}_S - k_{si}\Theta_S\,, \tag{3}$$

$$\dot{\hat{R}}_r = k_{rp}\dot{\Theta}_R + k_{ri}\Theta_R\,, \tag{4}$$

$$\dot{\hat{R}}_m = -k_{mp}\dot{\Theta}_m - k_{mi}\Theta_m\,, \tag{5}$$

$$\dot{\hat{\omega}}_r = -k_{\omega p}\dot{\Theta}_\omega - k_{\omega i}\Theta_\omega\,, \tag{6}$$

where $\Theta_S = \Theta_S\left(S_i\,,\hat{i}_s\right) = S_i^T\hat{i}_s$,

$\Theta_R = \Theta_R\left(S_i\,,\hat{\psi}_r\,,\hat{i}_s\right) = S_i^T\left(\hat{\psi}_r - M\hat{i}_s\right)$,

$\Theta_m = \Theta_m\left(\hat{s}\,,S_i\,,\hat{\psi}_r\right) = \left(L_r - \hat{s}M\right)S_i^T\hat{\psi}_r$,

$\Theta_\omega = \Theta_\omega\left(S_i\,,\hat{\psi}_r\right) = S_i^T\left(J + \dfrac{R_m}{\omega_s L_r}I\right)\hat{\psi}_r$,

and whole PI gains must be only positive values (k_{sp}, k_{rp}, k_{mp}, $k_{\omega p} > 0$ and k_{si}, k_{ri}, k_{mi}, $k_{\omega i} > 0$). When an integral type of four adaptive mechanisms is replaced by the PI portion, an enhanced Lyapunov function is selected and written in

$$\begin{aligned} V \;=\; & \frac{1}{2}S_i^T S_i + \frac{1}{2\sigma L_s k_{si}}\left(\Delta R_s - k_{sp}\Theta_S\right)^2 \\ & + \frac{1}{2\varepsilon L_r k_{ri}}\left(\Delta R_r + k_{rp}\Theta_R\right)^2 + \frac{1}{2\varepsilon M L_r k_{mi}}\left(\Delta R_m - k_{mp}\Theta_m\right)^2 \\ & + \frac{1}{2\varepsilon k_{\omega i}}\left(\Delta\omega_r - k_{\omega p}\Theta_\omega\right)^2 \;\geq\; 0 \end{aligned} \tag{7}$$

For simplifying the derivative of this Lyapunov function, a product [1] between the transpose of the surface vector and the derivative of the same with respect to time is expressed as

$$\begin{aligned} S_i^T\dot{S}_i = \; & S_i^T\left\{\left(A_{11}+K\right)e_i + \left(A_{12}+D_1\right)e_\psi - U_o\right\} \\ & - \frac{\Delta R_s}{\sigma L_s}\Theta_S + \frac{\Delta R_r}{\varepsilon L_r}\Theta_R - \frac{\Delta R_m}{\varepsilon M L_r}\Theta_m - \frac{\Delta\omega_r}{\varepsilon}\Theta_\omega \end{aligned} \tag{8}$$

Subsequently, differentiating the Lyapunov function along time brings forth

$$\begin{aligned} \dot{V} = \; & S_i^T\dot{S}_i \\ & + \frac{1}{\sigma L_s k_{si}}\left(\Delta R_s\Delta\dot{R}_s - k_{sp}\dot{\Theta}_S\Delta R_s - k_{sp}\Theta_S\Delta\dot{R}_s + k_{sp}^2\Theta_S\dot{\Theta}_S\right) \\ & + \frac{1}{\varepsilon L_r k_{ri}}\left(\Delta R_r\Delta\dot{R}_r + k_{rp}\dot{\Theta}_R\Delta R_r + k_{rp}\Theta_R\Delta\dot{R}_r + k_{rp}^2\Theta_R\dot{\Theta}_R\right) \\ & + \frac{1}{\varepsilon M L_r k_{mi}}\left(\Delta R_m\Delta\dot{R}_m - k_{mp}\dot{\Theta}_m\Delta R_m - k_{mp}\Theta_m\Delta\dot{R}_m + k_{mp}^2\Theta_m\dot{\Theta}_m\right) \\ & + \frac{1}{\varepsilon k_{\omega i}}\left(\Delta\omega_r\Delta\dot{\omega}_r - k_{\omega p}\dot{\Theta}_\omega\Delta\omega_r - k_{\omega p}\Theta_\omega\Delta\dot{\omega}_r + k_{\omega p}^2\Theta_\omega\dot{\Theta}_\omega\right) \end{aligned} \tag{9}$$

