

# Preisach Hysteresis Modeling and Applications

APHRODITE KTENA and CHRISTOS MANASSIS

Department of Electrical Engineering  
Technological Education Institute of Chalkis  
Psahna, Euboea 34400  
GREECE

---

*Abstract:* - Preisach modeling, long known in the area of magnetics, has introduced mathematical abstraction to the modeling of the highly nonlinear and complex phenomenon of hysteresis. The 2D Preisach-type models presented here, departing slightly from the classical formulation, waive some of its limitations while maintaining the major advantages of simplicity and speed in calculations. Results on different types of ferromagnets are shown, as well as on magnetostrictive materials and shape memory alloys.

*Keywords:* - hysteresis, energy systems, Preisach, shape memory alloys, magnetostriction, ferromagnets

## 1 Introduction

Hysteresis is a non-linear phenomenon encountered in problems in magnetism, in plasticity, in economics, in biosystems even. The etymology of the word suggests that in a system exhibiting hysteresis (means *delay* in Greek) the output lags the input but the lag is a non-linear function of the past history as well as present input. It can be observed in a large number of input-output pairs in several systems and materials in nature, e.g. the magnetization, the magnetoimpedance or the strain may lag the applied magnetic field in magnetic and magnetostrictive materials; the strain may lag the stress in elastic materials, or the temperature in shape memory alloys; examples from systems include the turbidity in lakes with respect to the amount of the existing nutrients or the number of existing firms in an economy with respect to the exchange rate.

Hysteresis is the result of a complex network of interactions and competing energies among a material's particles, grains and phases or a system's constituent parts and their interaction with the external stimulus. It is related to energy dissipation, memory properties and metastability. From the stability point of view, there are more than one possible equilibrium states for a given input value. The state that is ultimately chosen by the system, as the one that minimizes its energy, depends on the history of the system, i.e., on previous equilibrium states, hence the memory property. Alternatively, an external stimulus may elicit a partly or totally irreversible response; therefore, the original state cannot be recovered by

levying the stimulus but further energy must be supplied in order to "assist" the system in returning back to the original state, hence the energy dissipation. Hysteresis may be a desired or undesired effect. In the first case it must be controllable. In the second it must be compensated. In both cases, modeling is required.

When we are interested in the stability of information or energy storage, as in the case of data storage media (tapes, disks) and permanent magnet applications, hysteresis is not only desired but also necessary [1]. Fig. 1 shows a typical hysteresis loop of a ferromagnet where the magnetic induction  $M$  is plotted against the applied magnetic field,  $H$ . The loop is characteristic of a given material. It is symmetric,  $B(H) = -B(-H)$ , and the field at which the magnetization becomes zero is known as the coercivity or coercive field,  $H_c$ . When  $H_c$  is negligible or zero the material exhibits no hysteresis and all processes are reversible. A high coercivity suggests strong hysteresis and therefore higher storage stability. For example, permanent magnets are used in motors and generators whose strength depends on the energy storage capability of the magnet which in turn is proportional to the energy product  $(\mathbf{B} \cdot \mathbf{H})_{\max}$  of the magnet's major loop, i.e. the bigger the hysteresis, the better the magnet. However, because of the nonlinearity of the response, hysteresis modeling is needed to determine the field  $H$  necessary to reach a certain induction value with relative accuracy. Modeling is also necessary to compute the core losses in transformers since the eddy currents

induced depends on the magnetization configuration of the material.

