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Probabilistic Approach in Estimating Groundwater Changes for Slope Stability Applications

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Groundwater rising due to rainfall events is one of the main factors contributing to slope instability. Rising groundwater levels reduces the shear strength of soil and increases seepage forces which can cause slope failure. A probabilistic method for estimating groundwater changes using available rainfall data can be used in many geotechnical engineering applications, including slope stability analysis. In this study, a probabilistic methodology for estimating groundwater level changes using rainfall and monitored groundwater wells is presented, and the method is used for predicting water-table changes in another area.

Keywords: *Probability, groundwater, precipitation, recursive, slope stability*

1 Introduction

In recent years, applied researchers have become increasingly interested in estimating groundwater level changes as a function of precipitation using a probabilistic approach, since a rise in water-table reduces the shear strength of soil inducing slope instability. Knowledge of water-table changes is of great importance for risk analysis in geotechnical engineering applications, such as slope stability analysis.

Sangrey et al. [4] presented a probabilistic approach for estimating groundwater level changes due to rainfall events in a period of time. In this method the change in groundwater table is a function of recharge to aquifer due to rainfall. They used the same methodology presented by Johnson [1] for predicting groundwater level changes as a function of rainfall. Johnson in his methodology, which consists of a two-part geohydrological and hydrometeorological method, evaluated the sensitivity of his analysis to aquifer response time, evapotranspiration and other characteristics. He suggested that recharge to groundwater is equal to precipitation minus evapotranspiration. As estimating evapotranspiration is related to too many functions such as temperature, soil moisture, surface vegetation, existing manmade structures on soil surface, etc, the calculated evapotranspiration amount is often not equal to actual evapotranspiration. Thus the amount of recharge estimated from precipitation minus evapotranspiration is not reliable for predicting groundwater level changes.

Rennolls et al. [3] presented a method to describe the response of the water level in a borehole to a series of rainfall events as follows:

$$h_i = c_1 h_{i-1} + c_2 R_i + e_i \qquad (1)$$

where h_i and h_{i-1} are groundwater levels in a borehole on days i and i-1, R_i is the rainfall on day i and e_i is a noise component. c_1 is the drainage coefficient and c_2 is the aquifer infiltration coefficient. Rennolls et al. assumed that c_1 and c_2 are time invariant, while this is not a valid assumption since drainage and infiltration, which depend on soil moisture in unsaturated region, are time variant.

Viswanathan [5] developed a model for the aquifer that estimates the water-table levels from the history of rainfall observations and past water-table levels recorded in a monitored well. Unlike Johnson's method, at which groundwater change is a function of estimated recharge to groundwater (recharge = rainfall-evapotranspiration), in this method groundwater level change is directly a function of rainfall throughout a time variant parametric function.

In Viswanathan's method, the model parameters that influence the recharge were not only assumed to be time dependent but also to have varying dependence rates for various parameters (equation (2)):

$$h_i = c_1 h_{i-1} + c_2 R_i + c_3 R_{i-1} \ldots + c_8 R_{i-6} + c_9 + e_i \quad (2)$$

where h_i and h_{i-1} are groundwater levels in a borehole on days i and i-1, R_i to R_{i-6} are rainfall on day i and six days before day i. In addition coefficients c_1 to c_9 are all considered to be time dependent parameters, and e_i is a noise component.

Although Viswanathan's method does not have the disadvantage of Johnson method, this method can not be used in risk analysis using a probabilistic approach, where only the rainfall peak magnitude on the day of event is available. In the current study, a relationship between changes in groundwater as a function of rainfall events on the day of peak event was studied. Subsequently, a probabilistic groundwater change estimate in the period of existing rainfall data was presented.

2 Theory

In this study, a simpler form of Viswanathan's method was developed, in which only rainfall on the day i is considered, as shown in equation 3. The main reason of this study was to develop a methodology to be used in a probabilistic approach of estimating groundwater changes due to rainfall peak events in a period of time. Meanwhile in a time events study, only the magnitude of rainfall on day of events is applicable; therefore, the effects of rainfall magnitude on days before day i, such as those used in Viswanathan's method, were ignored.