By substituting the product in Eq. (8) and four adaptive laws in Eqs. (3) to (6) into Eq. (9), and assuming that R_s, R_r, R_m and ω_r are almost constant in comparison with system dynamic of state variables, then $\dot{V}$ is truncated into a shorter form as

$$\begin{aligned} \dot{V} = \; & S_i^T\left\{\left(A_{11}+K\right)e_i + \left(A_{12}+D_1\right)e_\psi - U_o\right\} \\ & - \frac{k_{sp}}{\sigma L_s}\Theta_S^2 - \frac{k_{rp}}{\varepsilon L_r}\Theta_R^2 - \frac{k_{mp}}{\varepsilon M L_r}\Theta_m^2 - \frac{k_{\omega p}}{\varepsilon}\Theta_\omega^2 \;\leq\; 0 \end{aligned} \tag{10}$$

where $S_i^T\left\{\left(A_{11}+K\right)e_i + \left(A_{12}+D_1\right)e_\psi - U_o\right\} \leq 0$. Eq. (7) and Eq. (10) signify that if an induction motor connected in alignment with its load rotates at a constant shaft speed, the speed observer always and usually remains stable because $\dot{V}$ is negative semidefinite. During transient state, the rotor angular acceleration or deceleration $\left(\dot{\omega}_r\right)$ is not zero, the observer may or may not be stable in the Lyapunov sense because $\dot{V}$ may be positive or negative.

The next section investigates the observer stability during transient state.

3 Investigation Regarding Stability during Transient State

Whenever the rate of change of the rotor speed varies, system mechanical dynamic will influence $\dot{V}$. This yields

$$\begin{aligned}\dot{V} = {} & S_i^T\left\{\left(A_{11}+K\right)e_i+\left(A_{12}+D_1\right)e_\psi-U_o\right\} \\ & -\frac{k_{sp}}{\sigma L_s}\Theta_S^2-\frac{k_{rp}}{\varepsilon L_r}\Theta_R^2-\frac{k_{mp}}{\varepsilon M L_r}\Theta_m^2-\frac{k_{\omega p}}{\varepsilon}\Theta_\omega^2 \\ & +\frac{1}{\varepsilon k_{\omega i}}\left(\Delta\omega_r-k_{\omega p}\Theta_\omega\right)\dot{\hat{\omega}}_r\end{aligned} \qquad ,(11)$$

, and the rotor speed is expressed by

$$\omega_r = -\frac{J_t}{B_t}\dot{\hat{\omega}}_r+\frac{pK_T}{2B_t}i_{eq}-\frac{p}{2B_t}T_L\,, \qquad (12)$$

where $i_{eq} = i_{s\beta}\psi_{r\alpha}-i_{s\alpha}\psi_{r\beta}$ and $K_T=\left(3p/4\right)\left(M/L_r\right)$. As an outcome, the Lyapunov function in Eq. (7) and its derivative in Eq. (11) become scalar functions. By superseding Eq. (12) into Eq. (11), one could obtain

$$\dot{V} = -\frac{J_t}{\varepsilon k_{\omega i}B_t}\dot{\hat{\omega}}_r^2+\frac{\Omega_E}{\varepsilon k_{\omega i}B_t}\dot{\hat{\omega}}_r+\dot{V}^*\,, \qquad (13)$$

where

$$\Omega_E = \Omega_E\left(T_e, T_L, \hat{\omega}_r, \Theta_\omega\right) = \frac{p}{2}\left(T_e-T_L\right)-B_t\left(\hat{\omega}_r+k_{\omega p}\Theta_\omega\right)$$

and

$$\begin{aligned}\dot{V}^* = {} & S_i^T\left\{\left(A_{11}+K\right)e_i+\left(A_{12}+D_1\right)e_\psi-U_o\right\} \\ & -\frac{k_{sp}}{\sigma L_s}\Theta_S^2-\frac{k_{rp}}{\varepsilon L_r}\Theta_R^2-\frac{k_{mp}}{\varepsilon M L_r}\Theta_m^2-\frac{k_{\omega p}}{\varepsilon}\Theta_\omega^2\end{aligned}\,.$$

