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Remarks on the Oseen Problem in Exterior Domains – Anisotropically Weighted Approach

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Abstract: - We consider the Oseen problem in exterior domains. We study existence and uniqueness of a solution in anisotropically weighted Sobolev spaces. We prove existence of a solution and its uniqueness in anisotropically weighted Sobolev spaces. For the proof of existence we use a localization procedure, see e.g. [KoSo].

Key-Words: - Anisotropically weighted Sobolev spaces, the Oseen problem, Exterior domains, Localization

1 Introduction

In a three-dimensional exterior domain Ω in $\mathbb{R}^3$, the classical Oseen problem [Os] describes the velocity vector $\mathbf{u}$ and the associated pressure p by a linearized version of the incompressible Navier-Stokes equations as a perturbation of $\mathbf{v}_\infty$ the velocity at infinity; $\mathbf{v}_\infty$ is generally assumed to be constant in a fixed direction, say the first axis, $\mathbf{v}_\infty = |\mathbf{v}_\infty|\,\mathbf{e}_1$. We consider the Oseen problem in an exterior domain Ω.

$$-\nu\Delta\mathbf{u} + k\partial_1\mathbf{u} + \nabla p = \mathbf{f} \ \text{ in } \boldsymbol{\Omega} \tag{1}$$

$$\operatorname{div}\mathbf{u} = g \ \text{ in } \boldsymbol{\Omega} \tag{2}$$

$$\mathbf{u} = \mathbf{0} \ \text{ on } \partial\Omega \tag{3}$$

$$\mathbf{u} \to \mathbf{0} \quad \text{as } |\mathbf{x}| \to \infty \tag{4}$$

For the case $\Omega = \mathbb{R}^3$ the respective problem (1), (2) and (4) was studied e.g. in [Fa1] and [KNoPo]. To extend the estimates derived in these papers the method of hydrodynamical potentials can be used. The application of this method to the Oseen problem without weights is well known. Moreover this method has been used for solution of the Oseen equations in weighted Sobolev spaces, see [Fa1]. We use another possibility to avoid technique of single layer and double layer potentials. We apply some localization procedure, see e.g. [KoSo], for the extension of anisotropically weighted estimates from whole $\mathbb{R}^3$ onto the case of exterior domains. This method is efficient for various modifications of the Oseen problem connected with additional assumptions (e.g. rotation of a body etc.), see [Fa2], [Hi], [KNP1], [KNP2]).

2 Function spaces, notation

The pair $(\mathcal{O},\mathcal{P})$ will denote the fundamental solution of the Oseen problem.

We introduce the following weight functions to reflect decay properties of a solution near the infinity:

$$w(\mathbf{x}) = \eta^\alpha_\beta(\mathbf{x}) = \eta^\alpha_\beta(\mathbf{x};\delta,\varepsilon) = (1+\delta r)^\alpha\,(1+\varepsilon s)^\beta\,,$$

$$r = |\mathbf{x}| = (x_1^2+x_2^2+x_3^2)^{1/2}, \quad s = s(\mathbf{x}) = r - x_1,$$

$$\mathbf{x}\in\mathbb{R}^3, \quad \varepsilon,\,\delta>0, \quad \alpha,\,\beta\in\mathbb{R}.$$

The weights η^α_β belong to the Muckenhoupt class A_2 of weights in $\mathbb{R}^3$ if $-1<\beta<1,\ -3<\alpha+\beta<3$, see e.g. [Fa1], [KNoPo]. Let us outline our notations:

We need to denote the special sets

$$B_R = \left\{\mathbf{x}\in\mathbb{R}^3;\ |\mathbf{x}|\le R\right\},$$

$$B^R = \left\{\mathbf{x}\in\mathbb{R}^3,\ |\mathbf{x}|\ge R\right\},$$

$B^{R_0}_R = B^{R_0}\cap B_R$, $\Omega^R = \Omega\cap B^R$, $\Omega_R = \Omega\cap B_R$, $\Omega^{R_0}_R = \Omega_R\cap\Omega^{R_0}$ for positive numbers $R_0<R$.

