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Stability of a Solution of the Navier–Stokes Equation in a Norm Induced by a Fractional Power of the Stokes Operator

PETR KUČERA
CTU, Fac. of Civil Engineering
Dept. of Mathematics
Thákurova 7, 166 29 Praha 6
CZECH REPUBLIC

JIŘÍ NEUSTUPA
CTU, Fac. of Mech. Engineering
Dept. of Technical Mathematics
Karlovo nám. 13, 121 35 Praha 2
CZECH REPUBLIC

Abstract: - We show that a strong solution $\boldsymbol{u}$ of the Navier–Stokes initial–boundary value problem which is in a certain sense bounded and integrable on the time interval $(0, +\infty)$, is stable with respect to small disturbances of the initial velocity in the norm $\|A^{1/4}.\|$ (where $\|\,.\,\|$ is the L^2–norm and A is the Stokes operator) and to small disturbances of the right hand side.

Key-Words: - Navier–Stokes equation, stability, Stokes operator

1 Introduction

Let $\Omega \subset \mathbb{R}^3$ be a bounded domain with the boundary $\partial\Omega$ of the Hölder class $C^{2+\beta}$ for some $\beta > 0$. Suppose that $0 < T \leq +\infty$. Put $Q_T = \Omega \times (0, T)$. We deal with the initial–boundary value problem for the Navier–Stokes equation

$$\frac{\partial \boldsymbol{u}}{\partial t} - \nu\Delta\boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} + \nabla p = \boldsymbol{f} \quad \text{in } Q_T, \tag{1}$$

$$\operatorname{div}\boldsymbol{u} = 0 \quad \text{in } Q_T, \tag{2}$$

$$\boldsymbol{u} = \boldsymbol{0} \quad \text{in } \partial\Omega \times (0, T), \tag{3}$$

$$\boldsymbol{u}(0) = \boldsymbol{u}_0 \quad \text{in } \Omega. \tag{4}$$

$\boldsymbol{u}$ denotes the velocity, p denotes the pressure, ν is the kinematic coefficient of viscosity and $\boldsymbol{f}$ is a specific body force. We further assume for simplicity that $\nu = 1$. (This assumption does not influence the validity of our results.)

We denote by $\boldsymbol{H}$ the closure of $C^{\infty}_{0,\sigma}(\Omega)$ (the space of infinitely differentiable divergence–free vector functions in Ω which have a compact support in Ω) in $L^2(\Omega)^3$. The norm in $L^2(\Omega)^3$ (and in $\boldsymbol{H}$) is denoted by $\|\,.\,\|$. Put $A = -P_\sigma\,\Delta$ where P_σ is the orthogonal projection of $L^2(\Omega)^3$ onto $\boldsymbol{H}$. A is the Stokes operator with the domain $D(A) = W^{2,2}(\Omega)^3 \cap W^{1,2}_0(\Omega)^3 \cap \boldsymbol{H}$. $D(A)$ is the Banach space with the norm $\|A\,.\,\|$ which is equivalent with the norm of $W^{2,2}(\Omega)^3$. It is well known that operator A is self–adjoint and positive (see e.g. Y. Giga [2] and [3]). Other properties of the Stokes operator are in greater detail described in the book [10] by W. Varnhorn. It makes therefore sense to consider fractional powers of A. It can be shown that $D(A^\mu)$ is not only the domain of A^μ, but it can be treated as the Banach space with the norm $\|A^\mu.\,\|$ (for $\mu > 0$).