Eq. (13) prescribes that, in addition to Eq. (10), whereas the rotor speed is changing, the estimated rotor speed $\left(\hat{\omega}_r\right)$, $\dot{\hat{\omega}}_r$, the mechanical parameters of the system, and the difference between electromagnetic torque ($T_e = K_T i_{eq}$) and load torque (T_L) affect stability of the speed observer through the function $\dot{V}$. Then, in order to elucidate the condition for transient stability, Eq. (13) is rearranged as an inequality

$$-J_t\dot{\hat{\omega}}_r^2+\Omega_E\dot{\hat{\omega}}_r+\varepsilon k_{\omega i}B_t\dot{V}^* \le 0 \qquad .(14)$$

where $\dot{\hat{\omega}}_r$ is a variable which must be further resolved to its solutions. Subsequently, Ineq. (14) is rewritten as

$$\dot{\hat{\omega}}_r^2-\frac{\Omega_E}{J_t}\dot{\hat{\omega}}_r-\frac{\varepsilon k_{\omega i}B_t}{J_t}\dot{V}^* \ge 0 \qquad .(15)$$

Within a period of time, if

$$\dot{\hat{\omega}}_r \ge \frac{\Omega_E}{2J_t}+\frac{\sqrt{\Omega_E^2+4\varepsilon k_{\omega i}B_tJ_t\dot{V}^*}}{2J_t}$$

or $\dot{\hat{\omega}}_r \le \dfrac{\Omega_E}{2J_t}-\dfrac{\sqrt{\Omega_E^2+4\varepsilon k_{\omega i}B_tJ_t\dot{V}^*}}{2J_t}$, Ineq. (15) is true, and $\dot{V}$ in Eq. (11) is negative. Thus, the speed observer is stable. However, if

$$\frac{\Omega_E}{2J_t}-\frac{\sqrt{\Omega_E^2+4\varepsilon k_{\omega i}B_tJ_t\dot{V}^*}}{2J_t} \le \dot{\hat{\omega}}_r \le \frac{\Omega_E}{2J_t}+\frac{\sqrt{\Omega_E^2+4\varepsilon k_{\omega i}B_tJ_t\dot{V}^*}}{2J_t}$$

, Ineq. (15) becomes false, and $\dot{V}$ in Eq. (11) becomes positive. In addition, the solutions $\dot{\hat{\omega}}_r$ must be real. Then, the observer becomes unstable. Hence, $\Omega_E^2+4\varepsilon k_{\omega i}B_tJ_t\dot{V}^*$ represents a discriminant. $\dfrac{\Omega_E}{2J_t}+\dfrac{\sqrt{\Omega_E^2+4\varepsilon k_{\omega i}B_tJ_t\dot{V}^*}}{2J_t}$ and $\dfrac{\Omega_E}{2J_t}-\dfrac{\sqrt{\Omega_E^2+4\varepsilon k_{\omega i}B_tJ_t\dot{V}^*}}{2J_t}$ stand for the upper bound and the lower bound of $\dot{\hat{\omega}}_r$, respectively. The term $\Omega_E/\left(2J_t\right)$ means a mid-point quantity between the upper and the lower bounds. While an induction motor with its load starts rotating from standstill or changing speed due to disturbances, the discriminant, the upper bound, the lower bound, and the mid-point quantity of $\dot{\hat{\omega}}_r$ are also varying along the instantaneously $\hat{\omega}_r$, the mechanical dynamic, and Eq. (10). Therefore, the speed observer may be either stable or unstable, depending upon the location of $\dot{\hat{\omega}}_r$ whether it is contained within the bounds. When time elapses adequately and the motor-load system rotates with $\dot{\hat{\omega}}_r$ decreasing successively till being less than a trivial level, the discriminant becomes smaller and commences to be negative. Regarding this, the distance between the upper and the lower bounds will become narrower, and then the two bounds meet together at a point in time before they vanish. So, if $\dot{\hat{\omega}}_r$ is further lower toward zero (i.e. the rotor speed tends to be constant.), $\dot{V}$ becomes continuously merely negative. Hence, the speed observer becomes stable because

