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Numerical solution of salt-dome by network method

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Abstract: - The salt-dome problem is numerically simulated by a network model whose design is based on the network simulation method (NSM). Streamfunction formulation is used and Boussinesq approach is assumed. The proposed model simultaneously provides the streamfunction and solute concentration variables with relatively low computing times in an ordinary PC. Mathematical manipulations inherent to this type of problem are not required with the network method since this work is done by the algorithms implemented in the simulation code.

Key-Words: - Density-driven flow, Solute transport, 2-D network model, Salt dome, Porous medium, Boussinesq approximation

1 Introduction

When the transport of solute does not affect fluid density, fluid and transport phenomena are not coupled and the problem is relatively easy to solve, but there are many cases in which fluid density is dependent upon concentration, as occurs, for example, in water flows near salt domes and for sea water intrusion. In these cases, the problem of solute transport is much more difficult to solve because it is highly nonlinear. Salt water affects fluid density which, in turn, affects the local velocity field.

Transport simulation for solute involves a variety of complex processes that interact with oneanother: advection, diffusion, dispersion and, sometimes, sorption and decay [1]. Numerical techniques must be resorted to especially as long periods of time involved.

The case of groundwater flow in geological formations overlying a salt dome has taken on special interest since salt formations have been considered from the disposal of hazardous waste in some countries –USA, Germany, Russia, Spain and others [2].

A salt dome is a geological formation in the subsurface in which incompressible salt of high plasticity rises (because of the geothermal gradients or other heat sources) through sediment layers towards the surface of the earth, forming a typical dome.

Numerical codes that use different approximation methods (FD, FE and integral finite differences), different time discretizations (predictor-corrector, semi implicit and fully implicit) and different coupling processes have been developed by many authors. Some of these codes are SWIFT [3], SUTRA [4] and FEFLOW [5]. Sometimes the results obtained by different codes can be contradictory [5]. Herbert et al. [6], Oldenburg and Pruess [7], and Johns and Rivera [8] studied the salt dome problem in the last decade.

An example of idealized flow over the salt dome is Case 5, Level 1 of the International HYDROCOIN ground water flow modeling project [9], a workshop organized by the Swedish Nuclear Power Inspectorate (SKI) with the objective of re-evaluating the procedures, assumptions and results of this benchmark case which is, however, far from the complexity of many actual problems.

The mathematical model is defined in terms of the stream function, an option also chosen by other authors. The code we use to simulate this problem is based on the network simulation method (NSM) [10]. A network model is designed from the finite difference differential equations that result from the spatial discretization of the partial differential equations (like in the known Lines Method). NSM, which has been successfully used for numerical simulation of other types of nonlinear problem makes use of the powerful capabilities of modern circuit simulation computer codes which use the more complex algorithms of calculus.

Since two dependent variables exist, two independent (electrically isolated) circuits form the model. Each

term of the discretized equations, whatever be its expression, is implemented in the model by a unique device: a resistor, a capacitor or a controlled current source, so that the design of the model is quite easy since very few terms form the equation. In addition, water and salt balances are assumed by the simulation code because these balances are the theorems of Kirchhoff (current Kirchhoff law).
Finally, boundary conditions are also easily implemented [10]. Pspice [11] is the code used for the simulation.

2 The governing equations

The 2-D coupled equations that define the mathematical model are those of the problem of density-driven flow and solute transport through porous media, already discussed in the communication of Soto et al. presented to this conference [12]. They are formulated in terms of the streamfunction, ψ, and assuming the Boussisnesq approach $\rho \cong \rho_o$. Streamfunction is defined as $\partial\psi/\partial x = q_y$ and $\partial\psi/\partial y = -q_x$, where q (m/s) is the specific discharge.
Defining the dimensionless variables

$$x' = x/H \quad (1)$$

$$z' = z/H \quad (2)$$

$$t' = t(D/H^2) \quad (3)$$

$$q_x' = q_x(H/D) \quad (4)$$

$$q_y' = q_y(H/D) \quad (5)$$

$$\psi' = \psi/D \quad (6)$$

$$c' = (c - c_o)/(c_s - c_o) \quad (7)$$

the resulting equations are:

$$(\partial^2\psi'/\partial x'^2) + (\partial^2\psi'/\partial y'^2) = Ra\,(\partial c'/\partial x') \quad (8)$$