2 Small perturbations of the initial velocity in the norm of $D(A^{1/4})$

Theorem 1 *Let $\boldsymbol{u}$ be a strong solution of the problem (1)–(4) and with the input data $\boldsymbol{u}(0) = \boldsymbol{u}_0 \in D(A^{1/4})$, $P_\sigma\boldsymbol{f} \in L^2(0, T;\, \boldsymbol{H})$. Let $\boldsymbol{u}$ satisfy*

$$\int_0^T \Big(\|A^{3/4}\boldsymbol{u}(t)\|^2 + \|A^{1/2}\boldsymbol{u}(t)\|^4 \Big)\,\mathrm{d}t < \infty. \tag{5}$$

Then to given $\epsilon > 0$, there exists $\delta > 0$ such that if $\boldsymbol{v}_0 \in D(A^{1/4})$, $P_\sigma\boldsymbol{g} \in L^2(0, \infty;\, \boldsymbol{H})$ are functions satisfying

$$\|A^{1/4}\boldsymbol{u}_0 - A^{1/4}\boldsymbol{v}_0\| + \int_0^T \|P_\sigma\boldsymbol{f}(t) - P_\sigma\boldsymbol{g}(t)\|^2 < \delta \tag{6}$$

then there exists a unique strong solution $\boldsymbol{v}$ of the problem (1)–(4) with the data $\boldsymbol{v}_0$ and $\boldsymbol{g}$ (instead of $\boldsymbol{u}_0$ and $\boldsymbol{f}$) on the time interval $(0, +\infty)$, satisfying

$$\|A^{1/4}\boldsymbol{v}(t) - A^{1/4}\boldsymbol{u}(t)\|^2 + \int_0^t \|A^{3/4}\boldsymbol{v}(s) - A^{3/4}\boldsymbol{u}(s)\|^2\,\mathrm{d}s \leq \epsilon \tag{7}$$

for all $t \in (0, T)$.

If $T = +\infty$ then Theorem 1 provides the information on stability of solution $\boldsymbol{u}$.

A similar result (with $T = +\infty$) was already proved by G. Ponce et al. in [4]. However, our assumption (6) is weaker because we measure the difference between the initial velocities $\boldsymbol{v}_0$ and $\boldsymbol{u}_0$ in the norm $\|A^{1/4}.\,\|$ while the authors of [4] were using the norm $\|A^{1/2}.\,\|$.

The assumption that the strong solution $\boldsymbol{u}$ satisfies (5) is not restricting because the strong solution on the interval $(0,T)$ usually belongs to $L^\infty(0,T;\,D(A^{1/2}))\cap L^2(0,T;D(A))$ and then it satisfies (5) automatically. In fact, it is sufficient if $\boldsymbol{u} \in L^\infty(0,T;\,D(A^{1/4}))\cap L^2(0,T;\,D(A^{3/4}))$ because then, using the obvious inequality

$$\|A^{1/2}\boldsymbol{\psi}\,\| \leq \|A^{1/4}\boldsymbol{\psi}\|^{1/2}\,\|A^{3/4}\boldsymbol{\psi}\|^{1/2} \tag{8}$$

which holds for every $\boldsymbol{\psi} \in D(A^{3/4})$, we get

$$\begin{aligned}
&\int_0^T \|A^{1/2}\boldsymbol{u}(t)\|^4\,\mathrm{d}t \\
&\leq \int_0^T \|A^{1/4}\boldsymbol{u}(t)\|^2\,\|A^{3/4}\boldsymbol{u}(t)\|^2\,\mathrm{d}t \\
&\leq \sup_{0<t<T} \operatorname{ess} \|A^{1/4}\boldsymbol{u}(t)\|^2 \int_0^T \|A^{3/4}\boldsymbol{u}(t)\|^2\,\mathrm{d}t \\
&< +\infty.
\end{aligned}$$

Proof of Theorem 1: Since $\boldsymbol{v}(0) \in D(A^{1/4})$, there exists $T^* > 0$ such that $\boldsymbol{v}$ is a strong solution on $(0,T^*)$. Then $\boldsymbol{w} = \boldsymbol{v} - \boldsymbol{u}$ satisfies the equation

$$\begin{aligned}
&\dot{\boldsymbol{w}} + A\boldsymbol{w} + P_\sigma(\boldsymbol{w}\cdot\nabla)\boldsymbol{w} + P_\sigma(\boldsymbol{u}\cdot\nabla)\boldsymbol{w} \\
&\qquad + P_\sigma(\boldsymbol{w}\cdot\nabla)\boldsymbol{u} \;=\; P_\sigma\boldsymbol{g} - P_\sigma\boldsymbol{f}
\end{aligned}$$

on $(0,T^*)$. Multiplying it by $A^{1/2}\boldsymbol{w}$ and integrating on Ω, we obtain

