Pricing the Resource in Computational Market Based on Bayes-game Model

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Abstract: - This paper uses Bayes-game model to analysis the incomplete-information dynamic game, in which the resource providers and users of the computational market ask and bid around the potential equilibrium price. The linear strategy is used to find the equilibrium solution of the Bayes-game, set the deal condition for the resource exchange, and acquire the cost function of resource providers and the benefit function of resource users.

Key Words: - Resource Pricing; Computational Market; Bayes-game Model

1.1 Introduction

It’s reasonable for the appearance of computational market (CM) [1, 2, and 3]. In the current day, IT has been developed rapidly, and more and more computing equipments are used in common enterprises and departments to form enormous computing power. While an phenomenon that could not be ignored is that resource is wasted so much due to the scarcity of applications, many equipments are left unused most of the time; while at the same time, many other enterprises or departments that have much computing applications could not get the necessary computing power because of limitation of budget or scale. Under this dilemma, a new exchange platform for computing resource, CM, could well solve the contrary problem of supply and demand of resource. In CM, it uses the pricing method and the mature mechanism of the real economic system to rationally allocate computing resources so as to promote the income of resource suppliers, reduce the cost of resource demanders, and finally add the whole social welfare.

Until now the research of pricing the resources in CM hasn’t advanced so much. In [4, 5, and 6], the authors use the theory of commodity market to exchange the computing resources. They suppose that the market is in a state of complete competition, so the clearing price could get from the equilibrium of supply and demand. But the resources in CM are heterogeneous, and the condition of indifference of goods couldn’t be satisfied, which is a necessary one for the complete competitive market. So the method they suggest is not well done in the CM. In [3, 7], the authors price the resources from the view of multi-unit combinatorial auction. They want to allocate the computing resource according to the outcome of auction, which is gained by the method of finding the winning bids set, and at the same time the price could be set. The limitation of these papers is that the resource suppliers are at a dominate position against the resource demanders with the rule of the auction because winner determination problem in multi-unit combinatorial auction aims to maximize the income of the resource suppliers, and it doesn’t consider the cost of the demanders.

In CM, the resource supplier and the resource demanders will try to maximize their own benefit separately, thus variation of the resource price will form the dynamic game with incomplete information. This paper uses the Bayes auction pricing model to lead the suppliers and the demanders to finish the resource deal around the potential equilibrium price so as to reach the goal of the maximized profit of both side and the maximization of social welfare.

There are many kinds of resources in CM. In [8], it uses the concept of “computon” to describe the abstract computing resource. “The concept for Computons is a bundle of CPU, bandwidth, and storage that you buy.” In [12], it gives a method to standardize each kinds of computing resource and to form computon. Here suppose the synthesizing process of computon has been done. Though in [12], it has given a method to pricing the computon, this paper will give another method to pricing the computing resource. It will use Bayes-game model to reach the goal.

The remainder of this paper is structured as follows: Section 2 gives the Bayes-game Model [9, 10] and how to get the solution of it; Section 3 gets
the equilibrium price of the auction model after the analysis to the cost function and the benefit function of the participants in CM with the Bayes-game model; Section 4 concludes with a discussion and introduces the future work.

2. The Bayes-game model and the equilibrium solution

2.1 The Bayes-game model

Here we focus on the economic behaviors among the participants in CM. In which, the suppliers are the providers of computing resources and their behaviors are selling the commodity of computing power; the demanders are the users of computing resources and their behaviors are using the computing power bought to finish their applications. The supply and demand strategies with which the two sides bargain for the resource price influence the deal price and the deal quantity.

We use the Chatterjee and Samuelson model [9], which is model of a two-side auction. In this model, the value of the commodity to the seller is \( c \), to the buyer is \( v \), and the seller asks for \( p_s \), and the buyer bids for \( p_d \). The auction rule matches the ask price and the bid price to decide whether the deal is done. After the process of standardization, \( c, v \) are equally distributed in the field between 0 and 1, just as \( c, v \in [0,1] \). (The process of standardization is given in the third section.) If \( p_s \leq p_d \), the deal is done at the midpoint of the ask price and the bid price, which is \( p = (p_s + p_d)/2 \), so the utility of the seller is \( u_s = (p_s + p_d)/2 - c \), and the utility of the buyer is \( u_d = v - (p_s + p_d)/2 \). The deal will fail and the utility of both sides will be zero if \( p_s > p_d \).