For $1\le q<\infty$ we denote

$$D^{m,q}(\Omega) = \left\{u\in L^1_{loc}(\Omega) : D^l u\in L^q(\Omega)\,, |l|\le m\right\}$$

with $|u|_{m,q} = \left(\sum_{|l|=m}\int_\Omega \left|D^l u\right|^q\right)^{1/q}$ as a seminorm. It is known that $D^{m,q}(\Omega)$ is a Banach space (and if $q=2$ a Hilbert space), provided we identify two functions u_1, u_2 whenever $|u_1-u_2|_{m,q}=0$.

Let $\left(L^2\left(\mathbb{R}^3;\, w\right)\right)^3$ be the set of measurable vector functions $\mathbf{f}$ on $\mathbb{R}^3$ such that

$$\|\mathbf{f}\|_{2,\mathbb{R}^3;\,w} = \left(\int_{\mathbb{R}^3} |\mathbf{f}|^2\, w\, d\mathbf{x}\right)^{1/2} < \infty.$$

We will use $\mathbf{L}^2_{\alpha,\beta}$ instead of $\left(L^2\left(\mathbb{R}^3;\,\eta^\alpha_\beta\right)\right)^3$ and $\|\cdot\|_{2,\alpha,\beta}$ instead of $\|\cdot\|_{2,\mathbb{R}^3;\eta^\alpha_\beta}$. Let us define the weighted Sobolev space $\mathbf{H}^1\left(\mathbb{R}^3;\,\eta^{\alpha_0}_{\beta_0},\eta^{\alpha_1}_{\beta_1}\right)$ as the set of functions $\mathbf{u}\in\mathbf{L}^2_{\alpha_0,\beta_0}$ with the weak derivatives $\partial_i\mathbf{u}\in\mathbf{L}^2_{\alpha_1,\beta_1}$. The norm of $\mathbf{u}\in\mathbf{H}^1\left(\mathbb{R}^3;\,\eta^{\alpha_0}_{\beta_0},\eta^{\alpha_1}_{\beta_1}\right)$ is given by

$$\|\mathbf{u}\|^2_{\mathbf{H}^1\left(\mathbb{R}^3;\,\eta^{\alpha_0}_{\beta_0},\eta^{\alpha_1}_{\beta_1}\right)} = \|\mathbf{u}\|^2_{2,\alpha_0,\beta_0} + \|\nabla\mathbf{u}\|^2_{2,\alpha_1,\beta_1}.$$

As usual, $\mathring{\mathbf{H}}^1\left(\mathbb{R}^3;\,\eta^{\alpha_0}_{\beta_0},\eta^{\alpha_1}_{\beta_1}\right)$ will be the closure of $(C_0^\infty)^3$ in $\mathbf{H}^1\left(\mathbb{R}^3;\,\eta^{\alpha_0}_{\beta_0},\eta^{\alpha_1}_{\beta_1}\right)$. For simplicity, we shall use the following abbreviations:

$\mathring{\mathbf{H}}^1_{\alpha,\beta}$ instead of $\mathring{\mathbf{H}}^1\left(\mathbb{R}^3;\,\eta^{\alpha-1}_{\beta-1},\eta^\alpha_\beta\right)$

$\mathbf{V}_{\alpha,\beta}$ instead of $\mathring{\mathbf{H}}^1\left(\mathbb{R}^3;\,\eta^{\alpha-1}_{\beta},\eta^\alpha_\beta\right)$

3 Results in $\mathbb{R}^3$

We recall results about weakly singular and singular integral operators in anisotropically weighted Sobolev spaces derived in [KNoPo] and in [Fa1], ($p=2$). We use here the original notation.

Theorem 1 *Let T be an integral operator with the kernel $|\mathcal{O}|$, $T: f\mapsto|\mathcal{O}|*f$, and let $1<p<\infty$. Then T is a well defined continuous operator:*

$$L^p(\mathbb{R}^3;\eta_\beta^{\alpha+p/2})\longmapsto L^p(\mathbb{R}^3;\eta_\beta^{\alpha-p/2-\varepsilon})$$

for $0<\beta<p-1<(\alpha+p/2)+\beta<3(p-1)$, $(\alpha+p/2)-\beta<p-1,\ 0<(\alpha+p/2)<2(p-1)$, $\varepsilon>0$.