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\frac{1}{2}\|A^{1/4}\boldsymbol{w}\|^2 + \|A^{3/4}\boldsymbol{w}\|^2 \\
&\leq \left|\int_\Omega P_\sigma(\boldsymbol{w}\cdot\nabla)\boldsymbol{w}\cdot A^{1/2}\boldsymbol{w}\,\mathrm{d}\boldsymbol{x}\right| \\
&\quad + \left|\int_\Omega P_\sigma(\boldsymbol{u}\cdot\nabla)\boldsymbol{w}\cdot A^{1/2}\boldsymbol{w}\,\mathrm{d}\boldsymbol{x}\right| \\
&\quad + \left|\int_\Omega P_\sigma(\boldsymbol{w}\cdot\nabla)\boldsymbol{u}\cdot A^{1/2}\boldsymbol{w}\,\mathrm{d}\boldsymbol{x}\right| \\
&\quad + \left|\int_\Omega (P_\sigma\boldsymbol{g} - P_\sigma\boldsymbol{f})\cdot A^{1/2}\boldsymbol{w}\,\mathrm{d}\boldsymbol{x}\right|
\end{aligned} \tag{9}$$

We will now estimate the integrals on the right hand side of (9). We shall denote by C a generic constant, i.e. the constant whose value may change from line to line. On the other hand, numbered constants will have a fixed value throughout the whole paper. The constants will always depend only on domain Ω. We shall also use the inequality

$$\|P_\sigma(\boldsymbol{\phi}\cdot\nabla)\boldsymbol{\psi}\| \;\leq\; c_1\,\|A^{1/2}\boldsymbol{\phi}\|\,\|A^{3/4}\boldsymbol{\psi}\| \tag{10}$$

which holds for every $\boldsymbol{\phi} \in D(A^{1/2})$ and $\boldsymbol{\psi} \in D(A^{3/4})$ (see [7], estimate (2.5)). Thus, we get the inequalities

$$\begin{aligned}
&\left|\int_\Omega P_\sigma(\boldsymbol{w}\cdot\nabla)\boldsymbol{w}\cdot A^{1/2}\boldsymbol{w}\,\mathrm{d}\boldsymbol{x}\right| \\
&\leq C\,\|A^{1/2}\boldsymbol{w}\|\,\|A^{3/4}\boldsymbol{w}\|\,\|A^{1/2}\boldsymbol{w}\| \\
&\leq C\,\|A^{3/4}\boldsymbol{w}\|^2\,\|A^{1/4}\boldsymbol{w}\|,
\end{aligned} \tag{11}$$

$$\begin{aligned}
&\left|\int_\Omega P_\sigma(\boldsymbol{u}\cdot\nabla)\boldsymbol{w}\cdot A^{1/2}\boldsymbol{w}\,\mathrm{d}\boldsymbol{x}\right| \\
&\leq C\,\|A^{1/2}\boldsymbol{u}\|\,\|A^{3/4}\boldsymbol{w}\|\,\|A^{1/2}\boldsymbol{w}\| \\
&\leq C\,\|A^{3/4}\boldsymbol{w}\|^{3/2}\,\|A^{1/4}\boldsymbol{w}\|^{1/2}\,\|A^{1/2}\boldsymbol{u}\| \\
&\leq \frac{1}{6}\,\|A^{3/4}\boldsymbol{w}\|^2 + C\,\|A^{1/4}\boldsymbol{w}\|^2\,\|A^{1/2}\boldsymbol{u}\|^4,
\end{aligned} \tag{12}$$

