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Trajectories of a Pair of Interacting Jets or Plumes Issuing Vertically Upwards into a Quiescent Environment

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Abstract: - The present work predicts analytically the trajectories of a pair of interacting turbulent round jets or plumes, which are discharged vertically upwards from two adjacent nozzles of the same diameter into quiescent environment slightly denser than the jet fluid. Trajectory prediction is based on the superposition solutions for the combined velocity field, considering that the jet or plume trajectories at a cross-section coincide with the local maxima of mean axial velocities. It was found that complete merging occurs at a distance approximately equal to ten times the spacing between the two jet sources. Beyond this distance, the behavior of the two interacting jets or plumes of the pair tends to return to the behavior of a single jet or plume identical to that belonging in the pair. Findings are discussed and compared well to available experimental data.

Key-Words: - Buoyant jets; jets; plumes; merging; interaction; superposition; trajectories; Coanda effect

1 Introduction

Experimental and physical evidence has shown in general that two buoyant jets, which are discharged from short-spaced sources, parallel or inclined to each other, tend to approach along their travel and finally match together. This phenomenon is observed in water bodies when wastewater is discharged by multiport diffusers [1, 2, 3] or in the atmosphere when smoke and other air pollutants are emitted by two or more chimneys or by cooling towers with a short spacing. For slot sources this phenomenon becomes rather more intense than for round sources [4]. Since the rate of plume rise depends on the magnitude of the buoyancy force, increase of this rate is expected with decreasing the port spacing or increasing the number of interacting plumes [5]. A wind tunnel experimental study has shown such a trend for two stack emission in a wind current parallel to the line connecting the centers of the stacks [6, 7]. The aforementioned phenomenon can be well justified by considering the secondary flow entrained by the main flow of these buoyant jets; this flow is directed from the environment towards each jet and therefore the entrainment starvation occurring between neighboring jets causes a pressure decrease resulting in their gradual attachment. Similar justification applies to the jet attachment to a wall parallel or inclined to the jet axis [8]. The tendency of air flows to remain attached to convex-shaped walls of mild curvature, like the wings of an airplane (Coanda effect), can be explained by the same reasoning [9]. The aforementioned effect is of great interest in studying buoyant jets flowing adjacent to walls or groups of interacting buoyant jets in either water bodies or atmospheric environment [10].

The present study utilizes the superposition solutions available in analytical forms for a row of two interacting jets or plumes [11] and computes their trajectories and the transverse concentration distributions along merging. The results are compared to available experimental findings and can be used for the detailed design of disposal systems involving multiport effluent emissions in water and atmospheric ambient environments.

2 Theoretical Development

A pair of interacting turbulent jets or plumes issuing vertically upwards from two adjacent round sources of diameter D and half spacing t, with an initial velocity V_0, initial fluid density ρ_0, into a little denser and quiescent environment of a uniform fluid density ρ_a, is considered. The flow and mixing fields created with respect to the Cartesian coordinate system and basic field properties are shown schematically in Fig. 1.

According to [11], the solutions for the fields of the mean axial velocities and mean concentrations can be expressed as

$$w_2 = w_m Lw_2 \tag{1}$$

$$c_2 = c_m Lc_2 \tag{2}$$

where w_2, c_2 are the mean axial velocity and the mean concentration of the combined field of interacting jets or plumes at a location (x,y,z); w_m, c_m are the mean axial centerline velocity and the mean centerline concentration of a single jet or plume of exactly the same characteristics with each jet or plume of the pair, given at same elevation z; and Lw_2, Lc_2, are mean axial velocities and mean concentrations ratios given by the following relationships [11]:

$$Lw_2 = \exp\left[-\frac{(x/t)^2}{K_w^2(z/t)^2}\right]\cdot$$

$$\left\{\exp\left[-j\frac{(y/t-1)^2}{K_w^2(z/t)^2}\right]+\exp\left[-j\frac{(y/t+1)^2}{K_w^2(z/t)^2}\right]\right\}^{1/j} \tag{3}$$

