ROLE OF SYNTHETIC STORMS ON PEAK FLOW ESTIMATION

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Abstract

In this study, synthetic annual maximum storms distributed as Gumbel with random effective storm durations and specific time distribution of precipitation for a given population mean and variance are generated by Monte Carlo method. Population means and variances are defined by a three parameter nonlinear regression model and by a cubic logarithmic model, respectively. Effective durations of the synthetic annual maximum storms are related to basin characteristics, length and harmonic slope of the main course through Kirpich's time of concentration relationship. Annual peak flow series of the hypothetical basins with physiographic characteristics are generated through a deterministic rainfall-runoff model, namely the Soil Conservation Service synthetic unit hydrograph method and discrete convolution technique. Sample statistics and frequency distributions of the generated annual maximum storms of random effective durations and of the resulting peak flow series are investigated. Eight well-known probability distribution models with moment and maximum likelihood parameters are tested by chi-square and probability plot correlation goodness of fit tests.

Key-words: synthetic rainfall, monte carlo, annual peak flow, SCS abstraction, frequency distribution, goodness-of-fit.

1. Introduction

The design of water resources structures for use in ungaged basins requires some estimate of flood flows and their frequency of occurence. If there is no historical streamflow data for these watershed areas, regional flood frequency analysis, or parametric rainfall-runoff event simulation can be used to estimate floods. Available precipitation data with maximum value are used as input variables in parametric rainfall-runoff models. If both rainfall and runoff data are not available, random input variables are generated by the Monte Carlo method and frequency analysis techniques are used to analyze output variables.

Sometimes rainfall data recorded at meteorological stations may be much longer than corresponding streamflow observations. Hence, frequency models fitted to extreme rainfall data are more reliable than the frequency models of peak flows. This advantage can be transferred into improving or performing decisions on the appropriate frequency distribution model for the peak flows. Of course, occurrences and frequencies of peak flows are also dependent on the time distribution or pattern of the rainfall, basin physiography, vegetation, land management, and antecedent moisture conditions of the watershed.

Methods of generating design storms are available and in wide use, but they are general in nature and assume storms occur with the same temporal distribution over a wide area. Because of extreme climatic differences between the areas, design curves are not likely to be representative of the actual time distribution of storms in semi-arid regions. For example, [12] decided to develop a new design storm generation procedure applicable to the State of Wyoming based on observed storm rainfall in Wyoming. Synthetic storm series, with similar statistical properties to observed series, were developed by [17]. Data of the Alcorta raingauge (with a 5-year series of data) were analyzed by means of five variables: duration of the rain, time between events, average and maximum intensity of the rain, and storm advance coefficient. The variables were classified as independent (the first three variables) and dependent (the last two variables). Probability distribution functions were fitted to the independent variables. Multiplicative relationships were proposed for dependent variables and their coefficients were adjusted. The statistical characteristics of the synthetic series were calculated and compared with the observed data series. A good agreement between calculated and observed series was obtained [17].

2. Scope of the Study

The major objectives of this study were to find satisfactory answers in the following questions and to give an insight into the problem of rainfall-runoff transformation on a probabilistic basis.

(1) Provided that the storm events of different durations have the same type of probability distribution (say, Gumbel), does the type of the probability distribution of mixed storm events remain unchanged?

(2) Does, or how does, a rainfall-runoff transformation process transfer the distributional characteristics of rainfall events into the output (peak flows)?

(3) Are there significant and physically meaningfull relations between the statistics of input and output?

(4) The storm events which are responsible for the annual peak flows may be drawn from different populations. Therefore, return periods of input and output may or may not be the same.

In order to find answers to the above given questions, a Monte Carlo simulation is followed in the study. Twenty-five sets of synthetic storms of different durations each N=100 size are generated. Kirpich's empirical relationship between the time of concentration of the small watershed and the length and slope of the main course is used for deciding the critical rainfall durations over the hypothetical watersheds. Synthetic storm inputs are transformed into peak flows by following Soil Conservation Service (SCS) triangular unit graph and discrete convolution procedures.