$$\begin{aligned}
&\left|\int_\Omega P_\sigma(\boldsymbol{w}\cdot\nabla)\boldsymbol{u}\cdot A^{1/2}\boldsymbol{w}\,\mathrm{d}\boldsymbol{x}\right| \\
&\leq C\,\|A^{1/2}\boldsymbol{w}\|\,\|A^{3/4}\boldsymbol{u}\|\,\|A^{1/2}\boldsymbol{w}\| \\
&\leq C\,\|A^{3/4}\boldsymbol{w}\|\,\|A^{1/4}\boldsymbol{w}\|\,\|A^{3/4}\boldsymbol{u}\| \\
&\leq \frac{1}{6}\,\|A^{3/4}\boldsymbol{w}\|^2 + C\,\|A^{1/4}\boldsymbol{w}\|^2\,\|A^{3/4}\boldsymbol{u}\|^2,
\end{aligned} \tag{13}$$

$$\begin{aligned}
&\left|\int_\Omega (P_\sigma\boldsymbol{g} - P_\sigma\boldsymbol{f})\cdot A^{1/2}\boldsymbol{w}\,\mathrm{d}\boldsymbol{x}\right| \\
&\leq \|P_\sigma\boldsymbol{g} - P_\sigma\boldsymbol{f}\|\,\|A^{1/2}\boldsymbol{w}\| \\
&\leq \frac{1}{6}\,\|A^{3/4}\boldsymbol{w}\|^2 + C\,\|P_\sigma\boldsymbol{g} - P_\sigma\boldsymbol{f}\|^2
\end{aligned} \tag{14}$$

which hold on $(0,T^*)$. We have also used the estimate $\|A^{1/2}\boldsymbol{w}\| \leq C\,\|A^{3/4}\boldsymbol{w}\|$ in (14). Using (9) and (11)–(14), we obtain

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\frac{1}{2}\|A^{1/4}\boldsymbol{w}\|^2 + \left(\frac{1}{2} - C\,\|A^{1/4}\boldsymbol{w}\|\right)\|A^{3/4}\boldsymbol{w}\|^2 \\
&\leq C\left(\|A^{1/2}\boldsymbol{u}\|^4 + \|A^{3/4}\boldsymbol{u}\|^2\right)\|A^{1/4}\boldsymbol{w}\|^2 \\
&\qquad + C\,\|P_\sigma\boldsymbol{g} - P_\sigma\boldsymbol{f}\|^2.
\end{aligned} \tag{15}$$

We denote

$$\begin{aligned}
\zeta(t) &= \|A^{1/2}\boldsymbol{u}\|^4 + \|A^{3/4}\boldsymbol{u}\|^2, \\
\vartheta(t) &= \|P_\sigma\boldsymbol{g} - P_\sigma\boldsymbol{f}\|^2.
\end{aligned}$$

Then we can write (15) in the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\|A^{1/4}\boldsymbol{w}\|^2+\Big(1-c_2\,\|A^{1/4}\boldsymbol{w}\|\Big)\,\|A^{3/4}\boldsymbol{w}\|^2 \\ \leq\ c_3\,\zeta(t)\,\|A^{1/4}\boldsymbol{w}\|^2+c_4\,\vartheta(t). \qquad (16)$$

The integral of ζ on each time interval which is contained in $(0,T)$ is less than or equal to c_5 where c_5 denotes the left hand side of (5). Let us compare function $\|A^{1/4}\boldsymbol{w}\|^2$ with function z such that $z(0)=\|A^{1/4}\boldsymbol{w}(0)\|^2$ and z satisfies the estimate

$$z'\ \leq\ c_3\,\zeta(t)\,z+c_4\,\vartheta(t). \qquad (17)$$

Integrating (17), we obtain that

$$\begin{aligned} z(t)\ &\leq\ \mathrm{e}^{\int_0^t c_3\,\zeta(\tau)\,\mathrm{d}\tau}\,z(0) \\ &\quad +\int_0^t \mathrm{e}^{\int_s^t c_3\,\zeta(\tau)\,\mathrm{d}\tau}\,c_4\,\vartheta(s)\;\mathrm{d}s \\ &\leq\ \mathrm{e}^{c_3c_5}\,z(0)+\mathrm{e}^{c_3c_5}\,c_4\int_0^t\vartheta(s)\;\mathrm{d}s \qquad (18)\end{aligned}$$

