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The Analysis of Stress and Velocity Fields in Axisymmetric Plastic Yielding Processes

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Abstract: - The method of the analysis of stress and velocity fields in axisymmetric plastic yielding processes based on yield zone mapping in deviator stress space is given. The method is illustrated by modeling of closing stage of indirect cold extrusion of the hollow cylindrical article.

Key-Words: - Plasticity, Stress, Plastic Yielding, Velocity, Deformation, Strain, Modeling, Manufacturing Methods

1 Introduction

Many complicated questions of the analysis, design and development of plastic forming manufacturing methods remain underinvestigated. Plastic deformation processes in which the work material is under three-dimensional axisymmetric strain conditions and compound loading with strong stressed state (stress phase) variation are especially difficult for the analysis and design [1, 2]. The technological capabilities analysis with computerization and prediction of finished products properties in non-steady plastic forming processes under axisymmetric strain conditions require using the reliable methods of spatial stress and strain field analysis from the point of view of fast convergence and split-hair accuracy.

Many problems of technology design such as justified selection of type and number of forming and concurrent operations, processing conditions, reliable tool strength provision, are related to the analysis of distribution of stresses, flow velocities and strains in produced metalware.

2 Problem Formulation

Axisymmetric plastic forming of materials in manufacturing processes of plastic working is described in cylindrical coordinate system r, z, θ by the following equations [1]

$$\frac{\partial\sigma_r}{\partial r} + \frac{\partial\tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_z}{r} = 0\,, \quad \frac{\partial\tau_{rz}}{\partial r} + \frac{\partial\sigma_z}{\partial z} + \frac{\tau_{rz}}{r} = 0\,, \tag{1}$$

$$(\sigma_r - \sigma_z)^2 + (\sigma_z - \sigma_\theta)^2 + (\sigma_\theta - \sigma_r)^2 + 6\tau_{rz}^2 = 6\tau_s^2\,, \tag{2}$$

$$\dot\lambda = \frac{\partial v_r / \partial z + \partial v_z / \partial r}{2\tau_{rz}} = \frac{\partial v_r / \partial z - \partial v_z / \partial r}{\sigma_r - \sigma_z}\,, \tag{3}$$

$$\dot\lambda = \frac{I_2(D_{\dot e})}{\sqrt{I_2(D_\sigma)}} = \frac{I_3(D_{\dot e})}{\sqrt[3]{I_3(D_\sigma)}}, \tag{4}$$

$$\frac{\partial v_r}{\partial r} + \frac{\partial v_z}{\partial z} + \frac{v_r}{r} = 0\,, \tag{5}$$

where σ_r σ_z σ_θ, τ_{rz} - nonzero components of the stress tensor; τ_s - a shear yield point of the material; $\dot\lambda$ - a positive scalar proportional to plastic deformation power; v_r, v_z - components of the vector of plastic yielding velocity ; $I_2(D_{\dot e})$, $I_3(D_{\dot e})$ and $I_2(D_\sigma)$, $I_3(D_\sigma)$ - square and cubic invariants of strain rate deviator $D_{\dot e}$ and stress deviator D_σ accordingly.

A difficulty of the analysis of metal plastic forming processes with axisymmetric stress and strain fields is concerned with a lot of sought parameters σ_r, σ_z, σ_θ, τ_{rz}, v_r, and local static indefinability of the equations system (1) - (5). The prevailing receptions to overcome these difficulties are a diminution of unknown stress and velocities components number (a condition of plane deformation, an assumption that some stresses or velocities components are known). The analysis methods based on a strong simplification of the main equations can lead to a loss of qualitative properties of the solutions.

Many technological problems of axisymmetric deformation are solved with the use of the hypothesis about complete plasticity [2, 3] which allows to obtain a statically definable set of equations. The assumption concerning full plasticity fixes a phase of the monoaxial stressed state. The similar condition is precisely fulfilled on a symmetry axis, and, generally speaking, approximately in all plastic area.

G. Lipman [4] used instead of Haar-Carman condition another condition: intermediate (by value) principal linear component of strain rate $\dot{e}_2 = 0$. By the law of flow it leads to a statically definable problem.

A finite element method, being actually grid-variational, is prevailing for the analysis of plastic working technological problems. When using the finite element method the main difficulties arise from basic equations nonlinearity and necessity of realization of incompressibility condition. A number of approaches are used for overcoming these difficulties. For example, flow function introduction allows to meet identically an incompressibility condition and to simplify a statement and solution of arising variational problems. Other approach is a generation of the equations for finite elements by means of Bunyakovsky-Schwarz inequality. A review of the scientific works shows that the finite element method is effective for a kinematic analysis and definition of related technological parameters. However, the detailed analysis in stresses without solution of differential equilibrium equations is problematic as the use of only nonholonomic connections between strain rates and deviator components of stresses does not enable determining a spherical tensor of stresses.