Theorem 2 *Let T be an integral operator with the kernel $|\nabla\mathcal{O}|$, $T: f\mapsto|\nabla\mathcal{O}|*f$, and let $1<p<\infty$. Then T is a well defined continuous operator:*

$$L^p(\mathbb{R}^3;\eta_\beta^{\alpha+p/2})\longmapsto L^p(\mathbb{R}^3;\eta_\beta^{\alpha})$$

for $0<\beta<3/2(p-1),\ -1+p/2<(\alpha+p/2)+\beta$, $(\alpha+p/2)<2(p-1),\ (\alpha+p/2)-\beta<p-1$.

Theorem 3 *Let T be an integral operator with the kernel $|\boldsymbol{\mathcal{P}}|$, $T: f\mapsto|\boldsymbol{\mathcal{P}}|*f$, and let $1<p<\infty$. Then T is a well defined continuous operator:*

$$L^p(\mathbb{R}^3;\eta_\beta^{\alpha+p/2})\longmapsto L^p(\mathbb{R}^3;\eta_{\beta-p/2}^{\alpha})$$

for $0<\beta<p-1,\ p-3<(\alpha+p/2)+\beta<3(p-1)$.

Theorem 4 *Let T be an integral operator in the value-principal sense with the kernel $\nabla\boldsymbol{\mathcal{P}}$, $T: f\mapsto\nabla\boldsymbol{\mathcal{P}}*f$. Then T is well defined continuous operator:*

$$L^p(\mathbb{R}^3;\eta_\beta^{\alpha+p/2})\longmapsto L^p(\mathbb{R}^3;\eta_\beta^{\alpha+p/2})$$

for $p>1,\ -1<\beta<p-1,\ -3<(\alpha+p/2)+\beta<3(p-1)$.

Remark 5 *An additional investigation shows that in the case $p=2$ the estimate in the Theorem 1 is satisfied also for $\varepsilon=0$, see [Fa1].*

Let us assume for the simplicity the case $p=2$. From the previous theorems we get in this case:

Corollary 6 *(Existence in $\mathbb{R}^3$)*

Let $0<\beta<1$, $|\alpha|<\beta$, $\varepsilon>0$, $\mathbf{f}\in\mathbf{L}^2_{\alpha+1,\beta}$, $g\in W_0^{1,2}$ with a compact support $K=$ supp g. Then there exists a weak solution $\{\mathbf{u},\,p\}$ of the problem (1), (2), (4) in $\mathbb{R}^3$, such that $\mathbf{u}\in\mathbf{L}^2_{\alpha-1,\beta}$, $\nabla\mathbf{u}\in\mathbf{L}^2_{\alpha,\beta}$, $p\in L^2_{\alpha,\beta-1}$, $\nabla p\in\mathbf{L}^2_{\alpha+1,\beta}$ and

$$\|\mathbf{u}\|_{2,\alpha-1-\varepsilon,\beta}+\|\nabla\mathbf{u}\|_{2,\alpha,\beta}+\|p\|_{2,\alpha,\beta-1}+\|\nabla p\|_{2,\alpha+1,\beta} \le C\left(\|\mathbf{f}\|_{2,\alpha+1,\beta}+\|g\|_{1,2;K}\right). \quad (5)$$

Theorem 7 *(Uniqueness in $\mathbb{R}^3$)*

Let $\{\mathbf{u},p\}$ be a distributional solution of the problem (1), (2), (4) such that $\mathbf{u}\in D_0^{1,2}$ and $p\in L^2_{loc}$. Then $\mathbf{u}=\mathbf{0}$ and $p=const$.

The proof of Theorem 7 is based on the Fourier transform, see [KNP2] for the proof.

Sketch of the proof of the Theorem 7:

We have $\nabla\mathbf{u}\in\mathbf{L}^2$, $\mathbf{u}\in\mathbf{L}^6$, $\mathbf{u}\in\mathbf{L}^2_{-2,0}$, $\mathbf{u}\in\mathcal{S}'$. Because $\triangle p=0$, we get using the Fourier transform

$$\triangle\left(-\nu\,\Delta\mathbf{u}+k\,\partial_1\mathbf{u}\right)=\mathbf{0},$$

$$|\xi|^2\left(-\nu\,|\xi|^2\,\widehat{\mathbf{u}}+k\,\xi_1\widehat{\mathbf{u}}\right)=\mathbf{0}.$$

From this relation we follows supp $\widehat{\mathbf{u}}\subset\{0\}$, $\mathbf{u}$ is a polynomial. $\mathbf{u}\in\mathbf{L}^6$, $\mathbf{u}\equiv 0$, $\nabla p\equiv 0$, $p\equiv const$.