Statistical descriptors of both input (rainfall) and output (peak flow) series and parameters of various probability distribution models by the methods of moments and maximum likelihood are estimated. Tests for the goodness of fit of those distribution models by the chi-square test, as well as the probability plot correlation test, are performed and frequency of acceptance of each model is found. Relations between the types of probability distributions of the input and the output are investigated on the basis of relative acceptance frequencies of the postulated probability distribution models.

3. Methodology

3.1 Probability Models, Parameter Estimations and Goodness of Fit

In this study; the normal (NOR), lognormal with two- and three parameter (LN2 and LN3), gumbel (GUM), log-gumbel (LGUM), gamma with two-and three parameters (G2 and G3), and the log-pearson III distributions (LP3) are used for synthetically generated storms and peak flows which are generated from these storms by using a rainfall-runoff models. Numerous methods of estimation are available and two most common approaches that method of moments (MOM) and maximum likelihood (ML) methods are used in this study. Detailed information about these probability models and parameter estimation methods are given by [3] and [8].

The selection of a distribution type is crucial and propounds a fundamental challenge to hydrologists of the flood frequency approach. For many cases, custom and convenience often play a significant role in the selection of a flood distribution. In this study, chi-square (χ^2) and probability plot correlation (PPC) test were used for goodness of fit.

3.2 Synthetic Unit Hydrograph

Determination of the unit hydrograph of a basin is very important for the design of water structures, because it gives information about peak discharge, time of the peak discharge, and duration of excess runoff. When it is necessary to determine a unit hydrograph for a ungaged basin, therefore, one of the synthetic unit hydrograph determination methods is used. Synthetic unit hydrographs can be estimated for ungauged drainage basins by means of relationships with parameters of characteristics of the drainage basin. The most commonly used methods are the Snyder (1938), the Mockus (1957), and U.S. Soil Conservation Service (SCS) (1972) methods. In this study the Mockus synthetic unit hydrograph method was used.

4. Computational Algorithm for Generation Synthetic Storms and Peak Flows

4.1 Generate Synthetic Storms

4.1.1 Time of concentration, effective storm duration and unit duration

A very simple event-based rainfall-runoff model is applied to a 100 km^2 hypothetical watershed area without baseflow. Durations of synthetic storms conforming basin lag are generated by considering that the length (L) and harmonic slope (S) of the main channel are dominating factors on the time of concentration. The effective storm duration, D_e, is given as [2] and [10];

$$D_e=2(t_c)^{0.5} \quad (1)$$

$$t_c=0.00032(L)^{0.77}/S^{0.385} \quad (2)$$

where L is the length of the main course in meters, S is the harmonic slope, t_c is the time of concentration of the watershed and D_e is the effective storm duration, both in hours.

Time of concentration (t_c) according to Kirpich's formula Eq.2 for various lengths of main course (L) and harmonic slopes (S) are given in Table 1. For $t_c \geq 4$ hour, effective storm durations are assumed to be equal to time of concentration ($D_e=t_c$).

Table 1 Time of Concentration (t_c; hour) According to Kirpich's Formula for Various Lengths of Main Course (L) and Harmonic Slopes (S)

	S	(Harmonic	Slope)		
L(m)	0.001	0.005	0.01	0.015	0.02
8000	4.63	2.49	1.91	1.63	1.46
12000	6.32	3.40	2.60	2.23	2.00
16000	7.89	4.25	3.25	2.78	2.49
20000	9.37	5.00	3.86	3.30	2.95
25000	11.13	5.99	4.58	3.92	3.51

A typical rainfall hyetograph is a composition of M number of discrete pulses of Δt time increments. Therefore the critical rainfall duration, D, generated in a random manner from a uniform distribution in the range $D_e \leq D \leq 2D_e$ is rounded off as multiples of Δt:

$$D = M. \Delta t \quad (3)$$

In order to simplify calculations of composite hydrograph M is assumed as 5, 6, or 7. This assumption is in accordance with unit graph durations used in hydrologic practice, $0.15t_c \leq \Delta t \leq 0.20t_c$.