3 Problem Solution

3.1 Construction of basic solution

For the analysis and mathematical modeling of axisymmetric plastic forming processes with a strong stress and strain rate phase variation it is necessary to use a method based on construction of basic solution in stresses and velocities and guided to exact solving of iterative process. For constructing of the basic solution it is expedient to temporarily add a property of local static definability to the basic equations system (1) – (5), for example, by using a side condition in stresses. The similar approach was applied when solving the axisymmetric problems by fixation of stress and strain rate phase.

The use of the "rigid" side condition fixing a stress and strain rate phase for construction of the basic solution can lead to greater difficulties when providing a similarity condition (4) for stress and strain rate deviators. Therefore, the use of "flexible" side condition non-limiting a phase of stresses and strain rates in basic solution is more effective. Side conditions should be universal, invariant concerning medial stress, and should allow to use the known solutions and experimental information for the analysis of technological problems.

Let's present components of the stress tensor in the following parametric form [1].

$$\sigma_r = \sigma + \sqrt{\frac{2}{3}}\tau_s\left[-m_z sign(\sigma_\theta - \sigma_r) - m_\theta \sin 2\varphi_\theta\right],$$

$$\sigma_z = \sigma + \sqrt{\frac{2}{3}}\tau_s\left[m_\theta \sin 2\varphi_\theta - m_r sign(\sigma_\theta - \sigma_r)\right],$$

$$\sigma_\theta = \sigma + \sqrt{\frac{2}{3}}\tau_s\left[-m_r sign(\sigma_\theta - \sigma_r) + m_z sign(\sigma_\theta - \sigma_r)\right]$$

$$\tau_{rz} = \tau_s m_\theta \cos 2\varphi_\theta \ , \qquad (6)$$

where φ_θ, m_r, m_z, m_θ - parameters introduced by the following relations

$$\varphi_\theta = \frac{1}{2} arctg\left(\sqrt{\frac{2}{3}} tg 2\delta_\theta\right), \qquad (7)$$

$$m_\theta = \frac{\tau_{\alpha\beta}}{\tau_s}\cos 2\delta_\theta \sqrt{1 + \frac{2}{3} tg^2 2\delta_\theta} \ , \qquad (8)$$

$$I_1(\overline{D}_\sigma) = m_r + m_z + m_\theta \sin 2\varphi_\theta = 0 \ , \quad (9)$$

$$I_2(\overline{D}_\sigma) = m_r^2 + m_z^2 + m_\theta^2 = 1, \qquad (10)$$

$\overline{D}_\sigma$ – directing deviator, *sign* – sign function.

The component $\tau_{\alpha\beta}$ represents a principal shearing stress in a meridional plane, and parameter δ_θ - an angle which is count off from an axis *r* up to the first direction of stress $\tau_{\alpha\beta}$ action. Directions of stress $\tau_{\alpha\beta}$ action form two sets α and β of interorthogonal glide lines.

The parameter φ_θ (or δ_θ) determines a direction of the normal to octahedral plane, and parameters m_i - orientation of the vector of shearing octahedral stress.

In many processes of deforming of axisymmetric articles a linear dimension *a* of plastic area in radial direction is known depending on natural boundary conditions. It allows to establish a side condition for the parameter m_θ determining in coordinate frame α, β a ratio between meridional maximum shearing stress and shear yield point. We shall introduce a characteristic dimension a/r_α for sections of the plastic area, where r_α - a radial coordinate of the section. On the symmetry axis, i.e. when $a/r_\alpha = 1$, where monoaxial condition is realized, a component $\tau_{\alpha\beta} = (\sqrt{3}/2)\tau_s$. A condition close to simple shear is realized in plastic area sections outlying from the symmetry axis, where $a/r_\alpha << 1$, and the component $\tau_{\alpha\beta} = \tau_s$. With decrease of a/r_α the material state verge fast towards simple shear. Therefore, connection of the component $\tau_{\alpha\beta}$ with the magnitude a/r_α is presented by exponential law, and the dependence (8) assumes the following form

$$m_\theta = \cos 2\delta_\theta \sqrt{1+\frac{2}{3}tg^2 2\delta_\theta}\left[1-\left(1-\frac{\sqrt{3}}{2}\right)\exp\left(1-\frac{r_a}{a}\right)\right], \quad (11)$$

which used as a side condition in the sequel.