4 Results in exterior domains

Let Ω be an exterior domain with the Lipschitz boundary $\partial\Omega$. Our main results in weighted Sobolev spaces in exterior domains are

Theorem 8 *(Uniqueness in exterior domains)*

Let $\{\mathbf{u}, p\}$ be a distributional solution of the problem (1)- (4) with $\mathbf{f} \equiv \mathbf{0}$ such that $\mathbf{u} \in \mathbf{V}_{0,0}(\Omega)$ and $p \in \mathbf{L}^2_{-1,0}(\Omega)$. Then $\mathbf{u} \equiv \mathbf{0}$ and $p \equiv 0$.

Theorem 9 *(Existence in exterior domains)*

Let $\Omega \subset \mathbb{R}^3$ be an exterior domain and $0 < \beta < 1$, $0 \le \alpha < \beta$, $\mathbf{f} \in \mathbf{L}^2_{\alpha+1,\beta}(\Omega)$, $g \equiv 0$. Then there exists a unique weak solution $\{\mathbf{u}, p\}$ of the problem (1)-(4) such that $\mathbf{u} \in \mathbf{L}^2_{\alpha-1,\beta}(\Omega)$, $\nabla\mathbf{u} \in \mathbf{L}^2_{\alpha,\beta}(\Omega)$, $p \in L^2_{\alpha,\beta-1}(\Omega)$, $\nabla p \in \mathbf{L}^2_{\alpha+1,\beta}(\Omega)$ and

$$\|\mathbf{u}\|_{2,\alpha-1,\beta;\Omega} + \|\nabla\mathbf{u}\|_{2,\alpha,\beta;\Omega} + \|p\|_{2,\alpha,\beta-1;\Omega} + \|\nabla p\|_{2,\alpha+1,\beta;\Omega} \le C\, \|\mathbf{f}\|_{2,\alpha+1,\beta;\Omega}. \qquad (6)$$

For the proofs of Theorems 8 and 9 see [KNP2]. The proof of Theorem 9 is based on the localization procedure, see [KoSo]. The important steps in this proof are

Lemma 10 *Let $\mathbf{f} \in D^{-1,2}(\Omega)$. Then there is a weak solution $\{\mathbf{u}, p\}$ of the problem (1)-(4) such that $\mathbf{u} \in D^{1,2}_0(\Omega)$ and $p \in L^2_{loc}(\Omega)$.*

Lemma 11 *Let $\Omega \subset \mathbb{R}^3$ be an exterior domain and $0 < \beta \le 1$, $0 \le \alpha < \beta$, $\mathbf{f} \in \mathbf{L}^2_{\alpha+1,\beta}(\Omega)$, and K is a compact subset of Ω. Then there exists a weak solution $\{\mathbf{u}, p\}$ of the problem (1)-(4) such that $\mathbf{u} \in \mathbf{V}_{\alpha,\beta}(\Omega)$, $p \in L^2_{\alpha,\beta-1}(\Omega)$, $\nabla p \in \mathbf{L}^2_{\alpha+1,\beta}(\Omega)$ and*

$$\left\|\nabla^2\mathbf{u}\right\|_{2;K} + \|\mathbf{u}\|_{2,\alpha-1,\beta} + \|\nabla\mathbf{u}\|_{2,\alpha,\beta} + \|p\|_{2,\alpha,\beta-1} + \|\nabla p\|_{2,\alpha+1,\beta} \le C\left(\|\mathbf{f}\|_{2,\alpha+1,\beta} + \|\mathbf{u}\|_{1,2;A(\rho)} + \|p\|_{0,2;A(\rho)}\right),$$

where $A(\rho) := B_\rho \setminus B_{\rho/2}$.