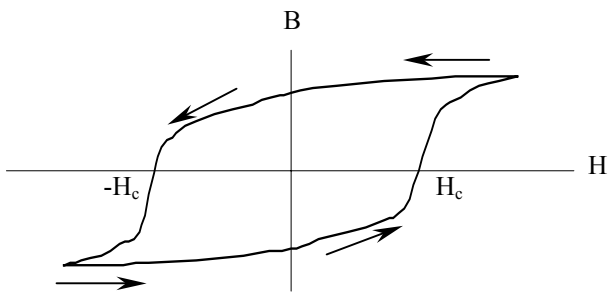


Fig. 1 – Hysteresis major loop of a ferromagnetic material

Hysteresis may be highly undesirable in sensing applications where the nonlinearity of the response adds to the uncertainty of the sensor [2]. Fig. 2 shows a typical hysteresis loop of a magnetostrictive material where the deformation  $\lambda$  is plotted against the applied magnetic field  $H$ . When a magnetostrictive material is exposed to an applied field, the magnetic dipoles, tending to align themselves with the field, apply stresses resulting in an elongation (positive magnetostriction) or shrinking (negative magnetostriction) of the material. This means that for a given elongation  $\lambda$  there are two possible field values causing it, depending on which branch the action is taking place. If a magnetostrictive material is used in a sensing arrangement, this uncertainty introduced by the inherent hysteresis must be compensated for.

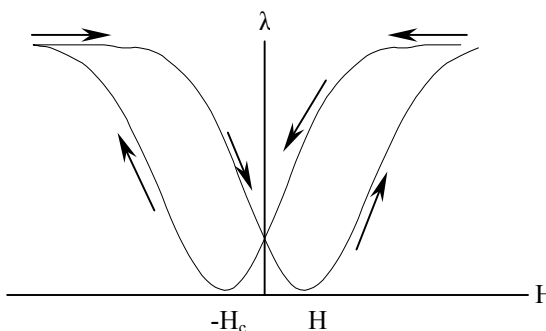


Fig. 2 – Hysteresis major loop of a magnetostrictive material

## 2 The model

Since hysteresis models are used as compensators, prediction tools or core models in simulations of a system's behavior, phenomenological models are better candidates than microscopic models based on

the minimization of the Gibbs energy equation. The Preisach formalism is probably the most popular class of phenomenological hysteresis models [3-4]. These models are abstract enough and therefore adjustable to a variety of materials; they account for the interactions in a statistical manner and therefore previous knowledge of the exact mechanism of hysteresis or energy terms involved is not necessary; they are efficient; they can be tuned to the material they model; and when Mayergoyz' theorem [4] applies, their accuracy is excellent.

The Preisach formalism postulates that hysteresis is the aggregate response of a distribution of elementary hysteresis operators. A hysteresis operator is the locus of the critical inputs at which irreversible switching (from one magnetization state to another, from one phase to another etc.) occurs. The classical Preisach model (CPM) predicts that, for an input  $u(t)$ , the output  $f(t)$  can be calculated as:

$$f(t) = \iint_{a \geq b} \rho(a,b) \gamma_{ab} u(t) da db \quad (1)$$

where  $\gamma_{ab}$  is a local hysteresis operator with switching points at  $a$  and  $b$  and  $\rho(a,b)$  is their probability density function.

### 1.1 Hysteresis operators $\gamma_{ab}$

The classical Preisach model (CPM) [4] is based on the relay-type operator shown in Fig. 3a. The output switches between  $+1$  and  $-1$  at the respective upper and lower switching points (fields),  $(a,b)$ . This pair of variables controls the width of the loop and its offset from the origin, thus incorporating the effect of interactions. The variations of this operator, shown in Figs. 3b and 3c, are mainly used in elastoplasticity. Notice, that the kp-operator (Fig. 3c) allows for a linear transition between the minimum and maximum values, and bi-directional horizontal movement at any point of the ascending or descending curve. These three operators are 1D allowing only for irreversible switching and therefore model only irreversible, scalar processes. When the hysteresis response of a system can be treated in a scalar manner, the models using these operators are quite reliable as long as the reversible component is added on.