The simpler form of predicting groundwater changes is:

$$h_i = c_1 h_{i-1} + c_2 R_i + c_3 + e_i \quad (3)$$

where h_i and h_{i-1} are groundwater levels in a borehole on days i and i-1, R_i is maximum magnitude of rainfalls on days i. In addition coefficient c_1 to c_3 are time dependent parameters, and e_i is a noise component.

equation 3 can be written as:

$$c_1 = c'_1 + 1 \quad (4)$$

by substituting equation 4 in equation 3:

$$h_i = (c'_1 + 1) h_{i-1} + c_2 R_i + c_3 + e_i \quad (5)$$

or:

$$h_i - h_{i-1} = c'_1 h_{i-1} + c_2 R_i + c_3 + e_i \quad (6)$$

using the expression of groundwater level changes on day i respects to day before, i-1:

$$\Delta h = h_i - h_{i-1} \quad (7)$$

Then:

$$\Delta h = c'_1 h_{i-1} + c_2 R_i + c_3 + e_i \quad (8)$$

Now in order to model a general methodology for predicting groundwater versus rainfall, also useful for another area that might have a different groundwater height, the term of $c'_1 h_{i-1}$ will be ignored in the analysis. This term will be considered as a time variant parameter but independent from groundwater height. Using new parameters:

$$b_2 = c'_1 h_{i-1} + c_3 \quad (9)$$

$$b_1 = c_2 \quad (10)$$

Therefore new form will be:

$$\Delta h = b_1 R_i + b_2 + e_i \quad (11)$$

In the equation 11, the term b_2, which is a substitution for $(c'_1 h_{i-1} + c_3)$, is independent from groundwater height. The reason of this assumption is to give the estimated groundwater model flexibility for use in other areas with different groundwater height. The term b_2 is included to account for any groundwater level variations subjected to external influences like gravitational effect, air pressure, etc. [2]. The error term, e_i is a noise term and assumed to have a zero mean random value, have a constant variance and is independent of R_i and Δh.

This equation can be written in following mathematical notation:

$$Y_i=a_1x_{1i}+a_2+e_i \qquad (12)$$

$$\text{Where } y_i=\Delta h_i,\ x_{1i}=R_i \qquad (13)$$

Equation 12 is expressed in vector notation as:

$$y_i=x^T{}_i a+e_i \qquad (14)$$

in which :

$$\underline{x}_i^T=[x_1 \quad 1]......and..........\underline{a}=\begin{bmatrix}a_1\\a_2\end{bmatrix} \qquad (15)$$

In this case, where parameters are time-dependent, the variation of parameters a_1 and a_2 can be calculated using recursive time series analysis presented by Young [6]. Further details on how to solve the equation 12 using recursive methodology can be found in Young [7].

3 Estimating groundwater level changes using available monitored groundwater wells

In this study four monitored groundwater wells from four locations in the North-East of city of Bath in the UK were selected as shown in Fig.1. Equation 12 for each of these four monitored groundwater wells and nearest rainfall records for the year 2004 was solved using Young's recursive method.

4 Simulating presented ground-water model in another area

In this study all four monitored groundwater wells were selected with the same geological condition, all located in Mudstone bedrock (Great Oolite series: Forest marble formation), in order for them to have almost the same geohydrological conditions. Estimated groundwater model for Binghams Melcombe wells was used to predict the groundwater level in Bramble Combe location. As shown in Fig.2, the predicted groundwater level matches the observed groundwater level in Bramble Combe location. The average groundwater depth in Binghams Melcombe and Bramble Combe are 4 and 5 meter, respectively.

Using groundwater analysis results as shown in Fig.2, writers illustrate that a groundwater model can be simulated for another area with an acceptable prediction, if both locations have the same geological condition and the same average depth of groundwater, or in the other way both areas have the same geohydrological conditions. This assumption is acceptable especially for the slope stability applications with shallow groundwater depth. This is because of two reasons: first stability of natural slopes is sensitive to shallow groundwater, as slopes usually fail with a shallow depths failure surface. The other reason is that simulating groundwater model for an area with the same geohydrological conditions is more acceptable, if groundwater is shallow. While in deep water-table, simulating groundwater model based on surface geological condition is less acceptable, as geohydrological conditions in deep depths might be totally different from shallow depths.

5 Probabilistic approach in estimating groundwater changes

Using the predicted groundwater level changes model and the accumulative number of rainfall events within period of available rainfall data, a probabilistic methodology in estimating groundwater level changes can be provided.

In this study the rainfall data from 1961 to 2005 was used. The accumulative number of rainfall events with a magnitude of larger than desired rainfall is shown in Fig.3. The accumulative number of events can be transferred in probabilistic form using Poisson method. The general form of Poisson model is shown in equation 16, in which μ is the mean and variance in Poisson model and x is an integer parameter.