When deciding whether the deal is done, only the seller (in CM, the seller is the supplier of computing resources) knows his own cost \( c \), and only the buyer (in CM, the buyer is the demander of computing resources) knows his own benefit \( v \) which he could get after having used the computing power bought. Because \( c, v \) are not common knowledge to both side, the auction period is an Bayes-game one. In this game, the seller’s strategy, which is the ask price \( p_s \), is an function of \( c \), we could describe it as \( p_s(c) \); the buyer’s strategy, which is the bid price \( p_d \), is an function of \( v \), we could describe it as \( p_d(v) \). The strategies of the ask price and the bid price form the Bayes-game. When the strategy profile \( (p_s^*(c), p_d^*(v)) \) fits the constraints followed, it is Bayes equilibrium.

For fixed \( p_s \), \( p_d^*(v) \) is the best solution of to following problem which could promise the seller’s maximized profit.

\[
A = \max_{v \in [0,1]} \left\{ \frac{1}{2} \left[ (p_s + E(p_d(v) | p_d(v) \geq p_s)) - c \right] \right\}
\]

In (1), \( E(p_d(v) | p_d(v) \geq p_s) \) is the seller’s expectation of the buyer’s bid price given the constraint that the ask price is lower than the bid price.

For fixed \( p_s \), \( p_d^*(v) \) is the best solution of to following problem which could promise the buyer’s maximized benefit.

\[
B = \max_{c \in [0,1]} \left\{ \left[ \frac{1}{2} (p_s + E(p(c) | p_s \geq p_s)) \right] \right\}
\]

In (2), \( E(p(c) | p_s \geq p_s) \) is the buyer’s expectation of the seller’s ask price given the constraint that the ask price is lower than the bid price.

Consider that in current CM, getting the detail information of the market participants is very difficult, so the equilibrium model with linear strategy is used in this paper to simple the complicated situation. (It will not lose generality from the view of doing research.) The linear strategy of both is defined as followed.

\[
p_s(c) = \alpha_s + \beta_s c
\]

\[
p_d(v) = \alpha_d + \beta_d v
\]

Because \( c \) and \( v \) are equally distributed in \([0,1]\), \( p_s \) and \( p_d \) are equally distributed in \([\alpha_s, \alpha_s + \beta_s]\) and \([\alpha_d, \alpha_d + \beta_d]\).

2.2 The solution of the equilibrium strategy

With the suppose that \( c \) and \( v \) are equally distributed in \([0,1]\), we could get the following conclusion.

\[
A = \max_{c \in [0,1]} \left\{ \left[ \frac{1}{2} (p_s + p_d + \alpha_s) - \frac{p_s - \alpha_s}{2} \right] \right\}
\]

\[
B = \max_{c \in [0,1]} \left\{ \left[ \frac{1}{2} (p_s + p_d + \alpha_d + \beta_d) - \frac{p_s - \alpha_d + \beta_d}{2} \right] \right\}
\]
Solving (5) and (6), we could get that:
\[ p_s = \frac{1}{3}(\alpha_s + \beta_s) + \frac{2}{3}c \]
\[ p_d = \frac{1}{3}\alpha_s + \frac{2}{3}v \]

With the constraint above, we could jointly get the linear equilibrium strategy, which is the equilibrium of Bayes-game.
\[ p_s^*(c) = \frac{1}{4} + \frac{2}{3}c \]
\[ p_d^*(v) = \frac{1}{12} + \frac{2}{3}v \]

The detailed analysis is in [9, 10].

2.2 The analysis to the deal
The supplier of computing resource and the demander of it game for the certain commodity - computing power. With (9) and (10), we could clearly analysis out that the behaviors of the two sides could be pointed out as Fig.1 under the linear equilibrium strategy. When \( c > \frac{3}{4} \), the supplier’s ask price is \( p_s(c) = \frac{1}{4} + \frac{2}{3}c \), which is lower than his cost, but is higher than the buyer’s highest bid price \( p_d(1) = \frac{3}{4} \), so the deal is failure.

When \( v < \frac{1}{4} \), the buyer’s bid price is higher than his valuation to the computing power he wants to buy, which is lower than the supplier’s lowest ask price \( p_s(0) = \frac{1}{4} \), so the deal is failure too.

If and only if \( p_d(v) \geq p_s(c) \), which means \( v \geq c + \frac{1}{4} \), the deal could be done. The field of successful deal under the linear equilibrium strategy is described in Fig. 2.

