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Energetic Aspects Referring to Servo Drive Systems

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Abstract:- Certain aspects referring to the energy consumptions in the servo drive system with D.C. motors are presented. The possibilities of the decrease of the energy losses are emphasized; these possibilities appear especially in the transient period of the speed change. A simple structure for suboptimal control is proposed.

Keywords: - servo system, energy consumption, optimal control, suboptimal control.

1 Introduction

There is nowadays an increased interest for energy saving and different methods and procedures are proposed for this purpose. One of the means of the achieving this goal is the optimal control [1], [2] of the drive and servo drive systems. Tacking into account that the more than 60% of the produced electrical energy is consumed by drive systems, this method is very important in order to ensure the reducing of the energy consumption. Unfortunately, the number of applications of the optimal control of electrical drives is very small and we appreciate that this is caused by the algorithm complexity. However, the methods proposed by authors [3], [4] allow to carry out an easy implementation and therefore, it is useful to apply the optimal control, since the diminution of the energy losses in the motor windings is up to 25…30% in the electromechanical transient process, by comparison with classical cascade control. These aspects are also interested for servo drive systems, tacking into account that they work very much time in transient state.

The aim of this paper is to present some energetic aspects regarding the electromechanical transient process of the electrical servo systems and to emphasize the great possibilities of the reducing energy consumption in this case. There are many papers and books which present the energy consumption of the electrical servo drives and their optimal control, but we consider that it is not sufficient underlined the possibilities of the energy losses decrease.

Since the goal is to emphasize the general energetic aspects and not an exact computing of the energy components, a simplified model will be considered. Mainly, the electromagnetic transient processes are neglected and therefore, the rotor current is adopted as control variable.

Starting from the results obtained on this basis, a simple structure for suboptimal control is proposed.

The main conclusions of this paper are valid for different types of the drive motors. For simplicity only the D.C. motors case is presented. A similar study for an electrical drive system was presented in [5].

2 Energy Consumptions

We shall consider the following model for an electrical servo drive system with a D.C. motor [6]:

$$\begin{aligned} &\dot{\theta}(t) = r\omega(t) \\ &u(t) = Ri(t) + L\frac{di(t)}{dt} + c_e\omega(t) \\ &c_m i(t) = J\dot{\omega}(t) + m(t), \end{aligned} \qquad (1)$$

where u(t) and i(t) are the rotor voltage and current, respectively, $\omega(t)$ is the rotor speed, m(t) is the load torque, R and L are the rotor winding resistance and inductance, respectively, J is the inertia, and c_e, c_m are motor parameters and r is the gear constant. We suppose that m(t) = constant on the interval [0,T] of the tracking process. Also, for simplification, we shall suppose that the electromagnetic transient process can be neglected (L=0). In this case, the current may be considered as an control variable. The simplified model of the servo system is

$$\dot{\theta}(t) = r\omega(t) \qquad (2)$$

$$\dot{\omega}(t) = \frac{c_m}{J} i(t) - \frac{1}{J} m(t) \qquad (3)$$

The energy losses in the rotor winding on the interval [0,T] are

$$E = R\int_0^T i^2(t)dt \tag{4}$$

The current i(t) can be expressed from (3)

$$i(t) = i_0(t) + i_s(t) \tag{5}$$

where

$$i_0(t) = \frac{J}{c_m}\dot{\omega}(t) \tag{6}$$

is the component which imposes the acceleration for the non load operation and

$$i_s(t) = m(t)/c_m \tag{7}$$

is the component established by the load torque.

Corresponding to these components, the energy loss is

$$E = R\int_0^T i_0^2(t)dt + R\int_0^T i_0 i_s(t)dt + R\int_0^T i_s^2(t)dt \tag{8}$$

If m = constant, i_s = constant, the last integral has the value $E_s = RTm^2/c_m^2$ and the second integral is

$$\begin{aligned} Ri_S\int_0^T i_0(t)dt &= R\frac{J}{c_m}i_S\int_0^T \dot{\omega}(t)dt = \\ &= R\frac{J}{c_m}i_S[\omega(t) - \omega(0)] \end{aligned} \tag{9}$$

The integral (9) has a fixed value for imposed terminal conditions ($\omega(0)$ and $\omega(T)$). If $\omega(0) = \omega(T) = 0$ (this situation frequently appears in the servo drive applications),

$$\int_0^T i_0(t)dt = 0 \tag{10}$$

Since the last two terms in (8) have imposed values for the imposed operating conditions, only the first integral in (9) can be modified by means of an adequate control. Therefore, in the sequel we shall consider especially this component and we shall denote in these cases $i = i_0$ and $E = E_0$.