$$Lc_2 = Lw_2^{-1}\exp\left[-\frac{(x/t)^2}{K_2^2(z/t)^2}\right]\cdot$$

$$\left\{\exp\left[-\frac{(y/t-1)^2}{K_2^2(z/t)^2}\right]+\exp\left[-\frac{(y/t+1)^2}{K_2^2(z/t)^2}\right]\right\} \tag{4}$$

where $K_2 = K_c/\sqrt{1+(K_c/K_w)^2}$; K_w, K_c, are the spreading rate coefficients for the fields of mean velocities and concentrations of a single jet or plume, correspondingly; j=2 for jets, and j=3 for plumes. Note that for the derivation of Eqs (3) and (4) all assumptions made do not introduce any kind of inconsistency to the actual flow field.

The definition that the axes locations $(x_m,\pm y_m,z)$ coincide with the positions of the local maxima of the transverse velocity distribution is adopted. These locations identify the trajectories of interacting jets or plumes and are shown schematically in Fig. 1 (point-dash lines).

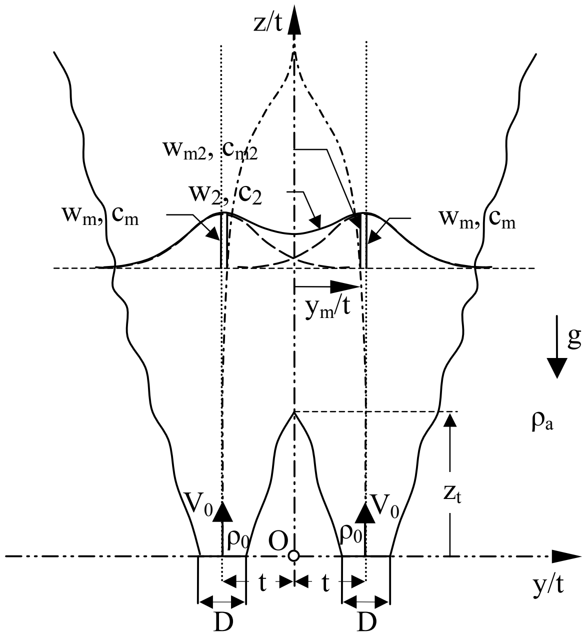

Fig. 1. Flow configuration of two interacting jets or plumes in a quiescent environment.

Then, these locations can be calculated under the condition of zeroing local derivative of the function describing the transverse velocity distribution of the two interacting jets or plumes. From the mathematical point of view, it can be observed that the aforementioned maxima occur always on the vertical plane connecting the centers of the two ports; thus x_m =0. Since the mean axial centerline velocity w_m is a non-zero function of z only, according to Eq. (1), the derivatives with respect to y of the functions w_2 and Lw_2, or equivalently Lw_2^j, will become zero at these locations, $\pm y_m$.

Consequently, for the trajectories of two interacting jets or two interacting plumes the following expression is derived from Eq. (3):

$$\frac{y_m/t-1}{y_m/t+1} = -\exp\left[-4j\frac{y_m/t}{K_w^2(z/t)^2}\right] \tag{5}$$

This relationship can be written in the form

$$\frac{y_m}{t} = \tanh\left[2j\frac{y_m/t}{K_w^2(z/t)^2}\right] \tag{6}$$

It is evident that the two solutions ($\pm y_m/t$) of Eq. (6) satisfy the relationship $\left|y_m / t\right| \leq 1$.

3 Results and Discussion

In each buoyancy behavior, one case of merging was examined. For the computations in the jet-like behavior of merging, a pair of two buoyant jets with the following parameter values was taken: j=2, K_w=0.10, K_c=0.12, t/D=0.75, F_0=100, T=0.0075, where F_0 is the densimetric Froude number, which is defined as $F_0 = V_0 (g_0' D)^{-1/2}$, $g_0' = g(\rho_a - \rho_0)/\rho_0$ is the apparent acceleration of gravity and T is the dynamic proximity number defined in [11] as $T = (t/D)F_0^{-1}$; since $T \leq 0.012$, a jet-like behavior of merging is expected.