for all $t\in(0,T)$. Obviously,

$$\|A^{1/4}\boldsymbol{w}(t)\|^2\ \leq\ z(t) \qquad (19)$$

on each interval $(0,T')$ such that

$$1-c_2\,\|A^{1/4}\boldsymbol{w}(t)\|^2\ \geq\ \frac{1}{2} \qquad (20)$$

also holds on $(0,T')$. This implies that provided

$$\mathrm{e}^{c_3c_5}\,\|A^{1/4}\boldsymbol{w}(0)\|^2 \\ +\,\mathrm{e}^{c_3c_5}\,c_4\int_0^{+\infty}\vartheta(s)\;\mathrm{d}s\ \leq\ \frac{1}{2c_2}, \qquad (21)$$

(19) holds as long as the solution $\boldsymbol{v}$ exists as a strong solution. However, using the well known theorem on the local in time existence of a strong solution (see e.g. O. A. Ladyzhenskaya [5]) and particularly the theorem which enables to consider the initial value in $D^{1/4}$ (G. P. Galdi [1]), we can now proceed with the interval of existence of the strong solution $\boldsymbol{v}$ up to T.

The inequality (21) is clearly satisfied if the left hand side in (6) is sufficiently small, i.e. if $\delta>0$ is sufficiently small. The inequalities (18) and (19) then imply the uniform estimate of $\|A^{1/4}\boldsymbol{w}\|^2$ on the time interval $(0,+\infty)$ by the right hand side of (18). The uniform estimate of the integral of $\|A^{3/4}\boldsymbol{w}\|^2$ on $(0,t)$ then easily follows from (16), if we integrate it with respect to time.

Thus, using the identity $\boldsymbol{w}=\boldsymbol{v}-\boldsymbol{u}$ and the properties of $\boldsymbol{u}$ (namely the estimate (5)), we get (7). Naturally, (7) implies that $\boldsymbol{v}\in L^\infty(0,T;\,D(A^{1/4}))\cap L^2(0,T;\,D(A^{3/4}))$, too. □

Remark 2 The inequalities (7) and (8) imply that under the assumptions of Theorem 1,

$$\int_0^T \|A^{1/2}\boldsymbol{v}(s)-A^{1/2}\boldsymbol{u}(s)\|^4\;\mathrm{d}s\ \leq\ \epsilon^2. \qquad (22)$$

3 Large perturbations of the initial velocity

We assume that $\boldsymbol{f}$ is a fixed specific body force which belongs to $L^2(0,+\infty;\,\boldsymbol{H})$. We shall only consider perturbations of the initial velocity in this section.

In [6] and [7], B. Scarpellini constructed a strong global solution (i.e. a strong solution on the time interval $(0,+\infty)$) of the Navier–Stokes equation with arbitrarily large initial velocity in the norm $\|A^{1/2}.\,\|$. One of possibilities how to extend this results is to construct a global strong solution with an initial velocity which is arbitrarily large in the norm $\|A^{\alpha}.\,\|$ for some $\alpha<\frac{1}{2}$. However, it can be easily done by means of Theorem 1: if $T=+\infty$ and $\boldsymbol{u}$ is a solution with the properties named in Theorem 1, if $\delta>0$ is the number given by Theorem 1 (corresponding e.g. to $\epsilon=1$), $\frac{1}{4}<\alpha\leq\frac{1}{2}$ and $R>0$ is an arbitrarily large real number then there exists $\boldsymbol{v}_0\in D(A^{\alpha})$ such that $\|A^{1/4}\boldsymbol{v}_0-A^{1/4}\boldsymbol{u}_0\|<\delta$ and $\|A^{\alpha}\boldsymbol{v}_0\|>R$. Due to Theorem 1 there exists a unique global strong solution $\boldsymbol{v}$ of the problem (1)–(4) with the initial velocity $\boldsymbol{v}_0$.