Equilibrium equations (1) subject to the parametric mode (6) assumes the following form in coordinate frame α,β

$$\frac{\partial\sigma}{\partial s_\alpha} + \sqrt{\frac{2}{3}}\tau_s\left[-sign(\sigma_\theta-\sigma_\alpha)\frac{\partial m_\beta}{\partial s_\alpha} - 2m_\theta\frac{\partial\varphi_\theta}{\partial s_\alpha}\right] -$$

$$2\tau_s m_\theta\frac{\partial\delta_\theta}{\partial s_\alpha} + \tau_s\frac{\partial m_\theta}{\partial s_\beta} -$$

$$-\sqrt{6}\tau_s\frac{m_\alpha}{r}\cos\delta_\theta sign(\sigma_\theta-\sigma_\alpha) - \tau_s\frac{\partial m_\theta}{r}\sin\delta_\theta -$$

$$-\sqrt{\frac{2}{3}}m_\beta sign(\sigma_\theta-\sigma_\alpha)\frac{\partial\tau_s}{\partial s_\alpha} + m_\theta\frac{\partial\tau_s}{\partial s_\beta} = 0, \quad (1a)$$

$$\frac{\partial\sigma}{\partial s_\beta} + \sqrt{\frac{2}{3}}\tau_s\left[2m_\theta\frac{\partial\varphi_\theta}{\partial s_\beta} + sign(\sigma_\beta-\sigma_\theta)\frac{\partial m_\alpha}{\partial s_\beta}\right] +$$

$$+2\tau_s m_\theta\frac{\partial\delta_\theta}{\partial s_\beta} + \tau_s\frac{\partial m_\theta}{\partial s_\alpha} -$$

$$-\sqrt{6}\tau_s\frac{m_\beta}{r}\sin\delta_\theta sign(\sigma_\beta-\sigma_\theta) + \tau_s\frac{\partial m_\theta}{r}\cos\delta_\theta +$$

$$+\sqrt{\frac{2}{3}}m_\alpha sign(\sigma_\beta-\sigma_\theta)\frac{\partial\tau_s}{\partial s_\beta} + m_\theta\frac{\partial\tau_s}{\partial s_\alpha} = 0, \quad (1б)$$

$$-m_\alpha sign(\sigma_\beta-\sigma_\theta) - m_\beta sign(\sigma_\theta-\sigma_\alpha) = 0, \quad (9a)$$

$$m_\alpha^2 + m_\beta^2 + m_\theta^2 = 1, \quad (10a)$$

The differential equations (1a, b) relate to hyperbolic type and have two sets of mutually orthogonal characteristics.

$$\frac{dz}{dr} = tg\delta_\theta \text{ (lines } \alpha\text{)}, \quad \frac{dz}{dr} = -ctg\delta_\theta \text{ (lines } \beta\text{)}. \quad (12)$$

The velocity equations (3) and (5) have characteristics (12) and are written in the fixed coordinates system α^* β^* coinciding with paths α, β as follows

$$\frac{\partial v_{\alpha^*}}{\partial s_\alpha} - v_{\beta^*}\frac{\partial\delta_\theta}{\partial s_\alpha} + \frac{v_{\alpha^*}\cos\delta_\theta - v_{\beta^*}\sin\delta_\theta}{r} = 0, \quad (3a)$$

$$\frac{\partial v_{\beta^*}}{\partial s_\beta} - v_{\alpha^*}\frac{\partial\delta_\theta}{\partial s_\beta} + \frac{v_{\alpha^*}\cos\delta_\theta - v_{\beta^*}\sin\delta_\theta}{2r} = 0. \quad (5a)$$

3.2 Construction of iterative process

The equations (1a, b), (9a), (10a) and a side condition (11) form a locally definable set of equations concerning five required components σ, δ_θ, m_α, m_β, m_θ. Two velocity relations (3a), (5a) form a closed system relative to two required components of velocities $v_{\alpha^*}, v_{\beta^*}$. The solution of these subsystems allows to map two positions of generatrix for each node of plastic area (M_1N_1 and M_2N_2) at the cylindrical surface of yielding flow in the space of principal stresses $\sigma_1, \sigma_2, \sigma_3$ (Fig.1), one of which corresponds to admissible stresses, and another - to admissible velocities.

The generatrix *MN* position corresponding to exact solution is between segments M_1N_1 and M_2N_2. Therefore, the iterative process directed to exact solution can be interpreted as counterrotation of generatrixes M_1N_1 and M_2N_2 around a hydrostatic axis $p = -\sigma$ until the condition (4) of phase coincidence of deviators $D_{\dot{e}}$ and D_σ is satisfied with given accuracy.