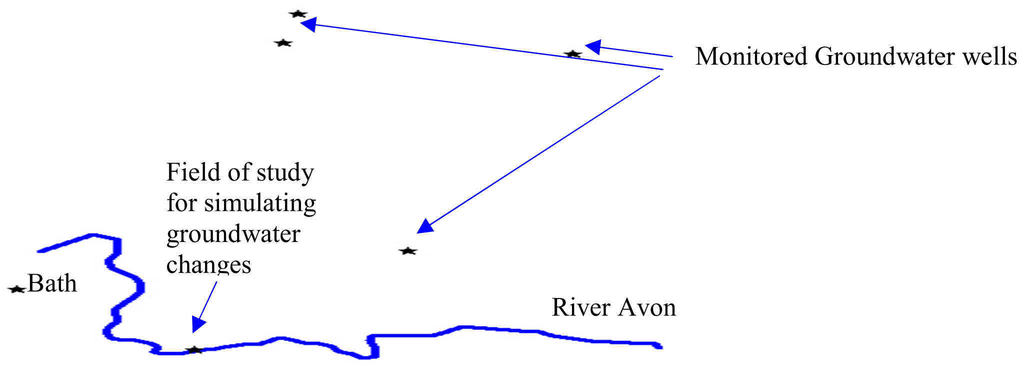

Figure 1: Location of monitored groundwater wells and filed of study

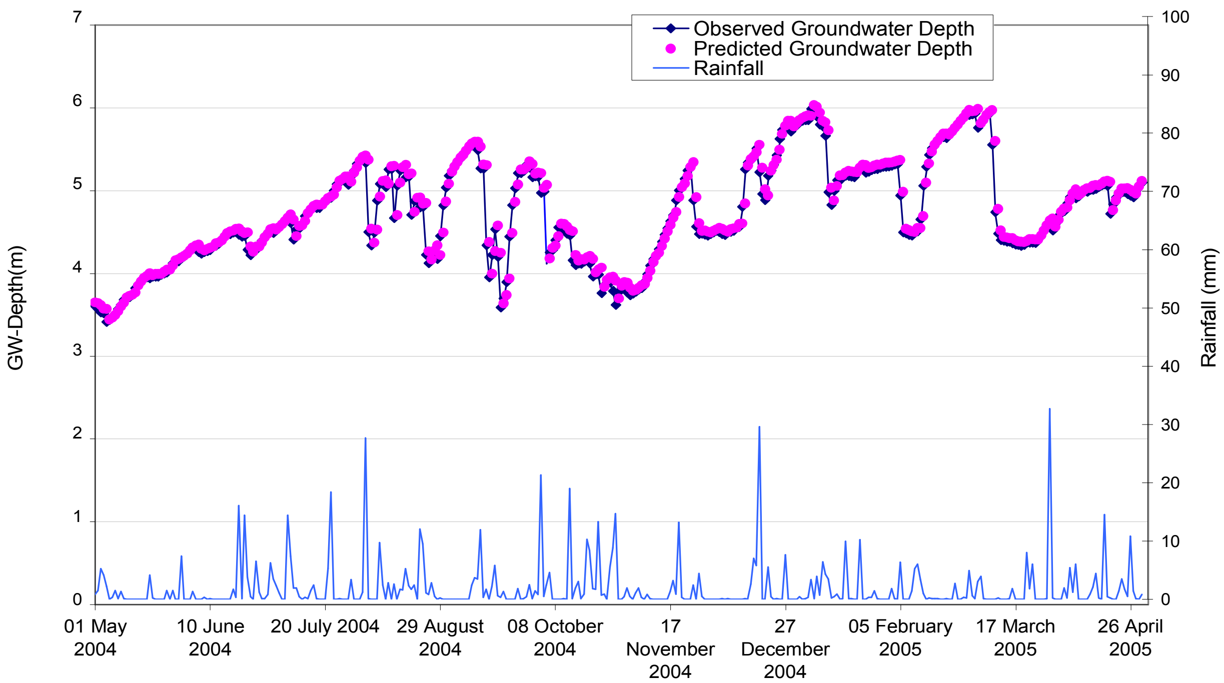

Figure 2: Predicted groundwater levels changes for Bramble Combe using simulated groundwater model from Binghams Melcombe's monitored well analysis