Note also that (10) indicates the mean values of the current for no-load system in the accelerating and decelerating period are equal, but with inverse sign.

3 Optimal Control

In order to obtain a good behaviour of the system and reduced energy consumption, it is recommended to adopt an optimal control, using a quadratic criterion

$$\begin{aligned} I = \frac{s_1}{2}[\theta_d - \theta(T)]^2 + \frac{1}{2}\int_0^T [q_1(\theta_d - \theta(t))^2 + \\ + q_2(\omega_d - \omega(t))^2 + q_3 i^2(t) + pu^2(t) + \\ + ru(t)i(t)]dt \end{aligned} \tag{11}$$

The criterion (11) is used in the problems with free end-point. The first term penalizes the difference between the desired values, θ_d and the final values $\theta(T)$. The first two terms in integral penalize the mean transient error of the angular displacement and of the speed and the third one refers to the energy losses. The next term penalizes the great value of the control variable u(t) and the last one refers to the global energy consumption. In the problems with fixed end-point, $s_1 = 0$ and it is imposed to achieve $\theta(T) = \theta_f$.

Since our goal is to study the energetic aspects of the drive system control, we shall consider only the criteria in the form

$$I_J = E = \int_0^T Ri^2(t)dt \tag{12}$$

or

$$I_T = \int_0^T i(t)u(t)dt \tag{13}$$

Note that the optimal control is equivalent in the both mentioned cases if the load torque *m* is constant (this property is proved in [5] for an electrical drive system), so that only the criterion (12) will be considered below.

The optimal control problem refers to the criterion (12) and the system (2), (3), with imposed terminal states $\omega(0)$, $\omega(T)$ and $\theta(T) = \theta$.

Since the initial and final conditions have not a major significance in the linear systems case, we shall consider the simplest conditions $\theta(0) = 0$, $\omega(0) = \omega(T) = 0$.

The Hamiltonian [1] of the problem is

$$H = Ri^2(t) + \lambda_1(t)r\omega + \lambda_2(t)\frac{1}{J}(c_m i(t) - m)$$

(λ_1 and λ_2 are the costate variables).

From the necessary conditions

$$\partial H/\partial i = 0, \partial H/\partial\theta = -\dot{\lambda}_1, \partial H/\partial\omega = -\dot{\lambda}_2,$$

One obtain the optimal current

$$i^*(t) = 6\frac{J}{rc_m}\frac{\theta_f}{T^2}(1 - 2\frac{t}{T}) + \frac{m}{c_m}, \tag{14}$$

where θ_f is the final imposed value of the angular displacement, and the corresponding optimal state variables

$$\omega(t) = 6\frac{\theta_f}{rT^2}(t - \frac{t^2}{T}) \tag{15}$$

$$\theta(t) = \frac{\theta_f}{T^2}(3t^2 - 2\frac{t^3}{T}), \quad \text{with } \theta(T) = \theta_f. \tag{16}$$

The minimum energy losses are from (12) and (14)

$$E^* = I_J^* = 12\frac{J^2}{rC_m^2}\frac{\theta_f^2}{r^2T^3} \tag{17}$$

In servo applications θ_f is imposed and we can diminish I_J^* by increase the transfer duration T, but a very long time is not acceptable in many cases.

From (14) and (15) we can compute the maximum values

$$\begin{aligned} i_M &= i(0) = \frac{6\theta_f}{rT^2}\frac{J}{C_m} + \frac{m}{C_m}; \\ \varepsilon_M &= \dot{\omega}(0) = \frac{6\theta_f}{rT^2}; \\ \omega_M &= \omega\left(\frac{T}{2}\right) = \frac{3\theta_f}{rT}. \end{aligned} \tag{18}$$

Also, we can define the mean values

$$\omega_m = \frac{\theta_f}{rT} = \frac{1}{3}\omega_M, \quad \varepsilon_m = \frac{\omega_M}{T/2} = \frac{3\theta_f}{rT^2}. \tag{19}$$