Our goal in this section is to prove the following theorem which shows that there exists a locally in time strong solution $\boldsymbol{v}$ of the problem (1)–(4) such that the norm $\|A^{\alpha}.\,\|$ (with $\frac{1}{4}<\alpha\leq\frac{1}{2}$) of $\boldsymbol{v}(0)$ is arbitrarily large and the norm $\|A^{\gamma}.\,\|$ (with $\frac{3}{4}<\gamma<1$) of $\boldsymbol{v}(\xi)$ can be arbitrarily small at a time instant ξ arbitrarily close to zero. In fact, we shall prove even something more: solution $\boldsymbol{v}$ has the property that its value $\boldsymbol{v}(\xi)$ belongs to an arbitrarily chosen open set U in $D(A^{\gamma})$.

Theorem 3 *Suppose that $\frac{3}{4}<\gamma<1$, $\frac{1}{4}<\alpha\leq\frac{1}{2}$, U is a nonempty open subset of $D(A^{\gamma})$, $R>0$ (arbitrarily large), $\chi>0$ (arbitrarily small). Then there exists $\boldsymbol{v}_0\in D(A)$, $T>0$ and a weak solution $\boldsymbol{v}$ of the problem (1)–(4) such that $\boldsymbol{v}\in C([0,T);\,D(A^{\gamma}))$,*

$$\|A^{\alpha}\boldsymbol{v}_0\|\ \geq\ R \qquad (23)$$

and

$$\boldsymbol{v}(\xi)\in U \qquad (24)$$

at some instant of time $\xi\in(0,T)$ such that $\xi<\chi$.

The theorem generalizes Scarpellini's result from [7] in the point which concerns the exponent α in (23): B. Scarpellini worked with the fixed $\alpha=\frac{1}{2}$.

Proof. Since U is an open set in $D(A^\gamma)$, there exist $\boldsymbol{u}_0 \in D(A)$, $T > 0$, $\mu > 0$ and a strong solution $\boldsymbol{u}$ of the problem (1)–(4) on a time interval $(0, T)$ such that $\boldsymbol{u} \in C([0, T);\, D(A^\gamma))$ and

$$B_\mu(\boldsymbol{u}(t)) \subset U \tag{25}$$

for every $t \in [0, T)$. (The symbol $B_\mu(\boldsymbol{u}(t))$ denotes the ball in $D(A^\gamma)$ with the center at $u(t)$ and with the radius μ.)

Let $\epsilon > 0$ be given. Due to Theorem 1, there exists $\delta > 0$ such that if

$$\|A^{1/4}\boldsymbol{v}_0 - A^{1/4}\boldsymbol{u}_0\| < \delta$$

then there exists a strong global solution $\boldsymbol{v}$ of problem (1)–(4) on $(0, T)$ with the initial velocity $\boldsymbol{v}_0$ and the right hand side $\boldsymbol{f}$ (which is not perturbed) such that (7) holds for all $t \in (0, T)$. $\boldsymbol{v}_0$ can be chosen so that it satisfies (23).

Then in each time interval whose length exceeds l, there exists τ such that $\|A^{3/4}\boldsymbol{v}(\tau) - A^{3/4}\boldsymbol{u}(\tau)\|^2 < \epsilon/l$. Hence if we again use the notation $\boldsymbol{w} = \boldsymbol{v} - \boldsymbol{u}$, we have

$$\|A^{1/2}\boldsymbol{v}(\tau) - A^{1/2}\boldsymbol{u}(\tau)\|^4 \;=\; \|A^{1/2}\boldsymbol{w}(\tau)\|^4$$
$$\leq\; \|A^{1/4}\boldsymbol{w}(\tau)\|^2\, \|A^{3/4}\boldsymbol{w}(\tau)\|^2 \;\leq\; \frac{\epsilon^2}{l}.$$