$$Px(x) = \mu^x e^{-\mu}/x! \quad (x = 0,1,2,3, ... ,\infty) \qquad (16)$$

The term μ can be substituted with Ny.t, in which Ny is the total annual number of events having a rainfall magnitude equal or greater than y* and t is the duration time. Now the probability of having no occurrence during the duration of t is given by:

$$Px(0) = \exp^{-\mu} \quad = \exp^{-Ny.t} \qquad (17)$$

And therefore the probability of at least one event occurring can be given by:

$$Px(\text{at least one event}) = P(Y>y^* |t)$$

$$= 1- \exp^{-Ny.t} \qquad (18)$$

In equation 18, if for example, the period of study is 50 years, then the probability of at least one rainfall event in this time period is:

$$P(\text{at least one rainfall event}) = 1- \exp^{-50Ny}$$

The probability of at least one rainfall event within 50 years for this study has been illustrated in Fig.3.

6 Modelling the probabilistic approach for the field of study

Using the predicted parameters of a_1 and a_2 in equation 12, the groundwater level changes can be estimated. A computer program, GISWaterRisk was written using visual basic

6 by the first author in order to estimate the groundwater changes parameters of equation 12 and subsequently to be combined with a GIS-based slope stability programme (SlopeSGA) to perform a risk analysis [8]. Groundwater level changes in GISWaterRisk are estimated throughout the whole year for each rainfall event. The maximum groundwater level changes versus rainfall magnitude are placed in a table and graph as shown in Fig.4. The corresponding probability for each rainfall magnitude can be placed in the same table or graph (Fig.4). Now as shown in Fig.3 each rainfall magnitude corresponds to a probability of occurrence and magnitude of raise in groundwater level. Fig.4 illustrates the probabilistic groundwater level changes estimation taken from GISWaterRisk program for the Binghams Melcombe's monitored well. The presented groundwater model using Binghams Melcombe's well data was simulated for analysing stability of natural slopes in the field of study (location shown with a triangle symbol in Fig.1).

The reasons for selecting this monitored well for the field of study were that they both have the same geological conditions and that the ground water depths in both locations were similar. Where there is no available monitored well on a particular site, the analysis using a monitored well with similar geological conditions and groundwater depth will provide a reliable approximation for predicting groundwater level changes.

7 Conclusion

A probabilistic approach can be used to create a link between rainfall events and geotechnical engineering applications, such as slope stability. In this study, a methodology for estimating groundwater changes in a probabilistic framework was presented.

This method is an approximate method but it can provide accurate probabilistic data over a study period.

The reliability of this methodology depends on the accuracy of data used. In this study the groundwater parameters were estimated in four different areas with the same geological conditions.

