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Optimal Load Dispatch in Power Plant under Uncertain Information

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Abstract: The principles of optimal load dispatch in condensing thermal power plant under uncertain information are presented in this paper. The uncertainty factors are the input-output characteristics of power units and the load demand of power plant. For optimal load dispatch under uncertainty conditions the criterion of min-max risk is recommended. The min-max optimal load dispatch problems are suitable to solve by the planned characteristic method. The method consists of two stages: 1) computation of planned characteristics of units; 2) solving the deterministic equivalent of min-max optimization problem.

Key-Words: Optimal load dispatch, thermal power plant, input-output characteristics of power units, probabilistic information, uncertain information, planned characteristic method

1 Introduction

The paper describes the principles of optimal dispatch of electrical power between the power units in a power plant under uncertain information. The computation problem of optimum active power generation schedules in a power plant for a certain time period (hour, day or week) is one of relevant optimization tasks in power plant control. Commonly regarded as a deterministic problem, its objective function, constraints and uncontrollable factors are single-valued [1], [2]. At the same time, are assumed, that the planned schedules of generation will be realized exactly. Actually the initial information is never complete and the planned generation schedules are subject to several corrections. Inconsideration of these circumstances decreases the efficiency of optimization. Therefore, it is necessary to elaborate power plant state optimization methods, which take into account really existing incompleteness of the initial information and deviations of factual states of a power plant from the planned ones. This problem is highly relevant in wind turbine based power systems.

2 Problem Formulation

2.1 Deterministic model

Let the power plant consists of n thermal-generating units. The input-output characteristics of power units can be represented as the composite functions:

$$C_i(P_i) = c_i B_{Bi}(Q_B\ (Q_{Ti}(P_i))),\quad i = 1,\ldots, n, \qquad (1)$$

where

i - unit index;

c_i - fuel price of unit i;

C_i - - fuel cost of unit i;

P_i- - power output of unit i;

$Q_{T\,i}$ - heat input of turbine i;

$B_{Bi}(Q_{Bi})$ - input-output characteristic of boiler i;

$Q_{Ti}(P_i)$ - input-output characteristic of turbine i;

$C_i(P_i)$ - input-output characteristic of unit i,

$Q_{Bi} = Q_{Ti}$.

Power units typically have continuous, piecewise smooth and strictly convex cost functions.

The optimal dispatch problem in a condensing power plant is to determine power unit loads to ensure the minimum of the production cost and meeting of all the constraints.

The initial deterministic optimization model is

$$\text{Minimize} \qquad C(P) = \sum_{i=1}^{n} C_i(P_i) \qquad (2)$$

Subject to the following constraints:

1) The power balance equation:

$$G = P_D + \sum_{i=1}^{n} P_{ai}(P_i) - \sum_{i=1}^{n} P_i = 0, \qquad (3)$$

2) The constraints of power limitation of power units (i = 1,…, n):

$$P_i^- \le P_i \le P_i^+ , \qquad (4)$$

where

P - vector of units outputs (load),

P_D - active power demand (load) of power plant;

P_i^-, P_i^+ - lower and upper limits of the power unit i,

$P_{ai}(P_i)$ - auxiliary power characteristic of the power unit i:

$$P_{ai}(P_i) = P_{auBi}(Q_{Bi}(P_i)) + P_{auTi}(P_i) , \qquad (5)$$

where P_{auBi} - auxiliary power of the boiler i,

P_{auTi} - auxiliary power of the turbine i.

The problem (2)-(4) enables to determine the minimum of cost in the ideal case, where all the initial data are given exactly and optimal power schedules are realized exactly too.

2.2 Min-max model

Let's assume now, that we don't have the exact information about input-output characteristics of units and about expected value of the power plant load demand. For that reason we will assume, that only the intervals of these characteristics and the interval of power demand are given.

Therefore, the input-output characteristics of power units can be regarded as the uncertain functions. We assume that the fuel cost characteristics may be presented in the following form:

$$C_i^-(P_i, Z^-) \le C_i(P_i, \tilde{Z}) \le C_i^+(P_i, Z^+) \qquad (6)$$

and the incremental fuel cost rate characteristics as

$$\beta_i^-(P_i, Z^-) \le \beta_i(P_i, \tilde{Z}) \le \beta_i^+(P_i, Z^+) ,$$
$$i = 1, \dots, n, \qquad (7)$$

where

$\tilde{Z}_i$ - the formal vector of uncertain factors of unit i, $Z^- \le \tilde{Z} \le Z^+$,

$C_i(P_i, \tilde{Z}_i)$ - uncertain fuel cost characteristic of unit i,

$\beta_i(P_i, \tilde{Z}_i)$ - uncertain incremental fuel cost rate characteristic of unit i.

The following functions $C_i^-(P_i, Z^-)$, $C_i^+(P_i, Z^+)$, $\beta_i^-(P_i, Z^-)$ $\beta_i^+(P_i, Z^+)$ are given as the deterministic functions.