However, hysteresis in materials is an inherently vector mechanism. In the case of ferromagnets, the response to an applied magnetic field has a reversible part, due to small rotations, as well as an irreversible part due to switching. Figs. 3d and 3e depict two 2D operators allowing both rotation and switching. The Stoner-Wohlfarth astroid (Fig. 3d), a well-known model in micromagnetics [5], is described by the

equation:  $h_x^{2/3} + h_y^{2/3} = 1$ , where  $h_x$  and  $h_y$  are the components of the applied field, normalized to the half-width of the astroid, along the easy and the hard axis of orientation of an anisotropic material. The normalized magnetization vector,  $\mathbf{m}$ , is the tangent to the astroid passing through the tip of the applied field vector,  $\mathbf{h}$ . Switching occurs when the magnetization vector, rotating from position  $\mathbf{m}_k$  to a new position  $\mathbf{m}_{k+1}$ , crosses the astroid from the inside out. The vector operator of Fig.3e is the first order approximation of the astroid and follows the same switching and rotation mechanism.

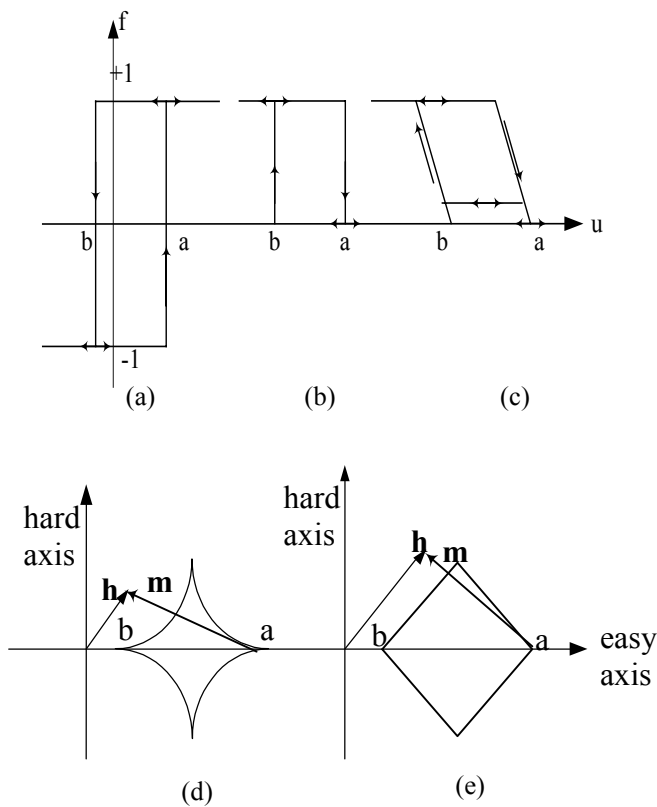


Fig. 3 – Hysteresis operators

### 2.2 The probability density function $\rho(a,b)$

The pair of variables  $(a,b)$  defines a half plane,  $a \geq b$ , known as the Preisach plane. The distribution of  $a$  and  $b$  is obtained from a probability density function  $\rho(a,b)$  defined over the Preisach plane, known as the Preisach or characteristic density.  $\rho(a,b)$  is characteristic of the material being modeled and as a consequence the identification of the model consists in determining this density of  $a$  and  $b$ . In other words, the shape of a material's major hysteresis loop, or any other trajectory inside it, is related to the shape of the

distribution, e.g. it is symmetrical in the cases of magnetic and magnetostrictive materials (Figs. 1-2), asymmetrical in the case of shape memory alloys (Fig. 6), and bimodal in AFC media (Fig. 8). It can be either reconstructed through detailed measurements of first order curves or as a weighted sum of bivariate normals whose parameters must be determined [6]. The first method is used in the case of the CPM [4]. It cannot be applied in the case of vector models or when these measurements cannot be carried out. The alternative method is based on the assumption that any pdf can be constructed as a sum of gaussians. It is more general because it needs only a major loop measurement and a curve-fitting procedure to determine the parameters of the pdf.