$$(\partial^2 c'/\partial x'^2) + (\partial^2 c'/\partial y'^2) - (\partial\psi'/\partial x')(\partial c'/\partial y') + +(\partial\psi'/\partial y')(\partial c'/\partial x') = (\partial c'/\partial t') \quad (9)$$

with Ra the Rayleigh number, $Ra = \kappa\,\Delta\rho\,g\,H/(D\mu)$. In these equations μ ($Nm^{-2}s$) is the fluid viscosity, k (m^2) the permeability of the porous medium, g (m^2s^{-1}) the gravitational acceleration, ρ (kg/m^3) the flow density, ε (dimensionless) the porosity c (kg/m^3) the salt concentration and D (m^2s^{-1}) the diffusivity. x (m), z (m) and t (s) are the independent variables space and time.

3 The network model

The steps for designing the network model are also discussed in the reference Soto et al. mentioned in the above section. More details about the design rules may be found in González Fernández et al. [10]. Figure 1 shows the network model of the volume element. Two independent networks make up the model, one related to the water flow problem, eq. (7), and the other related to solute transport, eq. (8). The coupled terms of these equations are implemented in the model by the current sources called "G" in the figure. These devices are able to provide a flow variable that can be defined by software and can be depend on the potential variables (concentration and streamfunction.

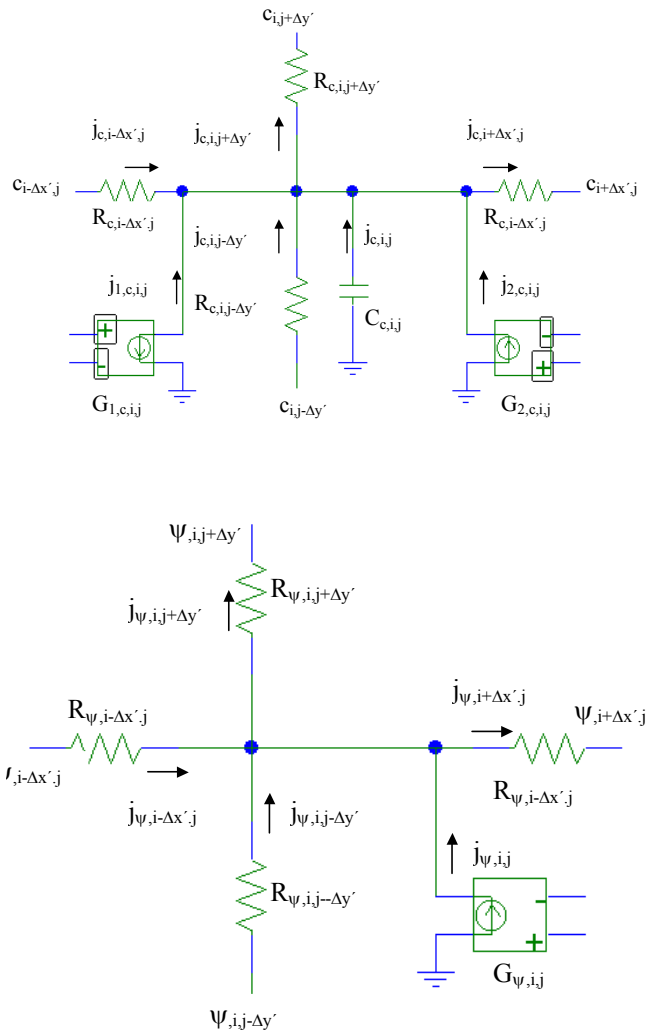

Figure 1. Network model of the volume element.
a) streamfunction variable,
b) saltwater concentration variable

4 Application. The salt dome problem

Parameters of the problem are:

H′= 1 (height of the aquifer, dimensionless)
L′ = 3 (length of the aquifer)
Ra = 1176.6 (Rayleigh number)
$v'_{o,x}$ = - 33.3 (prescribed horizontal velocity)
$\psi'_o = 0$

while the boundary and initial conditions are:

$\psi' = \psi'_o$, $\partial c'/\partial y' = 0$, $0<x'<1$, $y' = 0$ (10)
$\psi' = \psi'_o$, $c' = 1$, $1<x'<2$, $y' = 0$ (11)
$\psi' = \psi'_o$, $\partial c'/\partial y' = 0$, $2<x'<3$, $y' = 0$ (12)
$\psi' = \psi'_o$, $\partial c'/\partial x' = 0$, $x' = 0$ and $x' = 3$ (13)
$\partial\psi'/\partial y' = - v'_{o,x}$, $c' = 0$, $y' = 1$ (14)
$c' = 0$, $t' = 0$ (15)