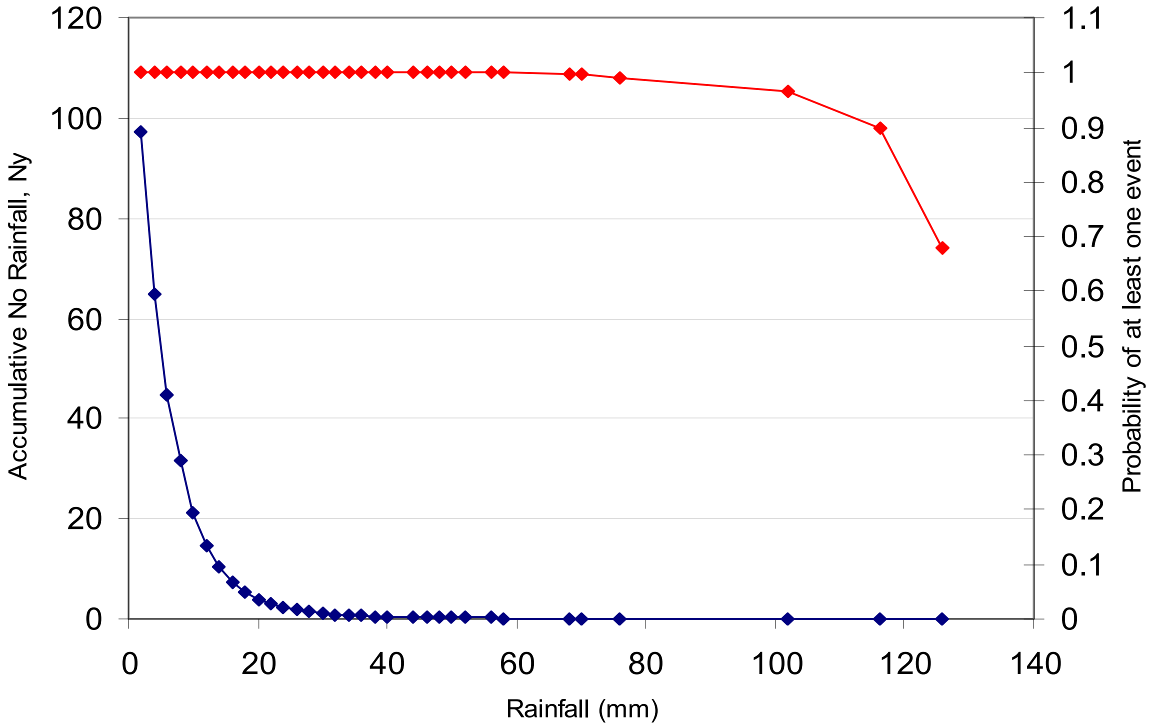

Figure 3: Accumulative number of rainfall and probability of at least one event in 50 years.

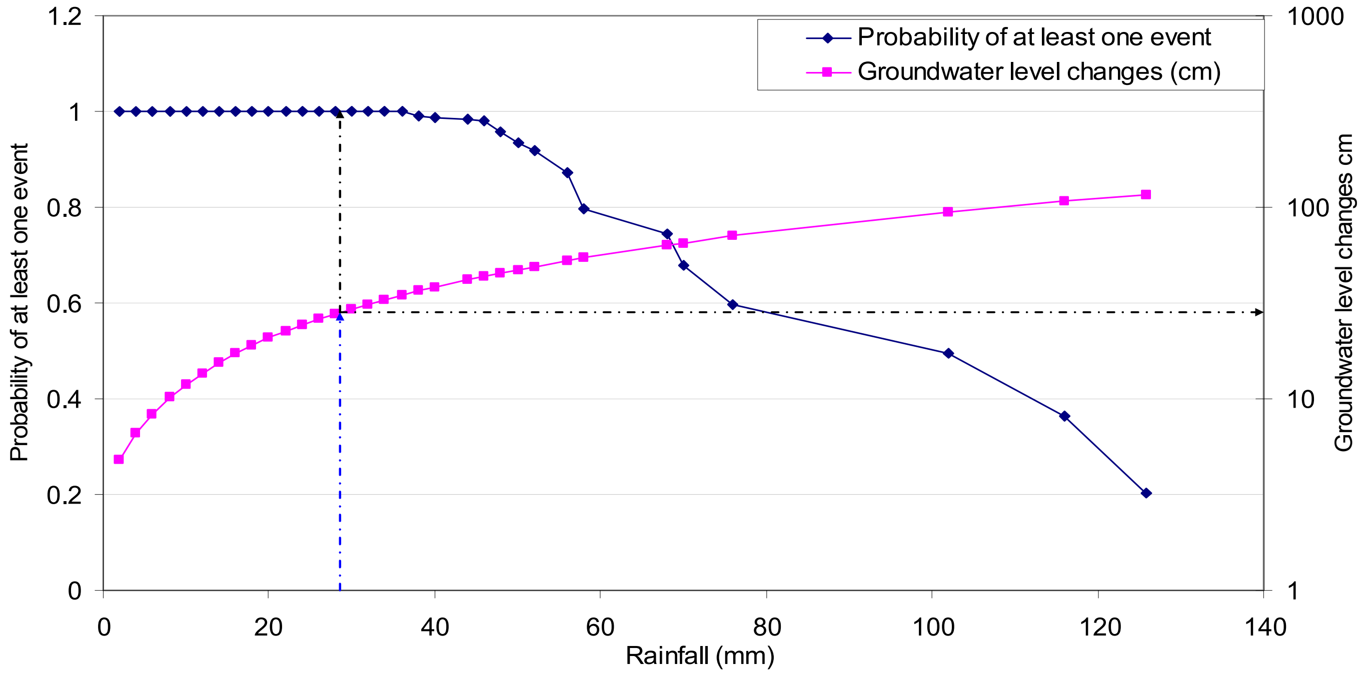

Figure 4: Probability of at least one event and groundwater level changes within 50 years.

The predicted groundwater model from one of monitored wells, with the same geohydrological conditions of field of study, was simulated for estimating the groundwater level changes for analysing stability of natural slopes in the field. The writers of this study illustrated that the groundwater model can be simulated for another area away from monitored well location, if both area have the same geological condition and the same average groundwater depth.

8 Acknowledgment

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9 Reference

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