### 2.3 Vector model

In the vector formulation, the 1D hysteresis operator is replaced by a vector one and an additional gaussian  $\rho(\theta)$  is used to account for the angular dispersion [1]:

$$\mathbf{f}(t) = \int_{-\pi/2}^{\pi/2} \rho(\theta) d\theta \iint_{a \geq b} \rho(a,b) \gamma_{ab} \mathbf{u}(t) da db \quad (2)$$

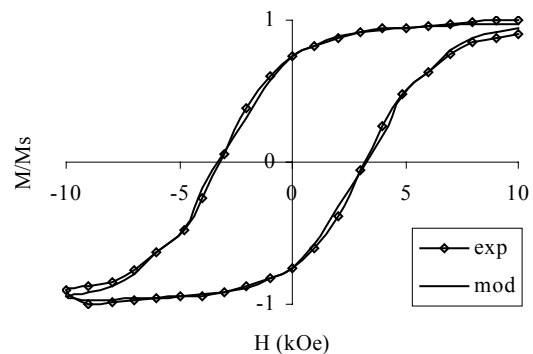


Fig. 4 – Experimental and calculated loop of a permanent magnet

## 3 Applications

In this section, applications of the presented model in the areas of ferromagnetism, magnetostriction and shape memory alloys are presented.

### 3.1 Ferromagnets

In ferromagnets, hysteresis occurs as the material switches from positive to negative magnetization and vice versa. An appropriate hysteresis model may be built using Eq. (2) along with a vector operator (Fig. 3e) and a bivariate gaussian. Fig. 4 shows the experimental vs. the calculated major loop of a SmFeN permanent magnet [6].

Another class of magnetic materials comprises thin films used for data storage, e.g. hard disks. Fig. 5 shows the experimental vs. the calculated ascending part of major loops of Gd-film samples that have been annealed at 610 °C and 560 °C prior to the hysteresis measurement. The higher the annealing temperature the sharper the anisotropy distribution is and the reversible component is minimized. The film annealed at 610 °C switches almost as a single dipole. Therefore, the scalar model is used. In the case of the other sample, a vector operator and the vector formulation must be used to capture the hysteresis behavior.

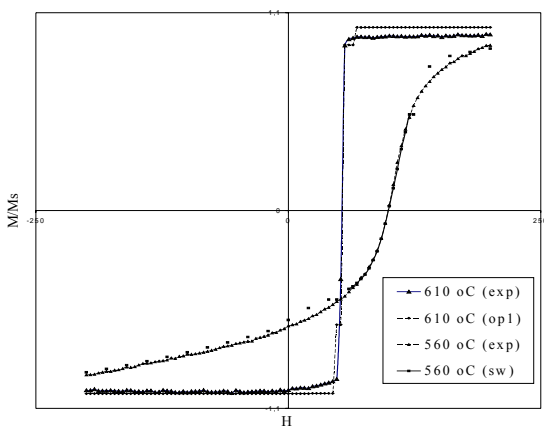


Fig. 5 – Experimental and calculated ascending major loops of thin film media annealed at 610°C and 560°C

Finally, the model is applied to antiferromagnetically coupled magnetic recording media. This is a relatively new class of materials promising higher recording aerial densities. AFC media consist of an upper and lower ferromagnetic layer, of thicknesses  $t_U$  and  $t_L$ , antiferromagnetically coupled through a nonmagnetic layer (fig.6a). It turns out, that, for this structure, the effective magnetic thickness parameter  $M_t t$  is the difference between the  $M_t t$  values of each layer. As a result, higher recording densities can be achieved while keeping higher thermal stability [7]. The upper layer is a hard magnetic medium with coercivity  $H_{CU}$  of a few kOe and the lower layer is soft with coercivity  $H_{CL}$  of a few hundred Oe. For large applied fields, the magnetizations of the two layers are parallel to each other but, as the field decreases below the value  $H_{EX}-H_{CL}$ , the antiferromagnetic coupling mechanism is activated and the lower layer switches (Fig.6b). The lower layer magnetization remains antiparallel to that of the upper layer until  $H_{CU}$  is reached and the upper layer switches as well. The effective exchange field  $H_{EX}$  is a measure of the AF coupling strength depending, among others, on the thickness of the

lower layer [7], i.e., the thinner the lower layer, the stronger the exchange bias and the resulting shift  $H_{EX}$ .