$$\left(\dot{\omega}_r - \frac{\Omega_E}{2J_t}\right)^2 - \frac{\Omega_E^2 + 4\varepsilon k_{\omega i} J_t B_t \dot{V}}{4J_t^2} \geq 0\,, \tag{16}$$

Ineq.(16), Ineq.(15) in arrangement of completing the square, is true. Whereas the observer enters steady state, $\dot{V}$ decays to zero.

4 Explanation Concerning Stability via an Invariant Set

In order to assert stability of the speed observer since prior ending of transient state up to complete steady state, a short review of the invariance principle attributed to LaSalle [2] is given. Consider an autonomous system

$$\dot{\vec{x}} = f(\vec{x}), \tag{17}$$

where $\vec{x}$ is a state variable vector. Theorem: Let V be a positive scalar function with its continuous first derivative for the system in Eq. (17). Let Ξ_C be the region or set containing all members of $\vec{x}$ such that $\dot{V} \leq 0$. Let Ξ_O be the region or set whose members are all $\vec{x}$ satisfying the condition that $\dot{V} = 0$ only. Moreover, let Ξ_I be the largest invariant region or set within Ξ_O. Owing to $\Xi_I \subset \Xi_O \subset \Xi_C$, then, every trajectory of $\vec{x}$ originating in Ξ_C approaches Ξ_I as time passes sufficiently long where V must be a Lyapunov function candidate.

According to the theorem, two error equations [1] between the motor-load system and the speed observer, dealing with obtaining three sets of Ξ_C, Ξ_O, and Ξ_I, can be expressed as

$$\begin{aligned}\dot{e}_i = {} & A_{11}e_i + (A_{12} + D_1)e_\psi + \Delta A_{11}\hat{i}_s \\ & + (\Delta A_{12} + \Delta D_1)\hat{\psi}_r - U_o\end{aligned} \quad ,(18)$$

$$\dot{e}_\psi = A_{21}e_i + (A_{22} + D_2)e_\psi + \Delta A_{21}\hat{i}_s + (\Delta A_{22} + \Delta D_2)\hat{\psi}_r \quad .(19)$$

Under the situation of $\dot{\omega}_r \neq 0$, the discriminant and $\dot{V}$ become negative simultaneously. Besides, the discriminant must be further negative successively. Thereafter, when the rotor speed reaches steady state, $\dot{V}$ is normally negative semidefinite. By this reason, a set of Ξ_C is given as

$$\begin{aligned}\Xi_C = \{ & e_i, e_\psi \in \Re^2 \mid \exists\Delta R_s, \exists\Delta R_r, \exists\Delta R_m, \exists\Delta\omega_r, \exists\dot{\omega}_r \\ & \text{if } \dot{\omega}_r \neq 0 \text{ then } \Omega_E^2 + 4\varepsilon k_{\omega i} B_t J_t \dot{V} \leq 0 \\ & \text{or if } \dot{\omega}_r = 0 \text{ then } \dot{V} \leq 0 \}\end{aligned} \quad ,(20)$$

where $\Re^2$ is a set of column vector with any two real numbers. From Eq.(7) and Eq. (10), while $\dot{\omega}_r$ becomes equal to zero, V is a decreasing function of t (i.e. $V(t) \leq V(0)$). When time goes by adequately long ($t \to \infty$), $S_i \to 0$, $e_i \to 0$, $\Theta_S \to 0$, $\Theta_R \to 0$, $\Theta_m \to 0$, and $\Theta_\omega \to 0$ as well as ΔR_s, ΔR_r, ΔR_m, and $\Delta\omega_r$ converge to their corresponding constant values in steady state. Thus, a set of Ξ_I is written as

$$\begin{aligned}\Xi_I = \{ & e_i, e_\psi \in \Re^2 \mid \exists\Delta R_s, \exists\Delta R_r, \exists\Delta R_m, \exists\Delta\omega_r, \\ & \dot{\omega}_r \to 0,\ e_i \to 0,\ \dot{e}_i \to 0,\ \dot{e}_\psi \to 0, \\ & \dot{V} \to 0 \}\end{aligned} \quad .(21)$$