These conditions are sketched in Figure 2. On the one hand, the constant value of ψ′ (Diritchlet) is implemented by a constant voltage source connected to the extreme of the volume elements in the network of the flow, while the condition $\partial\psi'/\partial y' = - v'_{o,x}$ (Neumann) is implemented by a current source. On the other hand, the constant value of c′ is also implemented by a constant voltage while the condition $\partial c'/\partial x' = 0$ is implemented by a resistor of very high value to prevent the flow salt.
The total number of volume elements used is 800, 40 horizontal and 20 vertical. A relatively low number in order to diminish the computing time necessary.

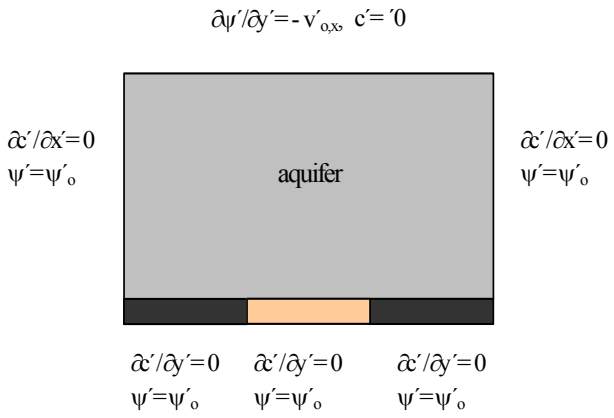

Figure 2. Schematic view and boundary conditions

An auxiliary C# program has been designed to import the tabulated data (output file) of Pspice code and to represent them in MATLAB.
Figures 3 and 4 shows the values of ψ′and c′, respectively, for a dimensionless time t′= 0.05, which is far from the steady time. Rayleigh number is 1176.6. Two eddies are formed in the bottom part of the aquifer (figure 3) turning in opposite directions. Most of solute is distributed at the lower region of the aquifer.
For a dimensionless time t′= 0.1, Figure 5 and 6 shows these same variables. We also see, for this time, recirculation in the lower region that transports the dense brine throughout the entire lower portion of the domain. These results also appears in the solution of other authors [6-7].