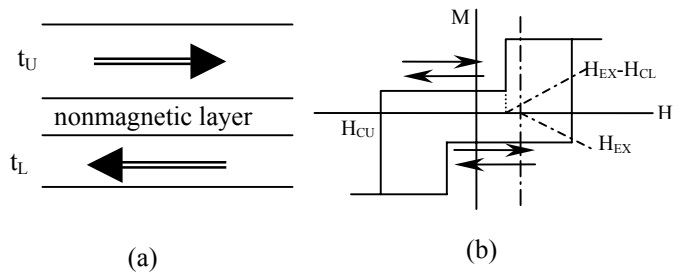


Fig. 6 – AFC media (a) layer structure (b) schematic of ideal hysteresis loop

Various approaches have been used to model this behavior. The approach presented here is to use the vector model of Eq. (2) and the operator of Fig. 3e along with a mixture of two normal pdfs, one for each layer:

$$\rho(a, b) = w_L \rho_L(a, b) + w_U \rho_U(a, b),$$

$$w_U = 1 - w_L \quad (3)$$

where  $\rho(a, b)$  is shifted by  $+H_{EX}$  for decreasing and  $-H_{EX}$  for increasing input fields  $H(t)$ . The results are shown in Fig. 7 for the major and as well as minor loops of a sample.

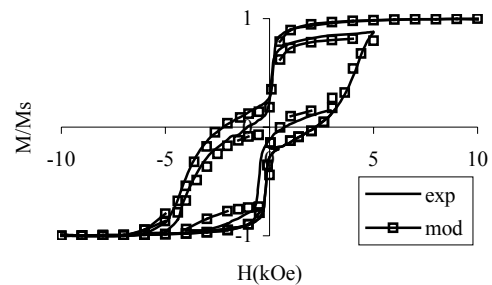


Fig. 7 – Experimental and calculated major and minor loops of an AFC sample

### 3.2 Magnetostrictive materials

Under an externally applied field, the change in the Zeeman energy density, due to the external magnetic field, is counterbalanced by a change in the elastic energy of the bonds [2]. This may result in an increase (positive magnetostriction) or decrease (negative magnetostriction) of the sample length along the direction of the applied field, which because of the changes in microstructure and the ensuing interactions is hysteretic. In order to generate the “butterfly” loop

of Fig. 2, the model of Eq. (1) has been used along with the operator of Fig. 3b and a density of the shape shown in Fig. 8.

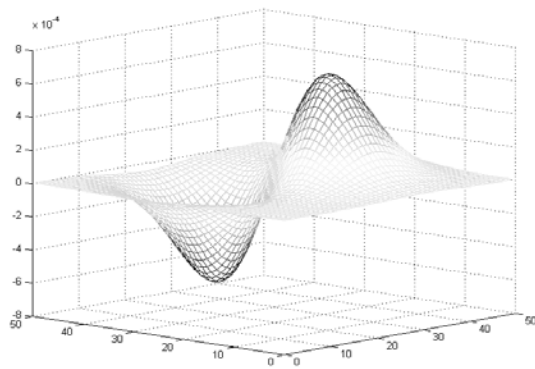


Fig. 8 –  $\rho(a,b)$  used in the case of magnetostrictive materials

### 3.3 Shape memory alloys (SMAs)

Hysteresis in SMAs is observed as the material undergoes a phase transformation from the martensitic to the austenitic phase and vice versa. The input variable is temperature,  $T(t)$ , and the output is strain  $x(t)$  [8]. In this case, Eq. (1) is used along with the operator of Fig. 3c and an asymmetrical bivariate gaussian [6]. Fig. 9 shows the calculated and experimental major loop of a NiTiInol sample.