3. The valuation of the cost of the supplier and the benefit of the demander
In reality the functional valuation of the cost of the supplier and the benefit of the demander is a very complicate and difficult research work, we will not do that in the current paper. To use the method suggested above in CM, we give the simple definition of the two functional valuations from the usable view.

3.1 The supplier’s cost function
In CM, the variation of the resource price is ruleless, but we could suppose that its varying field could be valued [11]. In [11], the author uses the method of autoregressive model to compute the possible price field with the help of historical time-series data. Here we suppose the possibly highest resource price is \( p_0 \). So the resource supplier could game in the Bayes model based on \( p_0 \) with the demander and ask for computing power in an auction period. We suppose that the ask price is \( p_s \), and it is a
descending ask $p_s$.

When using the Bayes-game model described above, first we should standardize the supplier’s ask price and his supplying cost. Because the highest price is $p_0$, obviously in which the supplier’s income is comprised. We could conclude that $c_s \leq p_0$; since the supplier is asking for descending price, we have $p_s \leq p_0$. Now we could do the standardizing procession based on $p_0$, and the formula is followed:

$$p_s^* = \frac{p_s}{p_0}$$

$$c_s^* = \frac{c_s}{p_0}$$

When the supplier’s standardized ask price $p_s^*$ is equal to his standardized cost $c_s^*$, his lowest ask price is his cost price, from (9), we could get that:

$$c_s^* = \frac{3}{4}$$

$$p_s^* = \frac{3}{4}$$

With 11 14, we revert the standardized ask price to the actually ask price:

$$p_s = \frac{3}{4}p_0$$

3.2 The user’s benefit function

Next we will define the benefit function. First we define the concept of “benefit per unit of the resource user”, which is the benefit he could get when consuming one unit of computing power, we use $u$ to substitute it. Then we get the following equation:

$$v = u - p_d - o$$

In (16), $v$ is the actually anticipant benefit when he use one unit of computing power, $u$ is the value created by consuming one unit of computing power, $p_d$ is the bid price, and $o$ is the rest cost in the value created by consuming one unit of computing power, in which the human maintain cost, the developing cost of the application, and so on are included.

In CM, we know that,

$$u = \frac{t}{q}$$

and,

$$o = \frac{p_d}{k} - p_d$$

In (17), $t$ is the actual value created by the application with the bought resource; $q$ is the resource quantity used.

In (18), $k$ is the ratio of the payout of expense to total cost when using the computing resources. So

$$v = u - p - o = \frac{t}{q} - p_d - \left(\frac{p_d}{k} - p_d\right) = \frac{t}{q} - \frac{p_d}{k}$$

Also when using the Bayes-game model described, we should do the standardization procession to the bid price $p_d$ and the benefit $v$ of the resource user. Because $u$ is the actual value, it is used as the standardizing base of $p_d$ and $v$, and the formula is following:

$$p_d^* = \frac{p_d}{u}$$

$$v^* = \frac{v}{u}$$

We use (19) and (20) in (21) to get:

$$v^* = 1 - \frac{p_dq}{tk}$$

When the bid price $p_d^*$ is equal to $v^*$, the highest bid is his anticipant benefit. From (10), we have that

$$v^* = \frac{1}{4}$$

According (22) and (23), we could get the actual bid price, which is:

$$p_d = \frac{3tk}{4q}$$

According the Bayes-game model described above, when $p_s \leq p_d$, the two sides will finish the deal at the mid-point $p = \frac{1}{2}(p_s + p_d)$. Considering the marginal deal situation in which the supplier’s ask for the lowest price and the user bid for the highest price, the deal price is:

$$p = \frac{1}{2}(p_d + p_s) = \frac{1}{2}\left(\frac{3tk}{4q} + \frac{3}{4}p_0\right)$$

4. Conclusion and the future work

In this paper, we first introduce the Bayes-game model and the solution of it, then analysis the cost
function of the resource supplier and the benefit function of the resource user in CM. With several constraints and the standardizing work, we well use them in the Bayes model. After solving the solution to the Bayes-game model, the reasonable price of the resource deal in CM could be found naturally.

Since the development of CM is not complete and there isn’t any actual exchange data, we couldn’t prove the model with example, but we will do the research in the following time and use the data formed in the later time to validate the conclusion of this paper. Additionally, this paper simply separate the participants of CM to the resource suppliers and the resource demander, and only consider the game between them, but in most situation, the existence of mid-agent could influence the game condition and promote the efficiency of resource exchange, we will consider the situation in the following work.

Reference: