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Cost Allocations of Transmission Network by Cooperative Game Theory; Nucleolus method[1]

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Abstract: - Given the restructure of the electric sector in the whole world, they have developed many methods to allocate cost of the transmission system that, independently we established the different characteristics for one country to another they try to pay to the sector, giving signs of technical and economic efficiency. This publication is to emphasize with in the discussion and describe a method to allocate cost of the system of transmission based on the cooperative game theory. The principle method is based in the agent's responsibility for the physical and economical use considering the utilization as everything. We have developed a mathematical program to be able to obtain the cooperative solution of the nucleolus, and we have applied the different methods to a system of tests. The results show a good result to the method presented in regards to the appropriate signal location and assigned fairness to the agent by way physical and economical terms

Key-Words: Coalition Formation, Cooperative Game Theory, Transmission Cost Allocation, Nucleolus, Shapley Value

1 Introduction

In the last years the reforms as changes in the electric business it has increased rapidly and many countries have started the disintegration of the sector, with the idea to introduce certain grades of competition to the industry, studies have been done extensively in a worldwide level [1],[2],[3],[4].

In this structure, and to be able to establish the competition in the generation, it is necessary to have a system of transmission to assure open access and no discrimination via red to any other agent that may want to enter in the business. Because of the special physical characteristics the transmission is considered as a regulated natural monopoly, pay in accord to each established regulation.

The payment of transmission system is based of two different price types: one calculated based in the original pricing and the other complementary payment and complementary payment or change to allow recovering the costs f operation maintenance investment and profitability of the company. The way to allocate costs among the transmission system users in a just way without giving sings that can alter the efficiency to the electrical market (in the sense of social and economically efficient) even it has not been solved. The evidences shown of approval recently in Chile the law 19.940, that modifies the transmission regulation, according to relative problems of investment shown in the crisis of 1998 – 1999 [5],[6].

Diverse methods based on different technical and economical aspects have been developed to resolve the problem. In this methods existing some theories based on cooperative games that pretend the existence of rational agents that interacting to maximize its benefits. The present work analyze the performance of a method based in cooperative games for the allocation of the transmission system costs, set against other methods implemented and at length studied in the literature [9][10], [11], [12], [13], [14].

2 Methodology

2.1 Formulation

The proposed method considers to the loads (or groups of them) connected to a bus as the agents of the game, which will have to distribute the cost of the network of transport in proportion to the use that they do of the system, taking in account that the coalition

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can be formed to enlarge its savings will use the system in its entirety.
The measurement of the use of the system is obtained by way of the DC flow for each line, to accomplish an optimal dispatch. The conditions of flow in each line determine a required their demand. In other words, this will determine its capacity and their cost of investment, operation and maintenance.
The participation of a certain agent in the cost of the system it's determined as a cooperative game (N,c), where N is a number of players and c their characteristics function, according to the definitions given in [15].

When the requirements of flow f_S^K for every line k on behalf of each of the possible coalitions S are possible of determining, then characteristic function $C(S)$ for each coalition S will be determined by:

$$C(S) = \sum_{k} f_k^s \quad ; \forall S \subseteq NA \, , k \in N_l \qquad (1)$$

Where N_l is the number of lines in the system. The characteristic function of each coalition is build by the dc flow that such coalition causes in the system., independently the flow will circulating in a opposite way.

This function differs from the used one in [14], where the responsibility decides analyzing the system for sections. This variation allows using the methodology in the new structure of the system of transmission in Chile, where the agents are not responsible for the utilization of certain lines, but of the utilization of a System of Main Transmission, for which they will have to pay all those who use it.

The possible coalitions that can be formed depending exclusively on its rationality of the potential of each agent (or coalition of agents), in other words, of the convenience in forming a coalition that allows them to obtain minor costs that those that they would obtain operating in an independent way. In the systems of transmission, and due to the presence of economies scale, the agents are ready to cooperate and in general the great coalition can be formed.
As we determined the function by the games, it will be applied to the nucleolus solution to determine the percentage of responsibility of each agent charge on the system of transmission. The responsibility of each agent in this physical requirement of the system will determined its participation in the total charges in a specific condition of operation. For the resolution of the cooperative games, it has being implemented algorithms in Mathematica, based in the formulation of savings established in [7].
In order that the alloction costs in function to the use of the system are representative of this one, it should be considered different conditions of operations relevant to its characterizations. For each condition it will establish its respective characteristic function and allocation. Its respective variation in each case will depend not only on the size of the demand, but also by all the charge used system. The final percentage of responsibility of each agent will be determinate by considering each scenario as a part of the total charge of each one of them in the following way:

$$PRF_i = \frac{\sum_k CT_k * PR_i^k}{\sum_k CT_k} \qquad (2)$$

Where k is the number of scenarios considered, i is the number of load of the system, CT_k is the total cost of the system in the scenario k and PR_i^k being the percentage of responsibility to the cost of load i in the scenario k.

2.2 Example

For effects and analysis it will be consider the system in Figure 1. The data of variable cost, lines and conditions of operations are shown in tables 1 and 2. The agents participating in the game will be the loads C1, C2, C3 y C4.

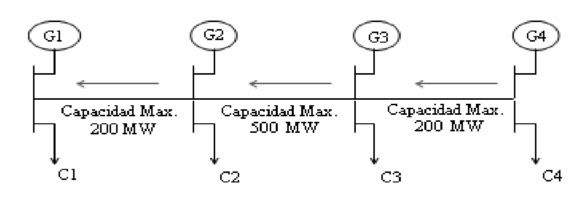

Fig.1. System of 4 buses with 4 generators and 4 charges

Table 1: System lines and generation data

Line	Z (p.u.)	Max. Capacity (MW)	Generator	Max. Capacity (MW)	Variable Cost (US$/MWh)
1-2	0+j0.15	200	1	50	70
2-3	0+j0.05	500	2	600	22
4-3	0+j0.14	200	3	400	12
			4	200	0

Table 2: System demand peak, medium and low (MW) data

Load	Peak (MW)	Medium (MW)	Low (MW)	Generator	Peak (MW)	Medium (MW)	Low (MW)
C1	120	84	48	G1	0	0	0
C2	500	350	200	G2	400	100	0
C3	300	210	120	G3	400	400	200
C4	80	56	32	G4	200	200	200

As an example, it will be considered the operation condition in a peak demand. For each possible coalition it will be determined the flow established in the proper system, according to optimal dispatch and demand conditions. In the case of the system, it exist 4 agents that can form 15 coalitions according to that has been established in the definitions given in [8].
If the load C1 it can satisfied its individual demands, this should be given by the generator located in bus 4,

it will be causes a total flow in the system of 360 MW, that represents the flow of line 1-2 (120 MW) , line 2-3 (120 MW) and line 3-4 (120 MW). According to these, it will be required to pay for the transmission system in the function of 360 MW that circulates by it.
In the same way, if the charge C3 wants to satisfy it's demand a individual way, the requirement of the flow over the system should be 200 MW, that circulate by the line 3-4 because of the generator located at its bus it gives 100 MW, and should be pay for the system in the function of the 200 MW that flow through it.
If the loads C1 y C3 considered formed a coalition {13} to support the costs of the transmission it will causes a flow of 440 MW and should pay for the system in function the 440 MW that circulate by it. By these way, to both agents is convenient to cooperate and confront the costs of the transmission system as per the sum of the costs should be confronted individually is greater than the joint cooperation. The stability of the coalition with in time will depend of how convenient the payments be assigned to each one by the utilization of the network of transmission. The process is done with each possible coalition by the cooperative game. The characteristic function for all possible coalitions are presented in Table 3.

Table 3: Characteristic function, demand and condition of generation for peak demand

Coalition	C(S)	Demand (MW)	Gen. Condition	Coalition	C(S)	Demand (MW)	Gen. Condition
{Ø}	0	0	-	{23}	500	800	2,3,4
{1}	360	120	4	{24}	620	580	3,4
{2}	700	500	3,4	{34}	120	380	3,4
{3}	200	300	3,4	{123}	620	920	2,3,4
{4}	0	80	4	{124}	760	700	2,3,4
{12}	920	620	2,3,4	{134}	360	500	3,4
{13}	440	420	3,4	{234}	340	880	2,3,4
{14}	360	200	4	{1234}	460	1000	2,3,4

Obtaining all the characteristic functions, of the resolution the cooperative game will done through the nucleolus, if this exists. The uniqueness is an advantage rather than other cooperative solutions [10],[11],[12],[13] even do it eliminates the problem and conflicts between agents. It is belongs to the core assures the solution to be efficient, fair and globally equal to accomplished its rationalities given of it. The result of the game is given in the table 4

Table 4: Assignments of the nucleolus in maximum demand

Load	Demand (MW)	Assignment %
C1	120,00	78,26
C2	500,00	21,74
C3	300,00	0,00
C4	80,00	0,00

The solutions shows the consumer locates in the points of lowest generation of the system it shouldn't pay for the lines of transmission not being used, if the capacity of generation exist to be used and respond to the demand. In case of the load located in bus 3, even do to use the system to its demand it's allocation is nil, because of it's savings that is presence located the furthest point of dispatch is at the lowest cost Even do the nil aspect of this problem can be being from the point of view in loads, the cooperation and allocation of the nucleolus assure to all participants that there is no better solution to the fact that its properties related to the nucleus. In the Table 5 it is shown the same results obtained through the GLDF and Shapley Value (SV)[2].

Table 5: Assignment in % in maximum demand

Load	Nucleolus	SV	GLDF
C1	78,26	37,10	44,13
C2	21,74	62,90	45,00
C3	0,00	0,00	10,87
C4	0,00	0,00	0,00
Total	100,00	100,00	100,00

Of the results obtained is possible to observe that in the method presented it applies the greatest responsibility to the load C1, even do it has a lower demand of load C2. To the different other methods, the allocation shown that, being the C1 furthest to the lowest generators, this will utilize in greater measure the network. Both cooperative solutions of loads C3 and C4 will receive nil charge. The allocation of the load C4 is explained by the existence of sufficient generation to satisfy its demands; The allocation of the load C3 shows that even do by using any amount of the system, its inclusion in the coalition of agents it causes great savings to other charges, which give a nil signal. Producing the same way other conditions of operations, it will have the following characteristics function shown in table 6.

Table 6: Characteristic function, demand and condition of generation

Coalition	Medium Demand			Low demand		
	$C(S)$	Demand (MW)	Gen. Condition	$C(S)$	Demand (MW)	Gen. Condition
{1}	252,00	84,00	4	144,00	48,00	4
{2}	550,00	350,00	3	400,00	200,00	4
{3}	200,00	210,00	3	120,00	120,00	4
{4}	0,00	56,00	4	0,00	32,00	4
{12}	718,00	434,00	3	496,00	248,00	3
{13}	368,00	294,00	3	264,00	168,00	4
{14}	252,00	140,00	4	144,00	80,00	4
{23}	550,00	560,00	3	400,00	320,00	3
{24}	494,00	406,00	3	368,00	232,00	3
{34}	144,00	266,00	3	120,00	152,00	4
{123}	674,00	644,00	2,3,4	496,00	368,00	3
{124}	662,00	490,00	3	464,00	280,00	3
{134}	312,00	350,00	3	264,00	200,00	4
{234}	478,00	616,00	2,3,4	368,00	352,00	3
{1234}	562,00	700,00	2,3,4	464,00	400,00	3

The assignments for each agent, in the different conditions of operations obtained through the Nucleolus, SV and GLDF are found in table 7. The proposed method has been compared to [14] for the

2 GLDF: General Load Distirbution Factor, see [12], Shapley Value: see [8]

different conditions of operations, considering the analysis line by line for each are of them. The results obtained through this method are found in table 8. The allocation through GLFD factors it doesn't differ from the results by the previous method.

Table 7: Assignment in % for the conditions of peak, medium and low demand

Load	Peak Demand (%)			Médium Demand (%)			Low Demand (%)		
	Nucleolus	SV	GLDF	Nucleolus	SV	GLDF	Nucleolus	SV	GLDF
C1	78,26	37,10	44,13	14,95	27,87	44,13	20,69	25,00	44,13
C2	21,74	62,90	45,00	59,43	64,53	45,00	79,31	62,50	45,00
C3	0,00	0,00	10,87	25,62	7,59	10,87	0,00	12,50	10,87
C4	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00

Table 8: Assignment in % for the conditions of peak, medium and low demand analyzing the system for line

Load	Peak Demand (%)			Médium Demand (%)			Low Demand (%)		
	Nucleolus	SV	GLDF	Nucleolus	SV	GLDF	Nucleolus	SV	GLDF
C1	85,00	47,00	44,13	61,00	44,00	44,13	40,00	44,00	44,13
C2	15,00	40,00	45,00	39,00	42,00	45,00	60,00	45,00	45,00
C3	0,00	13,00	10,87	0,00	14,00	10,87	0,00	11,00	10,87
C4	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00

It is possible to observe that the principal difference in both methods to realize the allocation that the nucleolus is to, the assigned responsibility in the second part with in loads C1 and C2, independently from the condition of operation. In the proposed method, the assignments in average demand are distributed among the loads C1, C2 y C3, and reflect the higher use in this condition of operation applied the load C3.

In the case of assigned through the SV, the method proposed shows a variation in the allocation for each condition of operation, with a difference in analysis line by line, where the variations are less. The shows a better analysis of the behaviour already proposed, with the different sense of use by the system to each load an each condition of operation reflects in the final assignments. Another advantage is given to the proposed method to the analysis per line which the existence of the nucleus. When the analysis is realized to the fullness of the system, the conditions of cost subaditivity are given more frequently, assuring that the existence of the nucleus gives are efficient and equal solution.

The results of the data by the flow of the system in different conditions of operation were obtained considering an optimum dispatch. These conditions of the flow of the system determine its cost, considering that the loads will give the requirements of the flow in such conditions of operations, as such is possible to establish a direct relation between costs of the system with it requirement of flow to each line, summed up in table 9.

The final percentage of responsibility by the use of the system that are determined considering it's responsibility in each scenario, as part at the total cost for each one of the according to the equation (2). The final assignments are obtained through the complete system and line by line, for the different concepts of the solution are presented in tables 10.

Table 9: Monetary units associated with the flow for the lines

Line	Peak demand (u.m)	Medium Demand (u.m)	Low Demand (u.m)
1-2	120,00	84,00	48,00
2-3	220,00	334,00	248,00
3-4	120,00	144,00	168,00
Total	460,00	562,00	464,00

Table 10: Final assignment in % (PFR) analyzing the system for line and entire

Load	Analysis Entire System			Analysis System by Line		
	Nucleolus	SV	GLDF	Nucleolus	SV	GLDF
C1	36,34	29,83	44,13	61,87	44,93	44,13
C2	53,97	63,39	45,00	38,13	42,32	45,00
C3	9,69	6,77	10,87	0,00	12,75	10,87
C4	0,00	0,00	0,00	0,00	0,00	0,00

According to the characteristics of the system in its operation and constitution, it's possible to establish the following conclusions: the load C2 is the one with the greatest responsibility for the cost of the system. This it is justified for two reasons: the size of its demand (500 MW, the biggest in hour peak) that causes greater requests of flow, due to that the generator located in its bus has a high variable cost. This causes even that, in low demand, this generator don't be dispatch, and the percentage for the use of the system and the percentage of responsibility by the use of the system of the load C2 be greater that in the other conditions of operation.

The load C1 has the second largest responsibility. Even though the demand of this bus is less then bus 3 it's further location makes that its requirements of use are greater. Plus, the variable cost of the generator in bus 1 is more expensive, this implies that in not one condition of the operation this should be dispatched, as such it will always have a percentage of responsibility in any conditions of operation, since it needs always to utilize the network to supply its demand.

The load C4 is free of any responsibility. This is justified because of its low demand and low variable cost of the generator located with in, allows not to utilize in any conditions of the operation not even in its maximum demand. The load C3 has allowed percentage of responsibility, because of the use of the system. Its justification radiates in great capacity of the generation in it's bus and the low variable cost of it implies that only in conditions of operation where the demand of the load C4 is smaller it utilize more the system.

3 Example of Application

A model simplified of the SIC is considered for the analysis, showed in the Figure 2.

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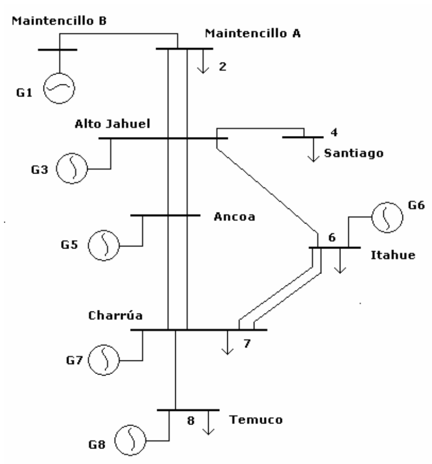

Fig. 2. System of 8 buss of the SIC

The data of the lines in the system and conditions of operation are found in tables 11 and 12.

Table 11: SIC Line Data

Línea	Z (p.u.)	Línea	Z (p.u.)
1-2	0.0009+j0.0013	**3-6**	0.0788+j0.3920
2-3	0.0274+j0.0390	**5-7**	0.0050+j0.0361
3-4	0.0001+j0.0027	**6-7**	0.0682+j0.2270
3-5	0.0008+j0.0170	**7-8**	0.0310+j0.1529

Table 12: SIC generation and demand data in MW

Bus	Operation 1		Operation 2		Bus	Operation 1		Operation 2	
	Gen.	Dem.	Gen.	Dem.		Gen.	Dem.	Gen.	Dem.
1	150	0	150	0	**5**	620	0	620	0
2	0	350	0	200	**6**	300	360	300	230
3	880	690	880	690	**7**	920	490	920	490
4	0	1100	0	880	**8**	260	140	260	100

The order of dispatch by the generators in accord with its respective variables cost are G8, G1, G6, G5, G7 y G3. The characteristics function is obtained according to the method previously explained, for each condition of operation and each possible coalition, showed in the table 13.

Table 13: Characteristics Function for both conditions of operation

Coalición	Operación 1	Operación 2	Coalición	Operación 1	Operación 2
{2}	1130,000	800,00	**{347}**	4169,56	3539,58
{3}	1645,530	1645,53	**{348}**	4084,81	3499,58
{4}	3159,140	2730,27	**{367}**	1240	1250
{6}	878,979	615,37	**{368}**	1086,77	1200
{7}	970,338	970,34	**{378}**	1330	1330
{8}	0,000	0,000	**{467}**	3000	2253,97
{23}	2052,17	1759,76	**{468}**	2860	2261,08
{24}	3648,54	2820,15	**{478}**	2813,83	2260
{26}	1078,93	903,95	**{678}**	965,69	951,77
{27}	1040	837,04	**{2346}**	4630	3890
{28}	864,358	680	**{2347}**	4219,56	3849,56
{34}	4224,81	3599,58	**{2348}**	4724,25	3767,98
{36}	1360	1389,95	**{2367}**	1830	1292,39
{37}	1330	1330	**{2368}**	1690	1290,57
{38}	1404,92	1480,62	**{2378}**	1653,32	1280
{46}	3000	2450,34	**{2467}**	3650	2522,37
{47}	2953,84	2360	**{2468}**	3610	2422,37
{48}	2894,17	2541,01	**{2478}**	3508,54	2497,01
{67}	980,729	962,51	**{2678}**	1150	740
{68}	783,915	556,42	**{3467}**	3600	3510
{78}	1064,910	1037,89	**{3468}**	4160	3490
{234}	4919,51	3867,98	**{3478}**	3771,68	3499,58
{236}	1830	1479,83	**{3678}**	1100	1150
{237}	1793,31	1530	**{4678}**	2860	2153,97
{238}	1787,21	1575,56	**{23467}**	3650	3410
{246}	3750	2540,22	**{23468}**	4210	3790
{247}	3648,54	2597,00	**{23478}**	3821,68	3565,36
{248}	3508,54	2630,89	**{23678}**	1690	1192,39
{267}	1160	740	**{24678}**	3230	2422,37
{268}	813,967	636,40	**{34678}**	3180	3210
{278}	1040	841,96	**{234678}**	3230	3110
{346}	4380	3590			

The results obtained are presented in table 14, for each condition of operation and every concept of solution. The final assignment of responsibility of its charge of the system is presented in Table 15.

Table 14: Assignment (%) for Operation 1 and 2

Operation 1				Operation 2			
Load	Nucleolus	SV	GLDF	Load	Nucleolus	SV	GLDF
C2	1,55	12,59	21.49	**C2**	8,63	6,27	21,38
C3	9,99	20,74	17,72	**C3**	22,11	29,86	17,51
C4	64,81	66,67	40,74	**C4**	69,26	63,57	40,41
C6	23,65	0,00	15,49	**C6**	0,00	0,00	15,60
C7	0,00	0,00	4,01	**C7**	0,00	0,29	4,42
C8	0,00	0,00	0,56	**C8**	0,00	0,00	0,68

Table 15: Final assignment of responsibility (%)

Load	Nucleolus	SV	GLDF
C2	5,02	9,49	21,44
C3	15,94	25,21	17,62
C4	66,99	65,15	40,58
C6	12,05	0,00	15,54
C7	0,00	0,14	4,21
C8	0,00	0,00	0,62

3.1 Discussion of the results

In the case by the method GLDF, the variations of the conditions of operations do not reflect a significant variation in the assignment for every agent in each of these conditions, and therefore of the use that each of them does of the net.

The solution through the nucleolus and SV reflects of better way the requirement in terms of flow that every agent does of the net, for each different conditions of operation. These requirements vary not only for the variation of the demand, but for the use that collectively is done of the net. This meets reflected, for example, in the highest assignment given to the load C4 even though it's less demand it increased its assignment in the second condition of operation, which shows a major requirement of use of the net in comparison to the first condition of operation.

The final assignment allows establishing results that demonstrate the variations of responsibility in the different scenarios, according to the magnitude of requirement in the use of the net for each of them. While the considered conditions of operation reflect of better way the functioning of the system, the assignments for every agent will be more just.

4 Conclusions

This work contributes to the discussion of the problem of how assigning the costs of the system, introducing elements based on economic aspects and considering technical aspects of the net, in order that the above mentioned assignment should allow that good economic signs should be delivered to the agents of the market, of way of having an efficient service from the technical and economic point of view.

The system of transmission has been analyzed thinking that the charges use the system in its entirety, and it has been achieved to establish differences in relation to a study realized in his utilization by parts. The application of cooperative game theory, especially the solution of the nucleolus, has allowed establishing just and stabling assignments in the time, from the point of view that the cooperation allows to the participants to assure minor costs to which they can obtain acting in a unilateral way.

The method applied showed to be consistent, in the sense of assigning to the loads that more used the net higher charges, not importing the size of his demand (minor demands can have top assignments that other major demands) but his requirements of flow. On having realized an analysis of the system in its entirety, and not for sections, it is possible to identify as the loads affect the whole system, assigning the responsibility of agreement to this total use. This consideration has given nil allocations in some cases to loads that, in comparison to its makes it utilizes

This consideration has delivered nil assignments in some cases for loads that, in comparison at his par, use in minor measurement the network and which presence in some coalition brings big benefits for all the agents. The void assignment does not re-dress major importance due to the fact that, physically, it can indicate that the charges do not use the network in certain scenario, or that his incorporation in a coalition brings important benefits for all. The final percentage of assignment raised will reflect finally the use that every agent realizes of the net, in the different scenarios and in function to the requirements in each of them.

The obtained results were compared by the solutions given by the SV and GLDF. In such comparisons, the similarity of results has happened as for the correct signs of location that deliver the assignments to the agents, important topic to the moment to think about the expansion of the system. The central differences take root in that the proposed method reflects of minor way the different levels of utilization of the system for different conditions of operation, and the stability and efficiency of the above mentioned assignment of agreement to the definitions raised by the theory of games. Especially, the SV presents the disadvantage of not assuring, as the nucleolus, the belonging to the nucleus, therefore the stability of the formed coalitions will not be insured.

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