4.1.2 Type-I Extremal Distribution of Annual Maximum Storms with Asymptotic Moment Parameters

In this study, the mean of rainfall depth-rainfall duration and standard deviation of rainfall depth-rainfall duration relationships of Uşak Meteorological Station developed by Benzeden (2001) are used as population statistics of the synthetic storms. The mean of rainfall depth-rainfall duration and the standart deviation of rainfall depth-rainfall duration relationships are as follows:

$$M_D = 4.154[\ln(D/1.1763)]^{1.0263} \quad (4)$$

$$S_D = \exp\left[-1.46047 + 1.79839\ln(D) - 0.3155\ln^2(D) + 0.01944\ln^3(D)\right] \quad (5)$$

where D is in minutes and M_D and S_D both are in mm. According to these relationships, synthetic rainfall depths, Y_D, distributed as the Extreme Value Type-I (Gumbel) with asymptotic parameters are generated through Eq.6.

$$Y_{D,T} = M_D + S_D.K_T \quad (6)$$

where

$$K_T = -\{0.45 + 0.7797 \ln[-\ln(1-1/T)]\} \quad (7)$$

is the frequency factor corresponding to the return period T or probability of nonexceedance P_T which can be calculated from generated uniform random numbers $0 < P_T < 1$, that is $T=1/(1-P_T)$ (Kite, 1977).

4.1.3 Time Distribution of the Generated Storms

The SCS dimensionless cumulative rainfall curves were developed for various storm types, storm durations and regions in the United States [10]. In this study, SCS 6-hour time distributions are used in order to calculate rainfall hyetographs of given synthetic storms.

A design storm, Y_D, is divided into increments ΔY_m using an appropriate time distribution curve for the project site. A time distribution curve represents the cumulative percentage of the precipitated rainfall $f_m = Y_m / Y_D$, during the percentage time $X_m = t_m / D$, where $D=M.\Delta t$ is the rainfall duration and $t_m=m\Delta t$ (m=1,2,...,M). Having f_m values, the cumulative rainfall amount (Y_m) precipitated during period 0 to t_m, and incremental rainfall amount (ΔY_m) can be computed as:

$$Y_m = f_m D, \qquad m=1,2,...,M \quad (8)$$
$$\Delta Y_m = Y_m - Y_{m-1}, \qquad m=1,2,...,M \quad (9)$$

where $y_0=0$ for m=1.

4.2 Genaration Synthetic Peak Flows According to Synthetic Annual Maximum Storms

The SCS-curve number method with CN=90 is applied in order to account for the initial abstractions and its role on the excess rainfall hyetograph and composite hydrographs. For the sake of simplicity, it is assumed that the Mockus unit hydrograph is a sufficient tool to simulate the rainfall-runoff transformation process. The triangular unit graph [10] of a unit duration Δt includes the size of the basin drainage area, A, the length of the main channel L, and slope, S because of the relation between time to peak, t_p, and time of concentration, t_c.

$$t_p = \Delta t/2 + 0.6 t_c = f_1(L, S, \Delta t) \quad (10)$$
$$q_p = KA/t_p = KA/f_1(s) = f_2(A, L, S, \Delta t) \quad (11)$$

Ordinates of Δt-hour triangular unit hydrograph at times Δt, $2\Delta t$, $3\Delta t$, ... are used in calculating superposed output resulting from given ERH of M number of incremental excess rainfall pulses, R_1, R_2, ... R_M.

$$Q_n = \sum_{m=1}^{n \le M} R_m U_{n-m+1} \quad (12)$$

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5. Results

5.1 Probability Distribution of Mixed-Storm Durations

When the results of χ^2 tests for 25 synthetic storm series are evaluated the most suitable model for the mixed-duration storm series cannot be determined clearly (Table 3). Though LN2 distribution for the case with MOM parameters has the highest acceptance frequency, GUM, G2, LN3 and G3 distributions are also compatible distributions. For the maximum likelihood parameters, LN2, G2 and GUM distributions are the most suitable distributions. On the other hand the PPC test results reveal that the most suitable models are LN2, G2, GUM, LN3 and G3 either with MOM or ML parameters.