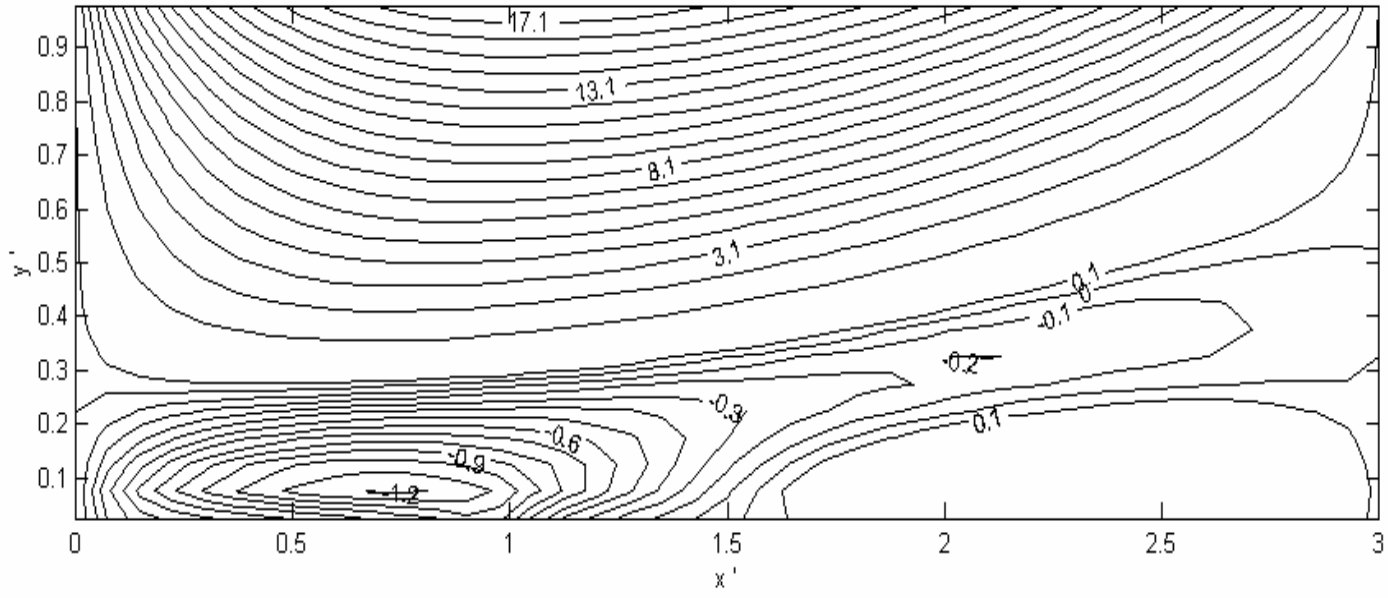

Figura 3. Unsteady streamfunction (dimensionless). t′ = 0.05

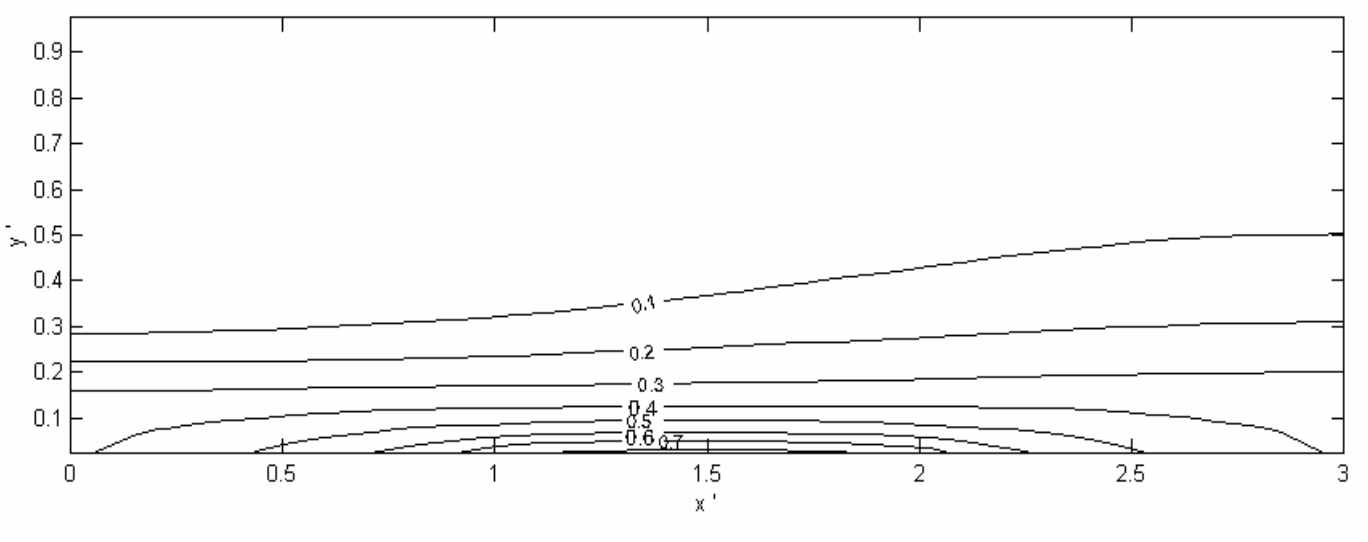

Figura 4. Unsteady solute concentration (dimensionless). t′ = 0.05

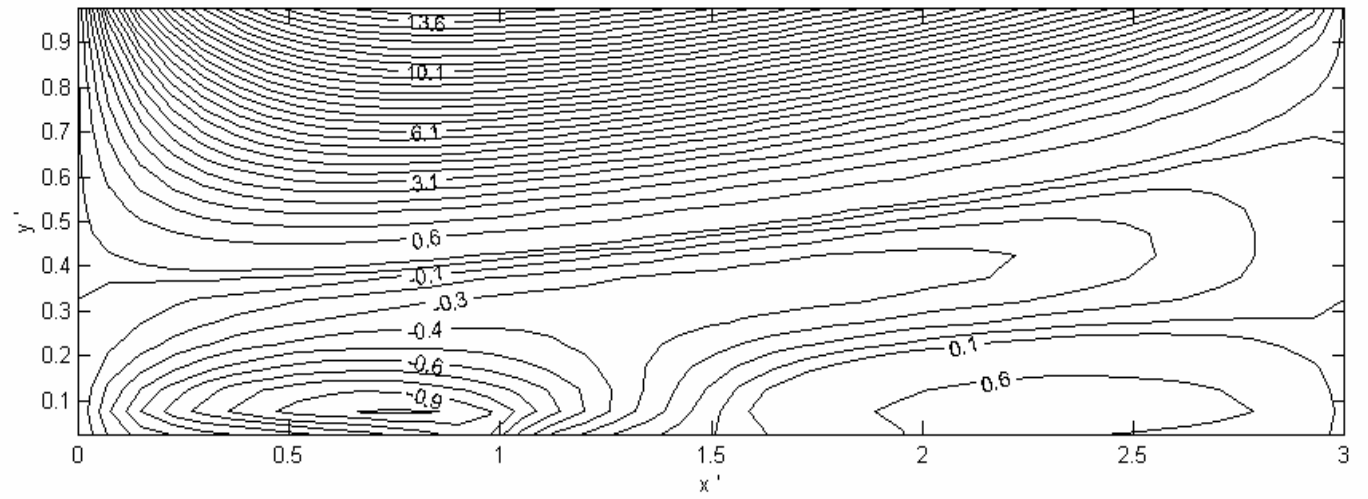

Figura 5. Unsteady streamfunction (dimensionless). t′ = 0.1

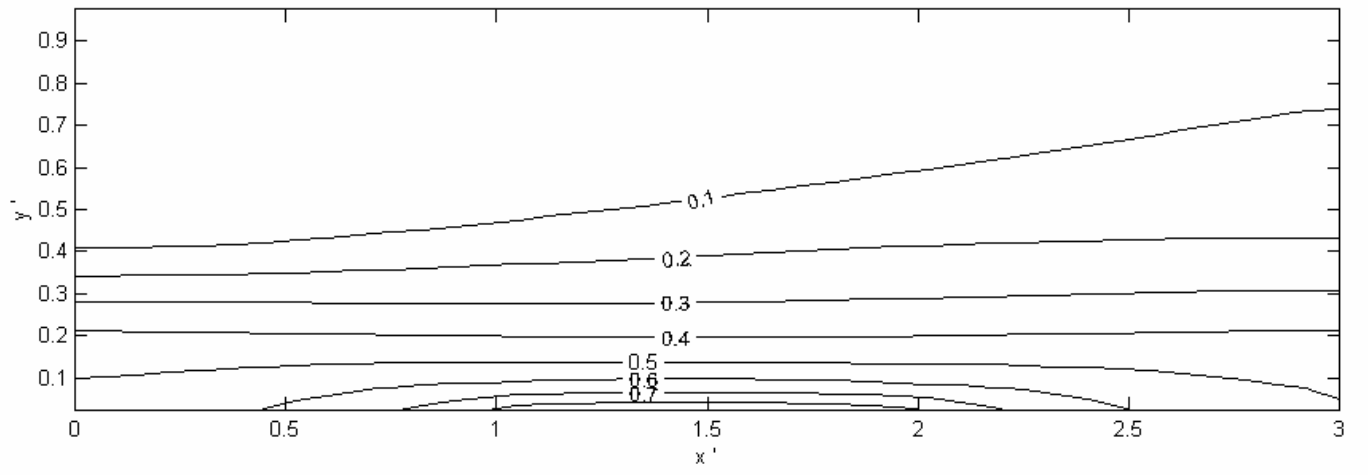

Figura 6. Unsteady solute concentration (dimensionless). t′ = 0.1

Streamfunction has a similar aspect for both times although the numerical values are different, particularly in the upper region. The salt concentration curves are, however, different since salt rises with time.
The computing times are in the order of 140 s in an ordinary PC.

2 Conclusions

The salt dome problem has been numerically simulated by a network model in a circuit simulation code (Pspice). Two independent circuits in each volume element are implemented, one for the fluid flow variable and one for the salt concentration variable. Simultaneous solution for both variables is provided by the code in tabulated and graphic form without the complex mathematical manipulations inherent in this type of coupled problem. The coupling between equations is carried out by a current source whose output is easily defined by software. Computing times are relatively short in a PC.

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