The mean or maximum value for speed and/or acceleration is imposed in certain applications. In this case, the duration T can be established for a given θ_f. The minimum value of the energy loss can be then expressed

$$I_J^* = \frac{2}{3}\frac{J^2}{C_m^2}\omega_M\varepsilon_M = 4\frac{J^2}{C_m^2}\omega_m\varepsilon_m. \tag{20}$$

Of course, if the speed and/or acceleration are restricted to the admissible values ω_a and ε_a, respectively, the control current (14) will be modified in the period $[t_1, t_2] \in [0, T]$ when the limit values ω_a or ε_a are achieved. A limit value for current will be adapted in this case, for instance

$$i_a = J\varepsilon_a / C_m + m / C_m. \tag{21}$$

If the emitted heat (17) in the period [0,T] overcome the admissible one, a motor with a bigger rated power must be adopted. But, if the motor is adopted in a first design step (usually from the heat conditions) considering a certain control law, a decrease of the emitted heat is obtained if the optimal control is preferred. Moreover, the reducing of the energy losses allows in many cases to adopt a motor with a smaller rated power.

By this way, the optimal control ensures not only the energy saving but allows to reduce the cost, weigh and volume. Note that in certain applications the decrease of weight of sub- ensemble leads to the diminish of the energy consumption of the all plant.

Non optimal control (in the sense of criterion (12)) leads to greater energy losses. The difference between the losses depends on the difference

$$\delta i(t) = i(t) - i^*(t) \tag{22}$$

between the non optimal and optimal current.

One can prove that the energy losses for non-optimal control are

$$E = E^* + \int_0^T (\delta i)^2 dt \tag{23}$$

and this expression can be asserted by the following

Application: We shall consider that the control variable has only two values:

$$\begin{aligned} &i(t) = i_1 > 0,\ t \in [0, t_1] \text{ and} \\ &i(t) = i_2 < 0,\ t \in [t_1, T]. \end{aligned} \tag{24}$$

For $m = 0$, $\dot{\omega}(t) = (C_m / J)i(t)$ and we find

$$\omega(t) = \omega_1(t) = \frac{C_m}{J}i_1 t \text{ for } \omega(0) = 0,\ t \in [0, t_1]$$

and

$$\omega(t) = \omega_2(t) = \frac{C_m}{J}[\varepsilon_2 t + (i_1 - i_2)t_1],$$
$$\text{for } \omega_2(t_1) = \omega_1(t),\ t \in [t_1, T]$$

The condition $\omega_2(\tau) = 0$ leads to

$$i_2 = -i_1\frac{t_1}{T - t_1} \tag{25}$$

Further

$$\theta(t) = \theta_1(t) = \frac{C_m}{J}i_1\frac{t_1^2}{2}, \text{for } \theta_1(0) = 0,\ t \in [0, t_1]$$

and, having in view (25),

$$\theta(t) = \theta_2(t) = \frac{C_m}{J}i_1\frac{t_1}{2(T - T_1)}(-t^2 + 2Tt - Tt_1),$$
$$\text{for } \theta_2(t_1) = \theta_1(t_1),\ t \in [t_1, T].$$

From the condition $\theta_2(T) = \theta_f$ and using (25), we obtain the necessary values for the control variable in order to achieve the final condition

$$i_1 = \frac{2J}{C_m}\frac{\theta_f}{Tt_1},\ i_2 = -\frac{2J}{C_m}\frac{\theta_f}{T(T-t_1)} \tag{26}$$

The energy losses are

$$E_n = R\left[i_1^2 t_1 + i_2^2 (T-t_1)\right] = \frac{4J}{C_n^2}\frac{\theta_f^2}{T^2}\frac{T}{t_1(T-t_1)} \tag{27}$$

One can easy to remark that E_n has a minimum value for $t_1 = T/2$ (the periods for acceleration and deceleration are equal), when

$$i_1 = -i_2 = \frac{4J}{C_m}\frac{\theta_f}{T^2} \text{ and } E_n = 16\frac{J^2}{C_m^2}\frac{\theta_f^2}{T^3} \tag{28}$$

It is well known [1] that (28) is the solution for the minimum time control problem for the given system and for the imposed maximum value for $|i(t)|$. It comes out that the final conditions can be reached in the same time T with greater energy losses by comparison with optimal control from energetic point of view (17). Although the control current (14) has a maximum value greater than (28), the quadratic mean value is smaller and this fact ensures a smaller energy loss.