Thus, the phase coincidence condition (4) is considered as a differential relation which should be satisfied by exact solution. From this point an advantage of Mises cylindrical surface selected as a loading surface is becoming obvious. Generatrix movement on yielding surface under compound loading can be interpreted as its hydrostatic axial rotation in the space of principal stresses, i.e. a phase change of the stress and strain rate deviators. Hence, each plastic particle of deformable material is mapped by generatrix moved on some surface $f = 0$ under complicated loading, in case of Mises condition (2) - surfaces of a square invariant of the stress deviator. The similar image of the deformation process conforms to principles of the deformation locality theory [5] considering it as the outcome of the action of elementary mechanisms (sliding, twinning, etc.) in a discrete polycrystal structure.

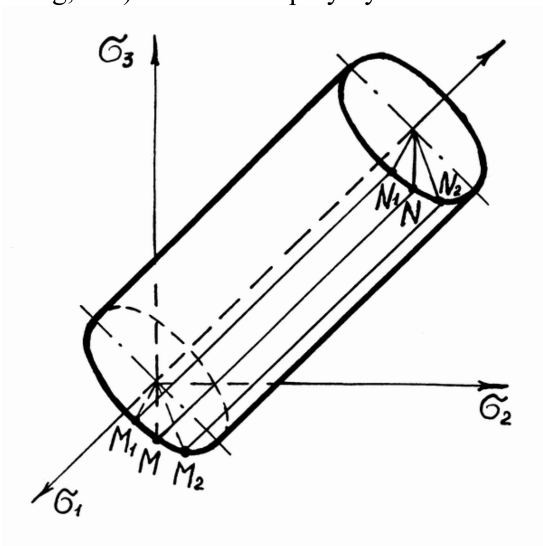

Fig.1 Mapping of solution in stresses and velocities in the space $\sigma_1, \sigma_2, \sigma_3$

3.3 Solution of applied problem

Let's consider a process of indirect cold extrusion of the hollow cylindrical detail from solid blank. The mild scheme of stressed state (σ / τ_s < -1) under extrusion promotes high plasticity of the worked material and, accordingly, greater operational strain extents. High hydrostatic pressure (p = -σ) in plastic area of strained material promotes deformation microflaws healing and damage level reducing for the finished articles in comparison with manufacturing operations with the stiff scheme of stressed state ($\sigma / \tau_s > 0$). Therefore, the exact information on arising technological stresses and yielding flow velocities of the strained material is important for prediction of its structural properties [6].

Let's analyze stresses and velocities at the closing stage of extrusion when deforming tool load amounts to the largest value (Fig. 2, 3).

When solving boundary value problems the convention $\tau_k = \tau_{\alpha\beta}$ of ultimate contact friction on the end surface of the punch and back surface of the container is made. Deformed metal normal pressure on the lateral surface of the container is much less than on the back surface. Therefore, a condition of non-ultimate contact friction is realized on the lateral surface of the container.

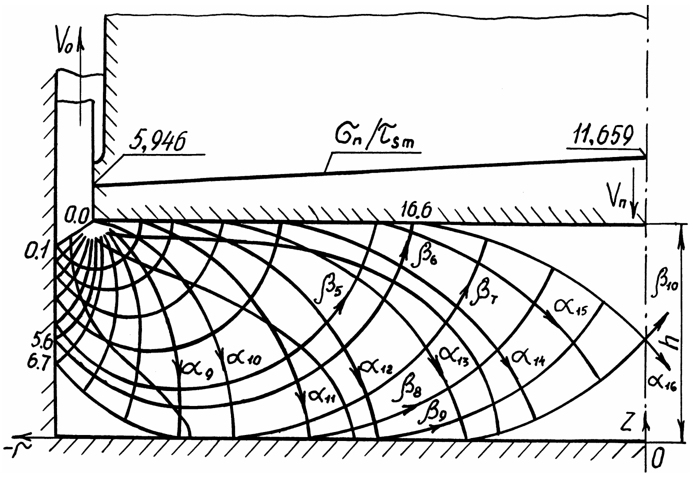

Fig.2 Paths α,β *field of maximum shearing stresses in the plastic area of strained half-finished product*