For the computations in the plume-like behavior of merging, a pair of two buoyant jets with the following parameter values was taken: j=3, K_w=0.12, K_c=0.12, t/D=0.75, F_0=2.44, T=0.3074; since $T \geq 0.12$, a plume-like behavior of merging is expected. All above values selected for either jet-like or plume-like case were taken according to [11]. The solutions of Eq. (6) were found applying the simple iteration method [12]. At relatively short distances z/D from sources, the trajectory of each buoyant jet coincides with the vertical axis of this jet considered as single buoyant jet. Therefore, the value y_m/t=1 is initially assigned. For getting solutions of satisfactory accuracy 100 iterations were made. The results of $\pm y_m/t$ with respect to the axial distance z/t are shown in Fig. 2(a) for the jet-like case and in Fig. 2(b) for the plume-like case of merging. It is interesting that complete merging of the case of two interacting jets occurs approximately at the same axial distance (z/t=20.2) with the case of two interacting plumes (z/t=20.6). This finding indicates that merging of two buoyant jets, which is caused by pressure decrease due to entrainment starvation in the region between them, is only slightly affected by buoyancy.

For two jets or plumes beyond the distance ($z/t \approx 20$) of complete merging, diagrams based on Eqs (3) and (4) regarding the longitudinal distributions of the ratios of mean axial velocities and mean concentrations, which have been plotted in [11], show that the flow and mixing fields start

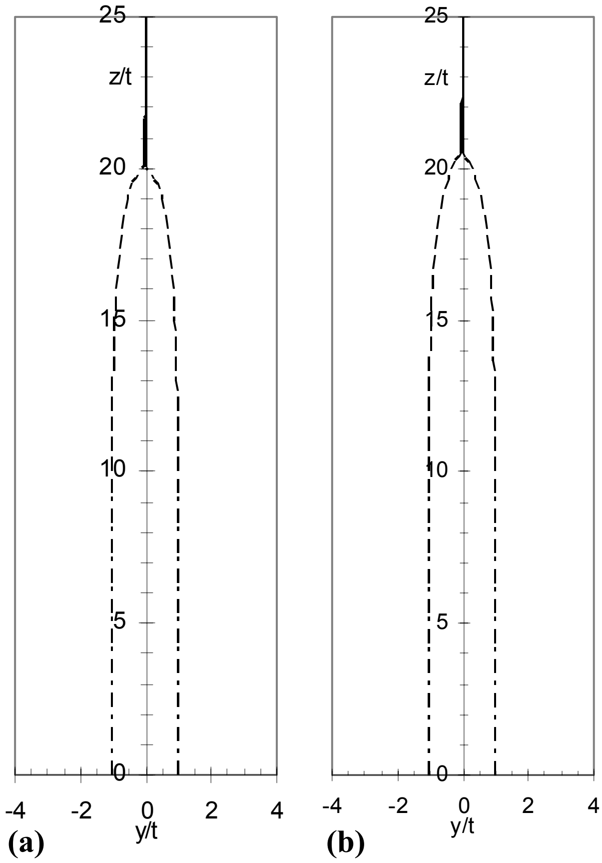

Fig. 2. Evolution of trajectories of two interacting buoyant jets: (a) the jet-like case; (b) the plume-like case.

returning gradually to their initial behavior being identical to the behavior of a single jet or a single plume of the pair, correspondingly. The distance $z/t \approx 20$ is approximately four times the distance z_t/t shown in Fig.1, where the internal visual jet boundaries intersect ($z_t/t \approx 5$). Experimental measurements shown in Fig. 3, concerning the transverse concentration profiles of a pair of two interacting plumes [11], indicate a gradual shift of plume trajectories towards the centerline of the pair of plumes (y/D=0). Complete merging is actually reached at the axial distance z/t=20, which is approximately equal to the distance predicted by the aforementioned mathematical analysis.