The difference between control current (28) and optimal control (14) is

$$\delta i(t) = \frac{2\theta_f}{T^2}\frac{J}{C_m}\left(1 - \frac{6t}{T}\right)$$

Tacking into account the symmetry of the control for $t < T/2$ and $t > T/2$, the corresponding difference (23) of energy consumptions is

$$E_n - E^* = 2\int_0^{\frac{T}{2}} (\delta i)^2 dt = 4\frac{J^2}{C_m^2}\frac{\theta_f^2}{T^3} \tag{29}$$

Obvious, this value is in concordance with (17) and (28).

A comparison with other non optimal control law (e.g. – a linear feedback control) can be also performed, but the expressions are more complicated. Note that in each case the energy losses provided by the no-load component of the current is in the form

$$E = \alpha\frac{J^2}{C_m^2}\frac{\theta_f^2}{T^3},\ \alpha \geq 12.$$

For analyzed non optimal control, $\alpha = 16$ and the losses increase with 33% by comparison with optimal control.

Remark: The above established current $i^*(t)$ is an ideal optimal control, since we have supposed that the current can be instantaneous modified. The real optimal control, such it is established in [3], [4], introduces a supplementary value of energy losses, depending on the difference between the ideal and non ideal optimal control (see (27)).

4 Optimal control implementation and simulation results.

The results were simulated for a servo drive system with a d.c. motor with following data: U_n=110 V, I_n=3.3 A, R=3,1Ω, L=0.16 H, C_e=0.58 Vs/rad, C_m=0.58 Nm/A, J=0.028 Nms^2/rad.

The optimal system behaviour (corresponding to equations (14), (15), (16)) is indicated in Fig. 1 (for m = 0) and Fig. 2 (for m = 0.78 Nm).

The energy consumptions are indicated on the figures.

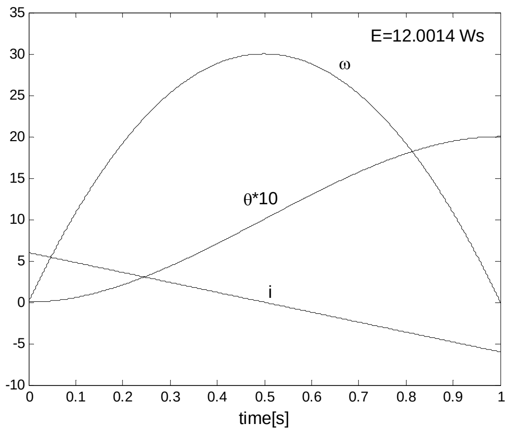

Fig. 1

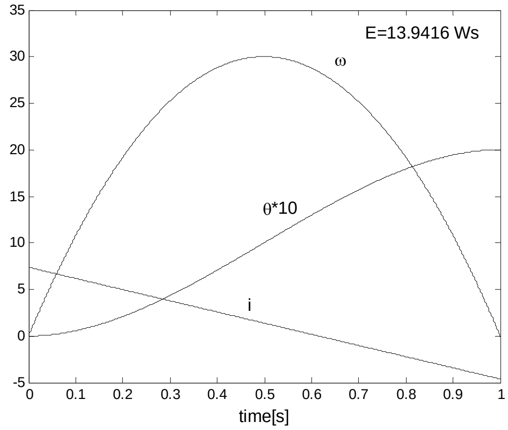

Fig. 2

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This optimal control was established above in certain idealized suppositions. A more realistic study implies to consider the system (1) and the criterion (11). This problem is solved in [3], [7] for free and fixed end point cases; the proposed procedures ensure a simpler implementable feedback solution by comparison with classical methods. The Fig.3 and 4 present the behaviour of such an optimal system for the fixed end-point case, for m = 0 and m = 0.78 Nm, respectively.

We can remark that the variables have similar variations as the ideal case (Fig. 2 and 3). Of course, the current has not a step variation at the initial moment and the energy losses are bigger (with about 11%) then in the ideal optimal case.

