Abstract: - Conservation of historical buildings is important to societies that are eager to perpetuate the collective memory. These societies protect their cultural resources and preserve their heritage. To promote the use and preservation of historic buildings, attention must be paid when incorporating modern building service systems. These systems must be adapted to functional peculiarities and demands of the building elements. Winter heating and summer cooling of monumental building can be very demanding and energy consuming task. Consequently, the detailed calculation of their envelope is very important. Masonry buildings, tend to regulate the external temperature fluctuations they are subjected to, either by storing heat in their thermal mass or by offering it back to the interior, with a time lag depending on the thermal capacity of the building and characterized mainly by the overall mass of the masonry. This time lag may vary from a few hours when lightweight masonry is concerned to several days for masonry buildings of large mass. The scope of this work is to calculate the thermal capacity of Byzantine (334 – 1453 AD) monuments in Greece and to show how their thermal capacity affects their internal temperature temporal profile. The analysis will be based on a typical 6th AD century Byzantine Church in Thessaloniki Greece, dedicated to the Holy Wisdom of God our Lord (Aghia Sophia).

Key-Words: - building, masonry, Byzantine monuments, thermal capacity, modelling, simulation

1 Introduction

The energy efficiency problem in building design has been paused very early in the history. The main parameters of this design were the orientation of the building, the solar heating and the thermal inertia of the building. From the Geometric Period of the Greek History (9th to 6th centuries BC) there is a tendency (e.g. small village of Vroulia in Rhodes) of taking advantage of the winter sunshine benefits and of avoiding the strong and cold Northen winds. For example, excavations prove that there is a strong tendency that the windows in the buildings in Athens, in the Holy Island of Delos, in Olynthos and in Priene were positioned to the South or Southeast, whereas their walls were oriented in a way that covered the building from strong winds. The same rules apply for urban planning too. Towns and even large cities were oriented to the South or Southeast and inclined areas were sought for to take full advantage of the wind and the sunshine. This philosophy is vividly presented in the ancient Greek historian Xenophon books, where Socrates described the so called ‘Solar House’ and gave instructions for its orientation. Moreover, vivid examples for thermal storage and thermal inertia building philosophy are also to be found in the ancient Greece building tradition. Santorini and Knossos are some excellent examples dating back to the 15th century BC. In these constructions solid bricks and stones were used. The same building tradition is evident through the Byzantine era too and can be observed in houses, palaces and churches.

Conservation of historical buildings is important to societies that are eager to perpetuate the collective memory. These societies protect their cultural resources and preserve their heritage. To promote the use and preservation of historic buildings, attention must be paid when incorporating modern building service systems. These systems must be adapted to functional peculiarities and demands of the building
elements. Winter heating and summer cooling of monumental building, especially of large churches may prove a very demanding and energy consuming task. On the other hand, good and detailed analysis and dynamic simulation of the building envelop may certainly assist the function of modern building service systems and help the energy efficient performance of the building.

In the present work, the thermal capacity of Byzantine (334 – 1453 AD) monuments in Greece is analyzed and the way their thermal capacity affects their internal temperature temporal profile is presented. The analysis is based on a typical 6th AD century Byzantine Church in Thessaloniki Greece, dedicated to the Holy Wisdom of God our Lord (Aghia Sophia).

1.1 Thermal Capacity

By definition the thermal capacity of masonry is its capability to save thermal energy, by storing it in its mass during the heating period. The amount of heat stored, depends on the temperature difference between the masonry and the air surrounding it, the thermal capacity of the masonry, and the masonry mass. The masonry thermal capacity is determined by:

i. The specific thermal capacity \( c \), which is determined as the amount of heat required to increase the unit mass by 1o K.

\[
c = \frac{q}{\rho \theta}
\]

(1)

ii. The thermal capacity coefficient \( C \), which is determined by the amount of heat stored in \( a \) m2 of masonry when the temperature difference is 1o K. It is usually defined, for a harmonic temperature variation, as the amplitude of the heat flow \( q \) divided by the temperature amplitude \( \theta \) on the surface of the masonry and the radial frequency \( \omega \) or the period \( T \) of the variation.

\[
a = \frac{\lambda}{\rho c}
\]

(2)

iii. The thermal diffusion \( a \) is defined as the thermal transmittance coefficient \( \lambda \) divided by the density \( \rho \) and the specific thermal capacity \( c \)

iv. The active masonry mass is defined by the surface of the masonry \( A \) and its thermal capacity coefficient \( C \) divided by an arbitrary specific thermal capacity \( c_c \) = 1000 J/KgK.

\[
m^* = A \frac{C}{c_c}
\]

(3)

1.2 Thermal Capacity Effects on Internal Temperature - Thermal Inertia

The thermal inertia of a building shell strongly affects the fuel consumption since it presents a so call time lag. When the solar radiation falls on a solid non-transparent surface e.g. walls, roof etc, part of the radiation will be absorbed as heat. Part of the heat in turn will be emitted to the external environment, whereas the rest, the amount of which depends on the thermal diffusion characteristics of the building, will propagate through the wall or roof inside. On the other hand when the external temperature drops, the external surface temperature decreases and part of the energy stored is emitted to the external environment. For example during the summer nights the temperature in the building is usually higher than the external temperature. The heat flow then is from inside out. Therefore, especially for the mountainous parts of the southern European countries, characterized by hot and dry climatic conditions, with large daily temperature fluctuations, the thermal inertia contribution to natural cooling is extremely important in energy saving. The masonry therefore affects the thermal wave propagation in the following manner:

i. Shifts its phase and exhibits a temporal lag expressed by a temporal thermal constant \( \tau \).

ii. Absorbs the temperature variability by a damping factor \( D \).

The higher the thermal capacity of the building the larger the phase lag and the damping factor \( D \). The temporal thermal constant \( \tau \) signifies the time necessary for the internal air temperature to reach the 63% of the external temperature when on the external side of the building element a temperature \( \theta \) is measured. This can be expressed by:

\[
\theta(0) - \theta(\tau) = \theta_0(1 - e^{-1})
\]

(4)

A simplified expression for a large mass masonry unit, provided that the air is considered still, is:

\[
\tau \frac{Q}{U} = \frac{T_0 d}{2\pi \rho c}
\]

(5)
where: \( Q \) is the thermal energy stored in the masonry and \( U \) is its thermal transmittance, \( d \) is the width of the masonry and \( d^\ast \) is the active masonry width. Of course it has to be pointed out that \( \tau \) is affected by many environmental parameters as for example the moisture content of the masonry, its orientation, its solar radiation absorption capacity etc. \[15\]

The Decrement factor is given by:

\[ D = e^{-d/d^\ast} \]  \hspace{1cm} (6)

The temporal constant of the building is used in the thermal balance calculation of a building. According to EN 832, the thermal balance is given by:

\[ Q_{ht}t = (Q_1 - nQ_1)t \]  \hspace{1cm} (7)

where:

\( Q_{ht} \) the energy provided by the heating system
\( Q_1t \) the heat losses
\( Q_{st} \) the thermal gains i.e. passive solar gains or gains of other types.

\( n \) utilization factor referring to the internal thermal gains which describes the thermal accumulation capacity in the masonry. It is given by:

\[ n = \frac{1 - \gamma^n}{1 - \gamma^n} \quad \text{if} \quad \gamma \neq 1 \]  \hspace{1cm} (8a)

\[ n = \frac{\alpha}{1 + \alpha} \quad \text{if} \quad \gamma = 1 \]  \hspace{1cm} (8b)

\[ \gamma = \frac{Q}{Q_1} \]  \hspace{1cm} (9)

Fig. 1 Decrement factor of masonry
Red: Stone, Blue: Fired Clay Bricks

Apparently, for a high thermal capacity building where

\[ \gamma \neq 1 \]

\[ \alpha = \alpha_0 + \frac{\tau}{\tau_0} \]  \hspace{1cm} (10)

where: \( \tau \) is the thermal constant of the building in seconds or hours. It is a function of its thermal capacity and it depends on the total mass of the building. The thermal mass constant varies from a few hours for light-weight building to several days for large mass ones. It is given by the expression:

\[ \tau = \frac{C_{eff}}{U_A} \]  \hspace{1cm} (11)

The active thermal capacity \( C_{eff} \) of a heated volume \( V \) describes the thermal energy stored when the internal temperature is sinusoidally changing with time amplitude of 1oK for a given period. It is given by:

\[ C_{eff} = \sum_{k=1}^{n} C_k A_k \]  \hspace{1cm} (12)

1.3 Thermal Capacity Calculation Method

1.3.1 Analytical method

The method used is based on heat transfer between building elements consisting of one or more flat, parallel and homogeneous levels. The building element exhibits a harmonic temperature fluctuation described by eq. 13.
\[ T(x,t) = \bar{\theta}(x)e^{i(\omega t + \phi(x))} \] (13)

whereas the heat flow fluctuations can be described by:

\[ q(x,t) = \bar{q}(x)e^{i(\omega t + \phi(x))} \] (14)

Using the transfer matrix \([Z]\) the amplitude of the temperature \(\theta\) and of the heat flow \(q\) on one side of the building element can be calculated when the corresponding quantities \((\theta_0, q_0)\) on its other side are known.

\[
\begin{bmatrix}
\bar{\theta} \\
\bar{q}
\end{bmatrix}
= 
\begin{bmatrix}
\bar{z}_{11} & \bar{z}_{12} \\
\bar{z}_{21} & \bar{z}_{22}
\end{bmatrix}
\begin{bmatrix}
\bar{\theta}_0 \\
\bar{q}_0
\end{bmatrix}
\] (15)

The matrix elements describe various thermal characteristics of the building element including its internal and external thermal capacity. The procedure followed when calculating the above parameters has as follows:

i. The temporal period for the temperature calculations is selected.

ii. The homogeneous layers are characterized by the materials contained.

iii. The homogeneous layers dimensions are given.

iv. The thermal characteristics of the specific materials are defined.

v. The thermal diffusivity coefficient for each material is calculated.

vi. The transfer matrix elements are calculated for each layer.

vii. The matrices are accordingly multiplied in order to determine the building element matrix.

viii. The thermal capacity and the thermal characteristics of the building element under consideration are calculated.

For a flat, parallel and homogeneous layer characterized by a thermal transmittance coefficient \(\lambda\) and a width \(d\), the thermal resistance \(R\) is calculated by:

\[ R = \frac{d}{\lambda} \] (16)

and its thermal diffusivity coefficient \(b\) by:

\[ b = \sqrt{\lambda \rho c} \] (17)

Then using the period \(T\) the harmonic thermal transmittance coefficient is calculated by:

\[ \kappa = b \sqrt{\frac{\pi}{T}} \] (18)

The \(z_{ij}\) elements are calculated by (19):

\[ z_{11} = z_{22} = ch(\kappa R)\cos(\kappa R) + jsh(\kappa R)\sin(\kappa R) \]

\[ z_{12} = -\frac{sh(\kappa R)\cos(\kappa R) + ch(\kappa R)\sin(\kappa R)}{2\kappa} \]

\[ j\left[ ch(\kappa R)\sin(\kappa R) - sh(\kappa R)\cos(\kappa R) \right] \]

\[ z_{21} = -\kappa\left[ sh(\kappa R)\cos(\kappa R) - ch(\kappa R)\sin(\kappa R) \right] \]

\[ -j\kappa\left[ sh(\kappa R)\cos(\kappa R) - ch(\kappa R)\sin(\kappa R) \right] \] (19)

Given that the thermal capacity is given by eq. 1, and solving the system of equations, the internal temperature fluctuations, when the external ones are considered insignificant, are given with respect to the temperature absorbed and offered to the internal environment.

\[ C_i = \frac{T|z_{11}|}{2\pi|z_{12}|} \] (20)

The external temperature fluctuations, when the internal ones are considered insignificant, are given by:

\[ C_e = \frac{T|z_{22}|}{2\pi|z_{12}|} \] (21)

Finally, the thermal capacity of the elements inside the building is given by:

\[ C_i = \frac{T\left|(1+z_{22})(1-z_{11})+z_{12}z_{21}\right|}{2\pi|z_{12}|} \] (22)
1.3.2. Penetration depth method
If the first layer of the building element, i.e. of the element that is in touch with the internal environment, is at least double than the penetration depth, i.e. if eq. 23 is applicable:

\[ d > 2\delta = 2\sqrt{\frac{aT}{\pi}} \]  

(23)

The thermal capacity of the building element may then be approximated by:

\[ C = \sqrt{2\delta \rho c} \]  

(24)

1.3.3. Effective Width Method
A homogeneous wall is also considered by this method. The wall is unilaterally heated and the heat flow is taken sinusoidal with time. If:

i. the heat flow to the other side can be neglected i.e. if this wall is thoroughly and strongly insulated

ii. If the temperature fluctuation on the opposite to the heated side is equal to the temperature fluctuation at the same distance on the front face of a wall of infinite width consisting of the same material (this is applicable to large mass walls. According to ASHRAE 7 BTU/ft²F or 143 KJ/m²K)

Then the heat transfer calculation can be drastically simplified. The above criteria are usually satisfied by the masonry used in the Byzantine monuments, therefore the heat storage in well insulated wall for a given sinusoidal temperature fluctuation of period T, can be simulated by an isothermal wall of infinite conductivity, of the same density and specific heat but smaller width. The resulting width is called active wall width. The asymptotic of the active width is plotted against the real width is given by:

\[ d^* = \sqrt{\frac{\lambda T_o}{\rho c \pi}} \]  

(25)

whereas the thermal capacity is given by:

\[ c = d^* \rho c \]  

(26)

Roughly speaking the active width equals the real width if the latter is smaller than the former, whereas it retains its value when the active width is smaller than the real one.

2 Historical Data and Description of the Church
The church of Aghia Sophia, as all Greek Orthodox churches, is oriented from West to East and is a very heavy cubic building with an almost square plane view, its basic dimensions being 30.92 m wide by 28.90 m long or 40.06 m if the Sanctus tri-folio niche is included. Its interior forms a cross-like nucleus covered by a large cupola that is based on masonry friezes supported by four cylindrical bows. These bows are based on four large masonry pillars. Lateral naves and church-porch with balconies surround the three sides of the central nucleus in a π-like shape. On the upper structure the cupola largely extends over the whole wooden roof, covered with ceramic roof tiles. It stands on a heavy masonry drum base, cubic-shaped outside and rather ellipsoid inside, its two N-S and E-W main diameters being 11.60 m and 10.95 m respectively. The average diameter of the cupola is 10.16 m and it is illuminated by twelve bow-like windows, three on each side of the cupola. The square base of the cupola is quite shorter than the cupola itself forming a balcony.

The three folio vaulted Holy Altar forms a somewhat architecturally independent unit with respect to the rest of the cubic church. The Sanctuary, paced at the center of the Holy Altar, is covered by a semicircular masonry bow (kamara) and it ends to a semi-hexagonal niche. On the left and right side of it are two squares covered by semi-spherical domes placed of fringes. Five large windows illuminate the Holy Altar.

The masonry of the church is distinguished in four periods.