Sketch of the proof of Theorem 8. We will prove that the solution $(\mathbf{u}, p)$ is unique in $\mathbf{V}_{0,0}(\Omega) \times \mathbf{L}^2_{-1,0}(\Omega)$. Let $\Phi = \Phi(z) \in C^\infty_0(\langle 0, +\infty))$ be a non-increasing cut-off function such that $\Phi(z) \equiv 1$ for $z < \frac{1}{2}$ and $\Phi(z) \equiv 0$ for $z > 1$. Let $|\Phi'| \le 3$. Let $\Phi_R \equiv \Phi_R(x) \equiv \Phi\left(\frac{|x|}{R}\right)$. We have $|\nabla\Phi_R(x)| \le 3\cdot\frac{1}{R}$ and $|\partial_1\Phi_R| \le 3\cdot\frac{1}{R}$ for $x \in \Omega$, $\frac{R}{2} \le |x| \le R$. Let $\{R_j\} \in \mathbb{R}$ be an increasing sequence of radii with the limit $+\infty$. So we have that $\mathbf{u}_j \equiv \mathbf{u}\cdot\Phi_{R_j} \in \mathring{\mathbf{H}}^1$. So, $\{\mathbf{u}_j\}$ is a sequence of functions with limit $\mathbf{u}$ in the space $\mathbf{V}_{\alpha,\beta}$. Using the (non-solenoidal) test functions $\varphi = \mathbf{u}\cdot\Phi^2_{R_j} = \mathbf{u}_j\cdot\Phi_{R_j} \in \mathring{\mathbf{H}}^1$ in we get:

$$\nu\int_\Omega \nabla\mathbf{u}\cdot\nabla\left(\mathbf{u}\Phi^2_{R_j}\right)\cdot d\mathbf{x} + k\int_\Omega \partial_1\mathbf{u}\cdot\mathbf{u}\cdot\Phi^2_{R_j}d\mathbf{x} + \int_\Omega \nabla p\cdot\mathbf{u}\cdot\Phi^2_{R_j}\cdot d\mathbf{x} = 0.$$

Using relations

$$\nabla\mathbf{u}\cdot\nabla\left(\mathbf{u}\,\Phi^2_{R_j}\right) = |\nabla\mathbf{u}_j|^2 - \nabla\Phi_{R_j}\cdot\nabla\Phi_{R_j}\,\mathbf{u}^2,$$

integrating by parts, we get after some evident rearrangements

$$\nu\int_\Omega |\nabla\mathbf{u}_j|^2\cdot d\mathbf{x} - \frac{k}{2}\int_\Omega \mathbf{u}^2\cdot\,\partial_1\Phi^2_{R_j}\cdot d\mathbf{x} - \nu\int_\Omega \left|\nabla\Phi_{R_j}\right|^2\mathbf{u}^2 d\mathbf{x} - \int_\Omega p\mathbf{u}\cdot\nabla\left(\Phi^2_{R_j}\right)d\mathbf{x} = 0,$$

$$\nu\int_\Omega |\nabla\mathbf{u}_j|^2\cdot d\mathbf{x} \le C\left(\int_{\Omega^{R_j/2}_{R_j}} \mathbf{u}^2\cdot\frac{1}{r}\cdot d\mathbf{x} + \int_{\Omega^{R_j/2}_{R_j}} |p|\cdot|\mathbf{u}|\cdot\frac{1}{r}\cdot d\mathbf{x}\right).$$

Because we have $\mathbf{u} \in \mathbf{L}^2_{-1,0}(\Omega)$, $p\mathbf{u} \in \mathbf{L}^2_{-1,0}(\Omega)$, for $j \to \infty$ we get

$$\int_{\mathbb{R}^3} |\nabla\mathbf{u}|^2\cdot d\mathbf{x} \le 0.$$

So, the function $\nabla\mathbf{u} = 0$ a.e. in Ω and this means $\mathbf{u}$ is a constant a.e. in Ω. From $\mathbf{u} \in \mathbf{L}^2_{-1,0}(\Omega)$ follows that $\mathbf{u} = \mathbf{0}$ a.e. in Ω. Substituting now arbitrary test function ϕ into the original equation, we get

$$\int_{\mathbb{R}^3} \nabla p\cdot\phi\cdot d\mathbf{x} = 0.$$

So, the function $\nabla p = 0$ a.e. in Ω and this means p is a constant a.e. in Ω. From $p \in L^2_{-1,0}(\Omega)$ follows that $p = 0$ a.e. in Ω. So, the uniqueness is proved.