Through Ineq. (16), at the instance of $\dot{\omega}_r \neq 0$, when the discriminant declines to zero, $\dot{\omega}_r$ equals a mid-point quantity as well as $\dot{V}$ equals zero momentarily. In other events, $\dot{V}$ tends to zero in steady state. Thereby, a set of Ξ_O is given as

$$\begin{aligned}\Xi_O = \{ & e_i, e_\psi \in \Re^2 \mid \exists\Delta R_s, \exists\Delta R_r, \exists\Delta R_m, \exists\Delta\omega_r, \exists\dot{\omega}_r \\ & \text{if } \dot{\omega}_r \neq 0 \text{ then } \Omega_E^2 + 4\varepsilon k_{\omega i} B_t J_t \dot{V} \leq 0 \\ & \text{and } \dot{\omega}_r = \frac{\Omega_E}{2J_t} \} \cup \Xi_I\end{aligned} \quad .(22)$$

Caused by $\Xi_I \subset \Xi_O \subset \Xi_C$, e_i, $e_\psi \in \Xi_C$ move onto e_i, $e_\psi \in \Xi_I$ as $t \to \infty$.

5 Simulation Results

Simulations are carried out to verify stability of the speed observer during transient state. According to direct-on-line starting, at the initial instant of time ($t = 0$) the motor previously de-energized at standstill is connected directly to a 220 V, 50 Hz three-phase ac sinusoidal supply [1]. An actual load torque is supposed to be constant. The speed observer receives measurable stator voltages and currents in order to on-line update stator, rotor, and core-loss resistances as well as estimate rotor angular speed and flux linkage of the induction motor. The resultant discriminant, the upper bound and the lower bound of $\dot{\hat{\omega}}_r$, and the mid-point quantity between two these bounds are revealed in Fig. 1 while the derivative of V ($\dot{V}$) in Eq. (13) is shown in Fig. 2. Because $\dot{\hat{\omega}}_r$ cannot be a complex number, the square root of the absolute value of the discriminant is computed in lieu of the square root of the discriminant. Thus, whenever this discriminant becomes negative continuously, $\dot{\hat{\omega}}_r$ is without the two above bounds being meaningless, for example, since the point 'K' of time in Fig. 1.

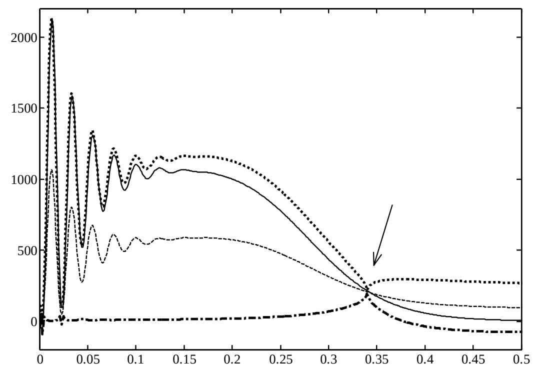

Fig. 1 The discriminant, the upper bound and the lower bound of $\dot{\hat{\omega}}_r$, and the mid-point quantity

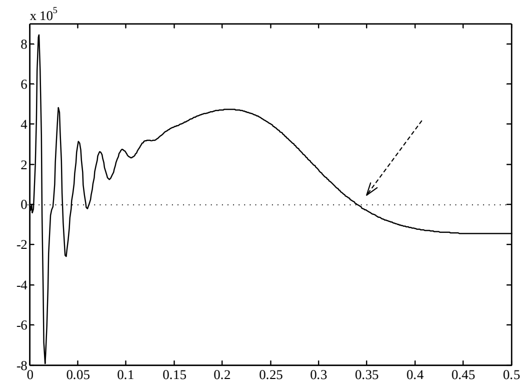

Fig. 2 $\dot{V}$ during transient state

When the motor speed ω_r increases or decreases, $\dot{V}$ can be positive or negative. During transient state, $\dot{V}$ and V may oscillate. As time goes by, the oscillation of $\dot{\hat{\omega}}_r$ dies down and decreases continually. $\dot{V}$ shows a similar pattern, and finally $\dot{V} \leq 0$, for example, since the point 'K' of time in Fig. 2.