5.2 Relations Between the Types of Probability Distribution Functions

In order to compare the probability distribution models most frequently accepted (that is, the goodness of fit test is passed) for the synthetic storm samples (input) and for the peak flows (output), relative acceptance frequencies of each model, f_I and f_O, computed from Eq.13 and 14, at a significance level α=5% for the χ^2 test are presented comparatively with and maximum likelihood parameters. Relative acceptance frequency of a specific model (f_I) is defined as:

$$f_I = 100\,(TNCH)/25 \tag{13}$$

where TNCH is total number of series that passed the χ^2 test for a specific distribution. Similarly, using parameters estimated by method of moments, relative acceptance frequencies of each model at a significance level α=5% are presented comparatively. The results of the PPC tests are evaluated in the same way except that the relative acceptance frequencies of the output series are calculated now from Eq.14.

$$f_O = 100\,(TNPP)/25 \tag{14}$$

where TNPP is total number of storm series that have a PPC coefficient greater than r_c=0.95. The PPC results are shown in Fig.5 and 6. Relative acceptance frequencies according to the chi- test results with MOM parameters LN2 is the most appropriate model. Similarly, with ML parameters; LN3 is the most appropriate model. For PPC test, it LN2 and LN3 models are most suitable both of parameter estimation methods.

Table 3 Results of godness-of fit tests of synthetic storms (input) and peak flow series (output) with MOM and ML estimators (GOF:Goodness-of-fit, PEM:Parameter estimation method)

GOF methods	PEM	Data	NOR	LN2	G2	GUM	LGUM	LN3	G3	LP3
Chi-square	MOM	Input	48	92	84	88	76	84	84	32
		Output	0	80	40	20	68	52	72	68
	ML	Input	48	96	96	96	28	92	88	28
		Output	0	76	56	48	72	84	56	36
PPC	MOM	Input	84	100	100	100	96	100	100	52
		Output	12	100	96	84	96	100	96	0
	ML	Input	84	100	100	100	96	100	100	28
		Output	12	100	84	84	96	100	96	40

5.3 Relations Between Distributional Characteristics of the Input and Output

Relation between the types of the probability distribution of input and output according to the chi-square tests are investigated on the basis of correlations (r) between the relative acceptance frequencies and are calculated with maximum likelihood parameters according. The correlations between the acceptance frequencies of input and output are calculated. The input-output relationships are 0.197 and 0.350 for chi-square goodnes-of fit test with MOM and ML, respectively. When PPCC model is used, input-output relationships are 0.350 and 0.625 for MOM and ML, respectively. As it can be seen, result of ML parameter estimation method is better than the MOM. But, the relationship between input and output distribution is not good in both of case.

6. Conclusions and Recommendations

As can be seen from Table 3, when the twenty-five synthetic storm series were evaluated with chi-square goodness of fit test, the most suitable distribution with MOM parameters is LN2; and LN2, G2 and GUM with ML parameters. This means that when the generated synthetic storms distributed as Gumbel are put into a mixed duration series the type of the appropriate distribution may change. Similarly, according to the results of probability plot correlation test given in Table 3, LN2, G2, GUM, LN3 and G3 distributions are the most suitable distributions either with MOM or ML parameters. These results of the study reveals that the probability distribution of the rainfall input may even diverge from their parent (Type-I Extremal) distributions because of the sampling, and since the generated input series is a mixture of rainfall events of variable durations. Provided that the storm events of different durations have the same type of probability distribution, the probability distribution types of mixed storm events have not been seen to change drastically.

The answer of the second question may be given as, a rainfall-runoff transformation process doesn't transfer the distributional characteristics of rainfall events into the peak flows. The answer of third question is there aren't significant and physically meaningfull relations between the probability distribution of storm and peak flow. The last answer is the storm events which are responsible for the annual peak flows may be drawn from different populations. Therefore, return periods of input and output may not be the same.

This study is based on various assumptions and simplifications and therefore the conclusions should be evaluated carefully. A rather complicated and comprehensive research should be conducted by taking into account for the complicated mechanism of rainfall-runoff transformation process which must include the storage effects, the nonlinearities in the watershed system, and alternative probability distribution models for the input. It is hoped that these conclusions of this study will be beneficial in small basins where significant information on precipitations is available in case of limited information on peak flows.

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