Sketch of the proof of the Lemma 11. Fix $\rho > 0$ so that $\mathbb{R}^3 \setminus \Omega \subset B_{\rho/2}$. Take $\Psi \in C_0^\infty$, supp $\Psi \subset\subset B_\rho$ such that $\Psi \equiv 1$ on $B_{\rho/2}$. By use of cut-off function Ψ we decompose the solution $\{\mathbf{u},\, p\}$ as

$$\begin{aligned} \mathbf{u} &= \mathbf{U} + \mathbf{V} \quad \text{where } \mathbf{U} = (1-\Psi)\,\mathbf{u}, \quad \mathbf{V} = \Psi\mathbf{u}, \\ p &= \sigma + \tau \quad \text{where } \sigma = (1-\Psi)\,p, \quad \tau = \Psi p. \end{aligned}$$

So, we get that $\{\mathbf{U}, \sigma\}$ is weak solution of the problem in $\mathbb{R}^3$:

$$\begin{aligned} -\nu \triangle \mathbf{U} + k\,\partial_1 \mathbf{U} + \nabla\sigma &= \mathbf{Z_1} \\ \operatorname{div} \mathbf{U} &= -\nabla\Psi\,\mathbf{u} \end{aligned}$$

where $\mathbf{Z_1} = 2\nabla\Psi \cdot \nabla\mathbf{u} + \mathbf{u} \triangle \Psi - k\partial_1\Psi\,\mathbf{u} - \nabla\Psi\,p + (1-\Psi)\,\mathbf{f}$. Analogously, $\{\mathbf{V}, \tau\}$ is a weak solution of the Stokes problem assumed in a bounded domain. Using now Theorem 7 and the estimate of a solution in $\mathbb{R}^3$ we deduce the assertion of Lemma 11.

Sketch of the proof of the Theorem 9. Let us assume that the estimate of Lemma 11 is not true without the additional terms on the right-hand side. This means that there is a sequence of functions $\{\mathbf{f}_k\}$, corresponding solutions $\{(\mathbf{u}_k, p_k)\}$ and $\{C_k\} \to \infty$ such that

$$\begin{aligned} 1 &\equiv \|\mathbf{u}_k\|_{2,2;\,K_1} + \|\mathbf{u}_k\|_{2,\alpha-1,\beta} + \|\nabla\mathbf{u}_k\|_{2,\alpha,\beta} \\ &\quad + \|p_k\|_{2,\alpha,\beta-1} + \|\nabla p_k\|_{2,\alpha+1,\beta} \\ &\equiv \|(\mathbf{u}_k, p_k)\|_{(2)} \geq C_k\ \|\mathbf{f}_k\|_{2,\alpha+1,\beta}\,. \end{aligned}$$

So we get $\left\{\|\mathbf{f}_k\|_{2,\alpha+1,\beta}\right\} \to 0$. The sequence $\{(\mathbf{u}_k, p_k)\}$ is bounded in the norm $\|\cdot\|_{(2)}$ so, there is a subsequence of this sequence (we will denote this subsequence using the same notation) with the weak limit $(\mathbf{u}, p)$ in the corresponding Hilbert space H_2. The additional terms on the right hand side we denote by the norm

$$\|(\mathbf{u}_k, p_k)\|_{(1)} \equiv \|\mathbf{u}_k\|_{1,2;\,A(\rho)} + \|p_k\|_{0,2;\,A(\rho)}$$

and the corresponding Hilbert space H_1. Because K_1 can be chosen taken such that $A\,(\rho) \subset K_1$, we have $H_2 \hookrightarrow\hookrightarrow H_1$, hence $\|(\mathbf{u}_k, p_k)\|_{(1)} \to 0$. So, $(\mathbf{u}, p)$ is a solution of the problem with zero right-hand side. Due to uniqueness from Theorem 8 we can conclude that $\|(\mathbf{u}, p)\|_{(2)} = 0$. From the Lemma 11 we get $\|(\mathbf{u}_k - \mathbf{u},\, p_k - p)\|_{(2)} \to 0$. So we have also $\|(\mathbf{u},\, p)\|_{(2)} = 1$ and we get the contradiction.

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