6 Conclusion

This article has shown that when the motor speed changes due to set-point change or load disturbance, the adaptive sliding-mode speed observer becomes unstable for a short moment before regaining stability. In practice, this unstable period can be shorten by increasing the gains of the PI adaptive laws such that the differences between motor dynamic and observer dynamic are decreased. However, care must be taken that high gains do not amplify chattering, noise, and harmonic so that speed estimation is ever increased. To obtain optimum gains is still an open question. Previous studies [1][3] have shown that the observer is always stable with positive PI-gains under steady state motor operation.

Appendix : Meaning of Each Term in the Speed Observer

The symbol '^' indicates the estimated values or vectors. The meaning of the symbols used is clarified in the Nomenclature, and the matrices of the speed observer are as follows:

$$\hat{i}_s = \left[\hat{i}_{s\alpha} \quad \hat{i}_{s\beta}\right]^T , \ \hat{\psi}_r = \left[\hat{\psi}_{r\alpha} \quad \hat{\psi}_{r\beta}\right]^T ,$$

$$\hat{A}_{11} = \hat{a}_{r11} I = -\frac{1}{\sigma L_s}\left(\hat{R}_s + \frac{M^2 \hat{R}_r}{L_r^2}\right) I , \ B_1 = \frac{1}{\sigma L_s} I ,$$

$$\hat{A}_{12} = \frac{1}{\varepsilon}\left(\frac{\hat{R}_r}{L_r} I - \hat{\omega}_r J\right) , \ \hat{A}_{21} = \frac{M\hat{R}_r}{L_r} I ,$$

$$\hat{A}_{22} = -\varepsilon \hat{A}_{12} , \ \hat{D}_2 = -\frac{\hat{s}\hat{R}_m}{L_r} I , \ \hat{\omega}_m = \frac{2\hat{\omega}_r}{p} ,$$

$$\hat{D}_1 = -\frac{\hat{R}_m\left(L_r - \hat{s}M\right)}{\varepsilon M L_r} I , \ \hat{s} = \frac{\hat{\omega}_{sl}}{\omega_s} = \frac{\omega_s - \hat{\omega}_r}{\omega_s} ,$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} , \ J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} , \ \omega_s = 2\pi f_s ,$$

$$\sigma = 1 - \frac{M^2}{L_s L_r} > 0 \text{ , and } \varepsilon = \frac{\sigma L_s L_r}{M} > 0 .$$

U_o is the correction vector laid to compel the estimation error to zero [1]. Let the mismatches between the estimated and the actual vectors as well as between the estimated and the actual parameters be

$$e_i = \begin{bmatrix} e_{i\alpha} \\ e_{i\beta} \end{bmatrix} = \begin{bmatrix} i_{s\alpha} - \hat{i}_{s\alpha} \\ i_{s\beta} - \hat{i}_{s\beta} \end{bmatrix} ,$$

on (rad/sec²)

I : Upp

II : Ro

III : M

IV : Lo

$$e_{\psi} = \begin{bmatrix} e_{\psi\alpha} \\ e_{\psi\beta} \end{bmatrix} = \begin{bmatrix} \psi_{r\alpha} - \hat{\psi}_{r\alpha} \\ \psi_{r\beta} - \hat{\psi}_{r\beta} \end{bmatrix}$$

$$\Delta R_s = R_s - \hat{R}_s \ , \ \Delta R_r = R_r - \hat{R}_r \ , \ \Delta R_m = R_m - \hat{R}_m$$
$$\text{and} \quad \Delta\omega_r = \omega_r - \hat{\omega}_r .$$

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