Here

$\beta_i(P_i, \tilde{Z}) = \dfrac{\partial C_i(P_i, \tilde{Z})}{\partial P_i}$ is the incremental fuel cost rate of the power unit i.

Under the market conditions, the load demand of power plant is also an uncertain factor, given by the inequalities

$$P_D^- \le \tilde{P}_D \le P_D^+ , \qquad (8)$$

There are several possibilities to optimize the processes and systems under uncertainty conditions [3, 10]:

1) Laplace criterion
2) Min-max cost criterion
3) Min-max risk or min-max regret criterion.

The best criterion for optimization under uncertain information is the min-max risk criterion [6]-[9].

The min-max risk model for optimization under uncertainty is following:

$$\min_P \max_Z R(P, \tilde{Z}) = C(P, \tilde{Z}) = \min_P C(P, \tilde{Z}) \quad (9)$$

Subject to corresponding constraints:

$$G = \tilde{P}_D + \sum_{i=1}^{n} P_{ai}(P_i) - \sum_{i=1}^{n} P_i = 0 , \qquad (10)$$

$$P_i^- \le P_i \le P_i^+ , \qquad (11)$$

where

R is the risk (loss) caused by uncertainty of information,

min C is the ideal deterministic minimum of cost:

$$\min C(P, \tilde{Z}) = C(P^0(\tilde{Z})) .$$

Here $P^0(\tilde{Z})$ is the optimal solution of the deterministic problem for every values of Z.

3 Conditions of optimality

3.1 Optimality conditions for deterministic problem

The Lagrange function for the deterministic problem is defined as the scalar function:

$$\Phi(P) = C(P) + \lambda G(P), \qquad (12)$$

where λ is the Lagrange multiplier.

Using Lagrange function the problem of load dispatch can be written as follows:

$$\min_P \max_\lambda \Phi \qquad (13)$$

Subject to the constraints (4).

The optimality conditions for the deterministic problem (13) are follows:

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$$\frac{\partial \Phi}{\partial P_i} = \frac{\partial C_i}{\partial P_i} + \lambda \cdot \left(\frac{\partial P_{ai}}{\partial P_i} - 1 \right) \begin{cases} = 0, when\ P_i^- \le P_i^o \le P_i^+ \\ \ge 0, when\ P_i^o = P_i^- \\ \le 0, when\ P_i^o = P_i^+ \end{cases}$$

$$i = 1,...,\ n, \quad (14)$$

$$G = P_D + \sum_{i=1}^{n} P_{ai}(P_i) - \sum_{i=1}^{n} P_i^0 = 0, \quad (15)$$

$$P_i^- \le P_i^0 \le P_i^+, \quad i = 1,..., n, \quad (16)$$

where P_i^0, i = 1,…, n is the optimal solution of deterministic problem.

3.2 Optimality conditions for min-max problem

For min-max problems with convex functions there exists the mixed optimal strategy for maximize the risk and the pure optimal strategy for minimize the risk R [3]:

$$\min_P \max_{W \in V} R(P, \tilde{W}) = \min_P \max_{\Omega} E\, R(P, W) \quad (17)$$

with constraints (10) and (11).

Here V is the parallelepiped of uncertainty and Ω is the mixed strategy for risk optimization:

$$\Omega = \langle \omega_1, ..., \omega_s \rangle. \quad (18)$$

The Lagrange function for the min-max problem is:

$$\acute{\Phi}(P, \tilde{W}) = ER(P, \tilde{W}) + \lambda EG(P, \tilde{W}) + + \delta(1 - \sum_{j=1}^{n} \omega_j) \quad (19)$$

The optimality conditions for the min-max problem (9)-(11) in a general form are:

$$\frac{\partial \acute{\Phi}}{\partial P_i} = \frac{\partial EC_i}{\partial P_i} + + \lambda \cdot \left(\frac{\partial P_{ai}}{\partial P_i} - 1 \right) \begin{cases} = 0, when\ P_i^- \le \hat{P}_i^0 \le P_i^+ \\ \ge 0, when\ \hat{P}_i^0 = P_i^- \\ \le 0, when\ \hat{P}_i^0 = P_i^+ \end{cases}$$

$$i = 1,…, n, \quad (20)$$

$$\frac{\partial \acute{\Phi}}{\partial \omega_i} = \frac{\partial EC}{\partial \omega_i} - \delta = 0, \quad i = 1,…, n, \quad (21)$$

$$\frac{\partial \acute{\Phi}}{\partial \lambda} = EG = E\tilde{P}_D + \sum_{i=1}^{n} E\tilde{P}_{ai}(P_i) - \sum_{i=1}^{n} P_i^0 = 0, \quad (22)$$

$$\frac{\partial \Phi}{\partial \varepsilon} = 1 - \sum_{i=1}^{s} \omega_i = 0. \quad (23)$$

The mixed strategy for maximize the value of $\acute{\Phi}$ may consist up to n+1 vertices of uncertainty parallelepiped.