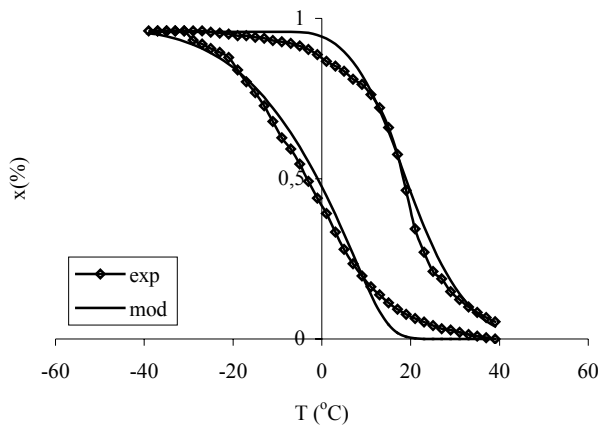


Fig. 9– Experimental and calculated loop of a shape memory alloy

## 4 Discussion

The Preisach formalism and the ensuing models presented here have performed well in all the cases studied. Although designed by Preisach [9] in the '30s to model hysteresis in ferromagnets, it has since been applied more or less successfully to a variety of

systems and materials. The models presented here attempt to levy some of the drawbacks of the classical formalism, such as the scalar response, and render it more flexible so that it adapts easily to different systems regardless of the underlying physical mechanism generating the hysteresis. Such a model cannot reveal much about the physics of the response and cannot be used to study the mechanism per se but it can be used in simulations as a core model since it yields results of acceptable accuracy, is implemented by fast algorithms and the only data needed for the identification are some points of the major hysteresis curve. Such a hysteresis model has already been used in a magnetic recording simulation, estimating the AC and DC erasure levels on a variety of commercially available magnetic tapes, with very good results [1]. The next goal is to use it in order to calculate transformer core losses.

### References:

- [1] A. Ktena and S. H. Charap, Vector Preisach Modeling and Recording Applications, *IEEE Trans. Magn.*, 29 (6), 3661-3663 (1993).
- [2] E. Hristoforou, Magnetostrictive Delay Lines: Engineering Theory & Applications, *Meas. Sci. & Technol.*, 14 R15 (2003).
- [3] S. H. Charap and A. Ktena, Vector Preisach modeling, *J. Appl. Phys.*, 73, 5818-5823 (1993).
- [4] I. D. Mayergoyz, Mathematical models of hysteresis, *Physical Review Letters*, 56(15), 1518-1521 (1986).
- [5] E.C. Stoner and E.P. Wohlfarth, A mechanism of magnetic hysteresis in heterogeneous alloys, *Phil. Trans. Roy. Soc.*, A240, 599-642 (1948).
- [6] A. Ktena, D.I. Fotiadis, P.D. Spanos, A. Berger and C.V. Massalas, Identification of 1D and 2D Preisach models for ferromagnets and shape memory alloys, *Int. J. Eng. Sci.*, 40(20), 2235-2247 (2002).
- [7] E.E. Fullerton, D.T. Margulies, N. Supper, Hoa Do, M. Schabes, A. Berger, and A. Moser, Antiferromagnetically coupled magnetic recording media, *IEEE Trans. Mag.*, 39(2), 639-644 (2003).
- [8] Zhonghe Bo, Dimitris C. Lagoudas, Thermomechanical modeling of polycrystalline SMAs under cyclic loading, Part IV: modeling of minor hysteresis loops, *International Journal of Engineering Science*, 37, 1205-1249 (1999).
- [9] F. Preisach, Über die magnetische Nachwirkung, *Zeitschrift für Physik*, 94, 277-301 (1935).