The initial masonry is made of five high, parallel, horizontal zones of roughly carved stones 1.15 cm high followed by five rows of fired clay solid bricks 40.0x(30.0 to 32.50) x (4.50 to 5.00) cm. Although the stones used are of limestone or sandstones of an inferior quality, the fired clay bricks are of excellent quality. The mortar consists of very cohesive well graded, milled fired clay of excellent quality. The milled fired clay offers excellent hydraulic performance of the mortar, together with fast humidity absorption.

The masonry of the 2nd period resembles that of the initial period, the only difference being in that the carved stones are substituted by non-carved ones. The bricks are of various dimensions and the bows are
exclusively made of fired clay bricks 3.00 to 3.50 cm high.

The masonry of the 3rd period consists of non-carved stones and joints filled with small stones and very small quantities of brick splinters.

The masonry of the 4th period is also made of series of non-carved stones and series of bricks. The mortar consists of lime, milled fired-clay, grit and ceramics broken to very small sizes. Compared to the mortar used during the 1st period, it contains smaller quantities of crusted fired-clay.

### 3 Results and Discussion

The thermal characteristics of the masonry units used are shown in Table 1.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Brick</th>
<th>Stone</th>
<th>Mortar</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$ (W/mK)</td>
<td>1.09</td>
<td>1.09</td>
<td>0.87</td>
</tr>
<tr>
<td>$\rho$ (kg/m3)</td>
<td>1900.00</td>
<td>1900.00</td>
<td>1900.00</td>
</tr>
<tr>
<td>$C$ (J/kgK)</td>
<td>790.00</td>
<td>790.00</td>
<td>1000.00</td>
</tr>
</tbody>
</table>

Table 1 Thermal characteristics of the masonry units

Table 2 shows the geometrical characteristics (m), the thermal resistance (m2K/W), the thermal transmittance (W/m2K), the active penetration depth (m), the thermal capacity (KJ/m2K) using the analytical method proposed by CEN TC89 WG4 and the time lag (h) of the masonry types mentioned above. Table 3 shows the total power losses (W/K), the total thermal capacity (MJ/K) and the total time lag (h) for the East, West, South and North sides, the Roof and for the whole building.

<table>
<thead>
<tr>
<th>Masonry Type</th>
<th>W</th>
<th>R</th>
<th>U</th>
<th>$d*$</th>
<th>c</th>
<th>$\tau$ (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bricks</td>
<td>0.85</td>
<td>0.780</td>
<td>1.055</td>
<td>0.141</td>
<td>203.23</td>
<td>22.98</td>
</tr>
<tr>
<td>Bricks</td>
<td>2.35</td>
<td>2.516</td>
<td>0.430</td>
<td>0.141</td>
<td>203.23</td>
<td>63.53</td>
</tr>
<tr>
<td>Stones</td>
<td>0.85</td>
<td>0.570</td>
<td>1.353</td>
<td>0.162</td>
<td>207.82</td>
<td>20.10</td>
</tr>
<tr>
<td>Stones</td>
<td>2.45</td>
<td>1.644</td>
<td>0.522</td>
<td>0.162</td>
<td>207.82</td>
<td>57.93</td>
</tr>
</tbody>
</table>

| Bricks*      | 0.85 | 0.814 | 1.018 | 0.141 | 203.23 | 22.98    |
| Bricks*      | 1.15 | 1.090 | 0.795 | 0.141 | 203.23 | 31.09    |
| Bricks*      | 1.30 | 1.227 | 0.717 | 0.141 | 203.23 | 35.15    |
| Bricks*      | 1.55 | 1.457 | 0.615 | 0.141 | 203.23 | 41.91    |
| Bricks*      | 1.70 | 1.594 | 0.567 | 0.141 | 203.23 | 45.96    |
| Bricks*      | 2.15 | 2.007 | 0.460 | 0.141 | 203.23 | 58.13    |
| Bricks*      | 2.35 | 2.190 | 0.424 | 0.141 | 203.23 | 63.53    |
| Bricks*      | 2.45 | 2.282 | 0.408 | 0.141 | 203.23 | 66.24    |
| Stones*      | 0.85 | 0.605 | 1.293 | 0.162 | 207.82 | 21.05    |
| Stones*      | 1.15 | 0.806 | 1.026 | 0.162 | 207.82 | 28.48    |
| Stones*      | 1.30 | 0.907 | 0.804 | 0.162 | 207.82 | 32.20    |
| Stones*      | 1.55 | 1.075 | 0.804 | 0.162 | 207.82 | 38.39    |
| Stones*      | 1.70 | 1.175 | 0.744 | 0.162 | 207.82 | 42.11    |
| Stones*      | 2.15 | 1.477 | 0.608 | 0.162 | 207.82 | 53.25    |
| Stones*      | 2.35 | 1.612 | 0.562 | 0.162 | 207.82 | 58.21    |
| Stones*      | 2.45 | 1.679 | 0.541 | 0.162 | 207.82 | 60.68    |
| Window       | 0.04 | 0.004 | 5.804 | 0.004 | 5.00   | 0.14     |
| Door         | 0.10 | 0.250 | 2.390 | 0.065 | 32.60  | 2.94     |
| Roof         | 0.30 | 0.435 | 1.657 | 0.129 | 71.73  | 0.89     |

Table 2 Geometrical and thermal characteristics of the masonry used

*the indoor side is covered with mortar.

The temperature fluctuation period equals 24 and the decrement factor $D$ ranges between $2.45\times10^{-3}$ and $6.02\times10^{-8}$ for the brickwork, between $5.20\times10^{-3}$ and $2.97\times10^{-8}$ for the stonework and equals 0.46 for the roof.

<table>
<thead>
<tr>
<th>Orientation</th>
<th>$\sum_{UF}$</th>
<th>$\sum_{cF}$</th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>West</td>
<td>894.05</td>
<td>81.46</td>
<td>25.31</td>
</tr>
<tr>
<td>South</td>
<td>749.09</td>
<td>97.21</td>
<td>36.05</td>
</tr>
<tr>
<td>East</td>
<td>785.99</td>
<td>120.61</td>
<td>42.62</td>
</tr>
</tbody>
</table>
Form Table 3 it is evident that the masonry walls largely contribute to the thermal capacity of the building and consequently to its thermal inertia. The overall thermal inertia of the building is 724.68 MJ/K, which for 18oC (291oK) equals 210881 MJ. This gives a time lag of 40.83 h or almost two days. Taking into account:

i. That for Thessaloniki the average maximum temperatures for July and August are 31.6oC their corresponding average minimum temperatures are 18.2oC and 17.9oC and the corresponding average absolute average temperatures (1% probability) are 36.6oC and 36.0oC respectively [19].

ii. The average duration of a Greek Orthodox Mass is of the order of two hours, twice-a-day it is evident that no artificial cooling is required in summer.

Moreover, taking into account that for Thessaloniki the absolute minimum temperature for the winter is -12.8oC whereas the corresponding design minimum temperature is -5oC, it can be seen that the 40.83 h thermal lag can regulate all external temperature short term fluctuations reducing the heating cost to a minimum, if of course the heating system is specifically designed and tuned for these large thermal capacities.

4 Conclusions
The above analysis has indicated that the temperature variation damping increases exponentially with the width, and consequently the mass, of the masonry. For wall of large width, as those of Aghia Sophia, the damping is very large. This leads to an almost constant temperature, which, if heating is not applied, is almost equal the average external temperature. On the other hand the temperature fluctuation hysteresis increases linearly with the masonry width. Taking these into account together with the contemporary tendency for bioclimatic building design and considering the empirical knowledge of our ancestors, heavyweight masonry, especially of fired clay bricks, seems to be the key to sustainable building.

References:
and heating load calculations. Athens 1987 (in Greek).

Fig. 2a East side view of the monument

Fig. 2b South side view of the monument

Fig. 2c North side view of the monument

Fig. 2d West side view of the monument

Fig. 2e Plane view of the monument