4 Solution Method

4.1 Deterministic equivalent

The deterministic equivalent for optimal dispatch problems may be presented in the following form [3]-[8]:

$$\min_{\overline{P}} \overline{C} = \sum_{i=1}^{n} \overline{C}_i(\overline{P}_i) \quad (24)$$

Subject to

$$\overline{G} = \overline{P}_D + \sum_{i=1}^{n} P_{ai}(\overline{P}_i) - \sum_{i=1}^{n} \overline{P}_i = 0, \quad (25)$$

$$\overline{P}_i^- \le P_i \le \overline{P}_i^+, \quad i = 1,…, n, \quad (26)$$

where the line on variables denotes their average values, determined by probabilistic information or by mixed strategies of uncertain factors.

Functions $\overline{C}_i(\overline{P}_i)$ are named planned functions or characteristics. They represent functions between the mathematical expectations (average values) of initial functions and their arguments in the case of mixed strategy of maximization.

4.2 Method of planned characteristics

The computation of optimal load dispatch in the power plant under the uncertain information consists of two stages:

1. Computation the planned characteristics of power units and construction the deterministic equivalents.
2. Solution of the deterministic equivalents.

Computation the planned characteristics and construction the deterministic equivalents under the uncertain information is made as follows.

The min-max planned characteristics could be calculated by various approximate methods.

The most simplistic method for calculation the min-max planned characteristics is the following:

1) Choose the different values of incremental cost rate of power plant.
2) Calculate the min-max load distribution by the chosen values of incremental cost rates.

Min-max incremental fuel cost characteristic of boiler is shown in Fig. 1. The point of the planned incremental cost characteristic $\beta(\overline{P})$ is found from the equation areas S_1 =areas S_2 .

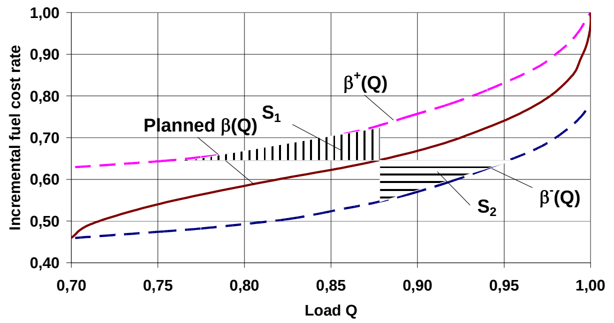

Fig. 1. The initial (lower and upper) and planned characteristics of a boiler

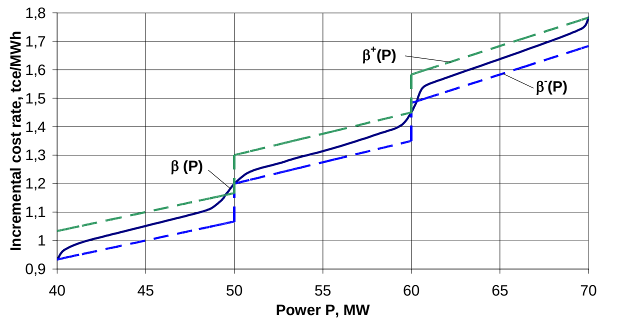

Fig. 2. The initial and the planned incremental fuel cost rate characteristics (in relative power units) of a power unit

Figure 2 shows the planned incremental fuel cost rate characteristic of a power unit.

After determination of planned characteristics for min-max problem, the common deterministic optimization problem with planned characteristics subject to the constraints will be solved.

The deterministic equivalent may be solved by the ordinary computer programs and methods, which have been elaborated for the solution of deterministic optimal scheduling problems in thermal power plants.

If the power demand is given by the interval of uncertainty $P_D^- \le \tilde{P}_D \le P_D^+$, then the optimal planned value $\overline{P}_D$ that satisfies the min-max R criterion have to be found.

The calculation is carried out as iterative procedure:

1. Choice the value of $\overline{P}_D$.
2. Calculate the deterministic optimal load distribution.
3. Calculate the risks $R(P_D^-)$ and $R(P_D^+)$.
4. Check the equality of risks.
5. If the risks are equal, then the min-max planned value $\overline{P}_D$ is found. If the risks are unequal, then this procedure must be repeated.

4.3 Computer programs

The methodology described above was realized in a program complex developed in Tallinn University of Technology. The modules for state optimization enable to compute planned input-output characteristics of power units under probabilistic and uncertain information and solve the optimization problem of power plants. The program may be used as a supplement for existing programs.

5 Conclusion

The methodology described here enables a rather simple handling of information in an uncertain form and to decrease the losses caused by the incompleteness of information. The method of planned characteristics is also used in the software for optimal scheduling of power generation in power system level.

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