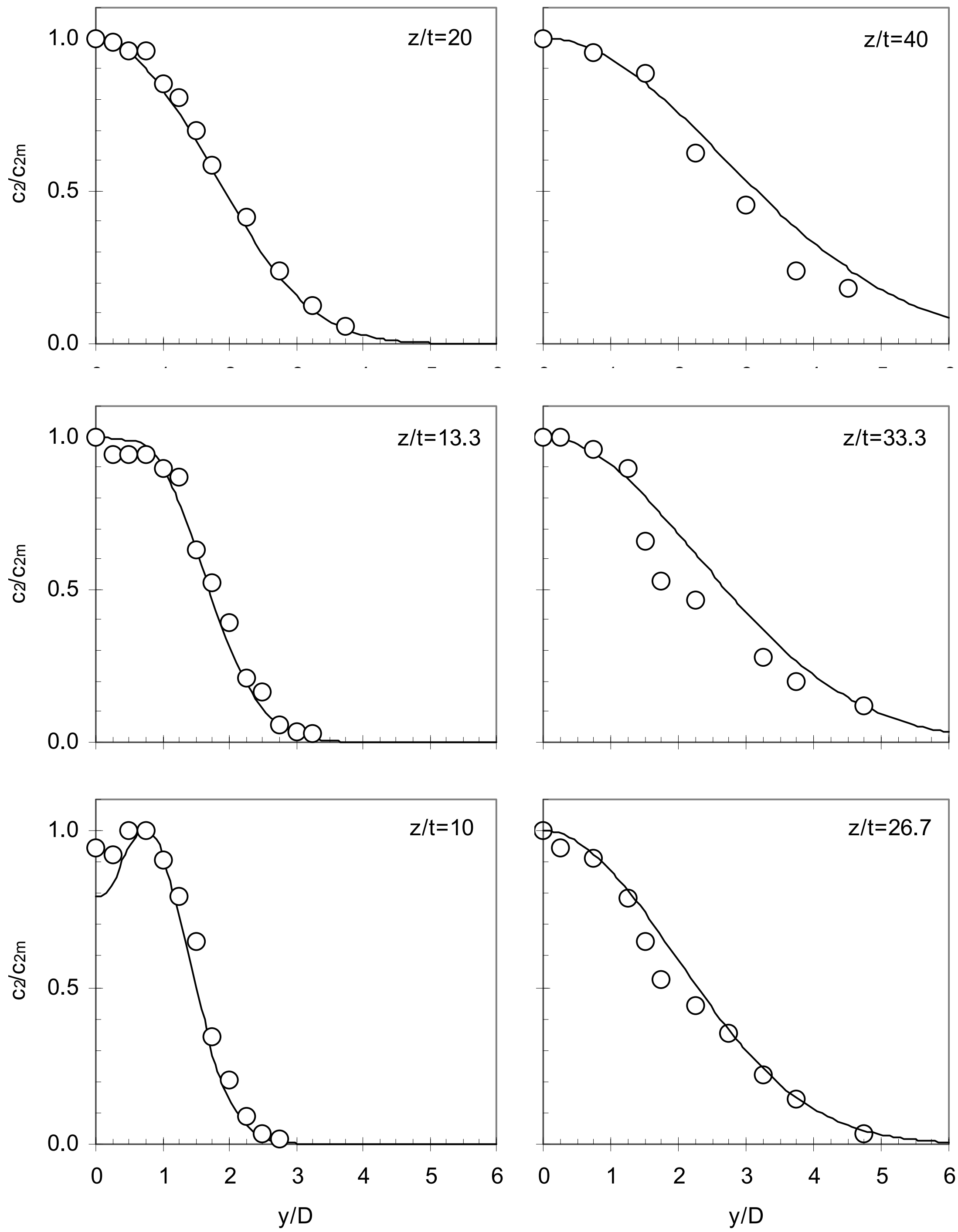

Fig. 3. Transverse concentration profiles for a pair of two interacting plumes: t/D=0.75, F_0=2.44. Concentrations c_2 of each particular profile have been normalized by the maximum values c_{2m} measured at the same profile.

4 Conclusion

Based on the superposition solutions valid for interacting jets or plumes, the trajectories of two interacting buoyant jets discharged vertically upwards were predicted with a satisfactory accuracy. It was found that merging is entirely integrated at a distance $z/t \approx 20$, which is approximately four times the axial distance of intersection of the internal optical boundaries of the two interacting jets or plumes. Beyond this distance, the behavior of the two interacting jets or plumes of the pair tends to return to the behavior of a single jet or plume identical to that composing the pair.

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