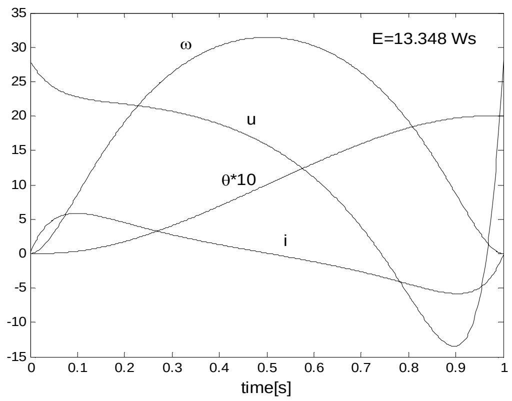

Fig. 3

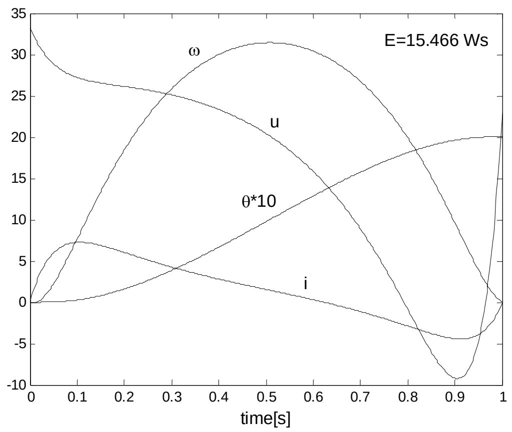

Fig. 4

The obtained equations for optimal current and speed suggest a very easy way for implementation of the optimal control.

A first possibility is to impose for the current the variation given by (14). The drawback is the dependence on the load torque *m* and this fact implies to estimate the torque at the beginning of the optimal process.

A simpler way is to impose the variation of speed in concordance with (15). The structure of this system is indicated in the fig. 5, where ω^* is the prescribed variation for speed (15), C is the controller and M is the motor.

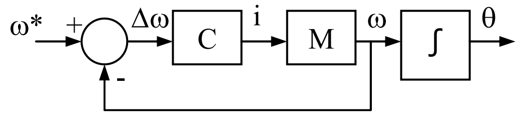

Fig. 5

Different type of controllers (C) can be chosen - for instance a predictive controller, an optimal one (in accordance with a criterion depending an $\Delta\omega$). The performed tests have indicated that a simple PI controller ensures a good concordance between ω and ω^*, as it is indicated in the Fig. 6and 7 for m = 0 and m = 0.78 Nm, respectively.

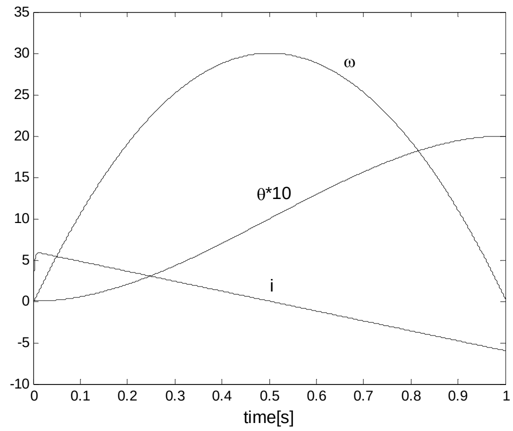

Fig. 6

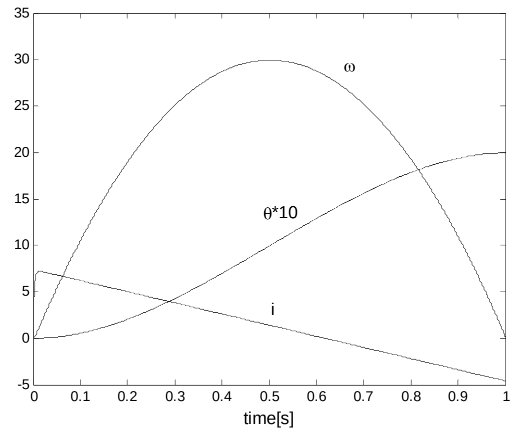

Fig. 7

The desired variations $\omega^*(t)$ is not presented on these figures because they are almost the same as $\omega(t)$

The system implemented by this way is suboptimal, but the difference between this system and an optimal one is very small.

On the other hand, the implementation is simple by comparison with other solution for optimal control. Of course, it is useful to carry out a closed loop system, using a supplementary angular displacement feedback.

5 Conclusions

An optimal control problem referring to an electrical servo drive version is studied. This case is approached as a linear quadratic optimal problem.

The winding energy losses represent a great amount of the total energy consumption of the servo drive system with D.C. motor in the transient process.

A suitable control of the rotor current leads to a significant decrease of the energy losses.

A simple suboptimal algorithm is presented and a comparison with the optimal solution is performed.

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