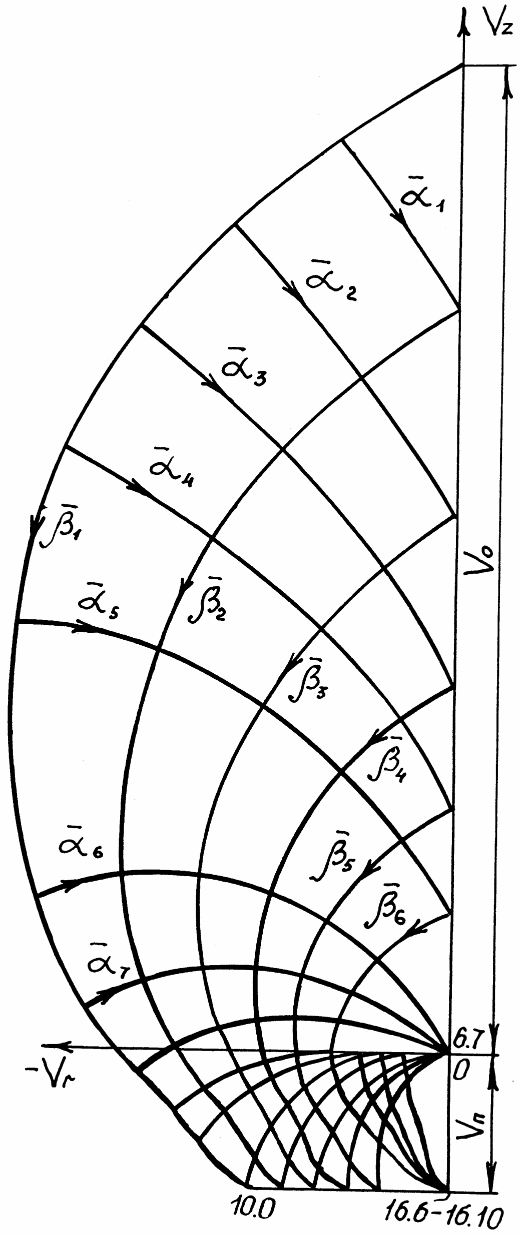

Fig. 3 Paths α,β *mapping in velocity plane (velocity field)*

According to the experimental data on detection of paths of maximum shearing stresses (by means of the method developed by A. Fri) glide lines (set α) approach to the lateral surface of the container at the angles of 54...57^0 and are reflected from the contact surface (as lines of set β) at the angles of 33...36^0. The established field of glide lines defines a directing deviator of stresses $\overline{D}_\sigma$.

Values of shear yield point (τ_s) calculated in nodes and average stress (σ) allow to evaluate the deviator and the spherical tensor of stresses. The field of velocities is determined on the basis of numerical solution of the equations (3a), (5a) for specified boundary conditions.

Degree of conformity between fields of stresses and velocities is established by means of differential connection (4) in the form $\omega_{\dot{e}} - \omega_\sigma \le [\Delta\omega]$, where $\omega_{\dot{e}}$, ω_σ – phase angles of deviators $D_{\dot{e}}$ and D_σ, $[\Delta\omega]$ – a permissible error in definition of phase angle. Error $\Delta\omega = \omega_{\dot{e}} - \omega_\sigma$ is evaluated in characteristic points of plastic area *m.n*. Thus, the iterative process directed to exact solution is reduced to the inequality

$$\Delta\omega\ (m.n) \le [\Delta\omega]. \qquad (13)$$

For solution of the inequality (13) the method of group relaxation is used. By regulating of the absolute value of discrepancy between parameters $m_\theta^{(\sigma)}$ and $m_\theta^{(\dot{e})}$ in the selected nodes (instead of its complete liquidation at the first stage) it is obviously possible to ensure the inequality (13) with a permissible error $[\Delta\omega]$ = 0,05 radians already at the first correction. The calculated field of paths of maximum shearing stresses (stress field) (Fig.2) and the field of yielding flow velocities (Fig.3) satisfy the inequality (13) at $[\Delta\omega]$ = 0,05 radians.

4 Conclusion

Let's consider the solution results. A kind of the stressed state varies within the limits of plastic area from monoaxial compression on symmetry axis to simple shear around a clearance between the punch and the container. It allows to consider an effect of all stress deviator invariants on technological plasticity of the material. The normal pressure diagram (Fig. 2) indicates non-uniform distribution of local loads on the contact end surface of the punch. Maximum pressure on the punch contact surface around symmetry axis amounts to $(11\ldots12)\,\tau_s$ at the closing stage of extrusion that leads to a necessity of using high-strength deforming tools. Fields of yielding flow velocities defined at stages of deforming process enable predicting a volume distribution of strains of finished articles as well as mechanical properties of their materials.

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