If the considered interval is $(0, \chi^*/2)$ (where $\chi^* = \min\{\chi;\, T\}$) then $\tau \in (0, \chi^*/2)$ and

$$\|A^{1/2}\boldsymbol{v}(\tau) - A^{1/2}\boldsymbol{u}(\tau)\| \;\leq\; \left(\frac{2\epsilon^2}{\chi^*}\right)^{1/4}. \tag{26}$$

According to Proposition 3.4 in [7], to $\mu > 0$, χ^* and τ (identical with the μ, χ^* and τ used above) there exists $\eta > 0$ such that the inequality

$$\|A^{1/2}\boldsymbol{v}(\tau) - A^{1/2}\boldsymbol{u}(\tau)\| \;\leq\; \eta$$

implies that

$$\|A^\gamma\boldsymbol{v}(t) - A^\gamma\boldsymbol{u}(t)\| \;\leq\; \mu \tag{27}$$

for all $t \in (\frac{3}{4}\chi^*,\, \chi^*)$. If $\epsilon > 0$ is chosen so small that the right hand of (26) is less than η then due to (25) and (27), $\boldsymbol{v}(t) \in B_\mu(\boldsymbol{u}(t)) \subset U$. Thus, we can finally put ξ to be equal to an arbitrary point from $(\frac{3}{4}\chi^*,\, \chi^*)$. $\square$

Remark 4 Choosing set U to be a sufficiently small neighborhood of zero (in the space $D(A^\gamma)$), Theorem 3 provides solution $\boldsymbol{v}$ which has the so called "big fall" at a very short instant of time $(0, \xi)$. If, in addition, we assume that the specific body force $\boldsymbol{f}$ is "sufficiently small" then solution $\boldsymbol{v}$, due to its smallness at time ξ, can be prolonged as a strong solution onto the whole time interval $(0, +\infty)$.

Further interesting theorems on global in time strong solutions which initially have "big falls" can be found in preprints [8] and [9] by Z. Skalák.

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References:

[1] G. P. Galdi: An Introduction to the Navier–Stokes initial–boundary value problem, in *Fundamental Directions in Math. Fluid Mechanics,* ed. G. P. Galdi, J. Heywood, R. Rannacher, series "Advances in Mathematical Fluid Mechanics", Birkhauser, Basel 2000, pp. 1–98.

[2] Y. Giga: Analyticity of the semigroup generated by the Stokes operator in L^r–spaces. *Math. Z.* 178, 1981, 297–329.

[3] Y. Giga: Domains of fractional powers of the Stokes operator in L^r–spaces. *Arch. Rat. Mech. Anal.* 89, 1985, 251–265.

[4] G. Ponce, R. Racke, T. C. Sideris, E. S. Titi: Global stability of large solutions to the 3D Navier-Stokes equations. *Comm. Math. Phys.* 159, 1994, 329–341.

[5] O. A. Ladyzhenskaya: *The Mathematical Theory of Viscous Incompressible Flow.* Gordon and Breach, New York 1969.

[6] B. Scarpellini: On a family of large solutions of Navier-Stokes. *Houston J. of Math.* 28, No. 3, 2002, 641–648.

[7] B. Scarpellini: Fast decaying solutions of the Navier-Stokes equations and asymptotic properties. *J. Math. Fluid Mech.* 6, 2004, 103–120.

[8] Z. Skalák: Fast decays of strong solutions of the Navier–Stokes equations. Preprint of the Czech Technical University Prague, 2005.

[9] Z. Skalák: On asymptotic dynamics of solutions of the homogeneous Navier–Stokes equations. Preprint of the Czech Technical University Prague, 2005.

[10] W. Varnhorn: *The Stokes Equations.* Akademie Vrlag GmbH Berlin, Mathematical Research, Vol. 76, 1994.