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Vertical turbulent fountains in a uniform calm ambient

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Abstract: Experiments have been conducted to investigate the penetration of a fountain of lighter fluid, issuing vertically downwards from a nozzle into a quiescent fluid of higher density. Our interest was focused in the maximum jet penetration below the source, as well as, in the subsequent terminal (steady state) depth of penetration of the fountain. The source Richardson numbers used, have been extended in the full range, i.e. from very low (jet-like flows) to about 1 (plumes). Two types of jets have been investigated, fresh water jets issuing into salt water (buoyancy flux is constant), and hot water jets issuing into fresh cold water (buoyancy flux decays). Besides the circular nozzles, three other different shape nozzles have been used for comparison. The normalized penetration depth plotted against the source Richardson number, was found to be constant for initially jet-like flows, while it decays exponentially in plumes.

Key-Words: turbulent buoyant jet, negative buoyancy, fountain, mixing, penetration depth

1 Introduction

The treated liquid or gas wastes of many societies are naturally ubiquitous in diluted form. For these wastes a rapid discharge to the environment is often the best means of recycling. The initial dilution can be accomplished by means of turbulent buoyant jets and plumes, because they entrain large volumes of ambient fluid and mix it with the discharge fluid. The actual discharge arrangement can often be simple, essentially the open end of a submerged pipe. In many cases though, much thought and expense must be given to design a structure, which enables us to achieve much higher initial dilution, in order to minimize the immediate effect of the discharge on the environment. The discharge devices which can be either ocean outfalls, or stacks, or deep water sea vents, are usually positioned at the lowest point of the receiving fluid. The discharged fluid density, in most of the cases is different from that of the receptor. It is usually lighter, and therefore it is positively buoyant, that means it can move vertically upwards. In many cases though, the density of the discharged fluid may be greater from that of the receiving fluid. When we discharge heavier fluid vertically upwards into a calm ambient lighter fluid, the heavier mixture settles at the bottom near the elevation of the origin of discharge. Negatively buoyant jets, for which gravity forces oppose their movement, are also called fountains. They have been investigated for a long time, since Turner's [17] pioneering work. Their mixing mechanics is of great interest for design engineers, who desire to optimize the dilution of heavier fluid into a volume of lighter one.

Turbulent fountains are formed when a continuous jet of dense fluid is injected rapidly upwards, into a less dense environment, or when a continuous jet of buoyant fluid is injected downwards, into a denser environment [1,17]. They arise in a number of important situations both in engineering and in nature. Applications include the forced heating or cooling of large enclosures, such as aircraft hangers, and buildings or rooms. During the heating or cooling of a room, a jet of air at different temperature may be forced into the room through a floor or ceiling vent [3]. Other applicatios include the disposal of brines, or heavier industrial wastes into the ocean, [9], and the improvement of water quality by forced mixing in reservoirs, small lakes and harbours, or fjords [11]. Jet mixing in a tank is used industrially to blend fresh fluid with the contents of a tank. Geophysical buoyant jets resulting from temperature (or salinity) differences can occur in the ocean and in magma chambers near the crust of the earth. Other examples of natural fountains are the evolution of volcanic eruption columns [19], and the replenishment of magma chambers in the Earth's crust [5, 18].

List [12] and Baines and Chu [2] provide excellent reviews of the literature on positively buoyant turbulent jets. When buoyancy forces and the jet momentum oppose one another, such as a when fluid injected upward into a lower density fluid, or downward into a higher density fluid, the jet is negatively buoyant. Upward negatively buoyant jets are often referred to as fountains, since the heavy fluid penetrates upward to a particular height and then flows back downward in an annular region surrounding the jet. Negatively buoyant jets are somewhat more interesting than positively buoyant jets, because the buoyancy force and momentum compete, but little is known about the transient character of either positively or negatively buoyant jets upon start-up.

The development of negatively buoyant jets is more implicated by the fact that buoyancy is acting in a direction opposite to that of the initial flow. Therefore, the reversing buoyancy reduces the flux of momentum, and the vertical velocity vanishes at some distance from the source. The jet reaches its maximum penetration length soon after it was started, then it reverses its direction and flows back on itself as a fountain. Turner [17] in a pioneering work, showed that the properties of a vertical dense jet issuing vertically upwards from a small source into a calm uniform environment, are mainly governed by the initial specific jet momentum and buoyancy fluxes. The turbulent interaction between the up-and down flows restricts the rise of fluid any further, thus immediately reducing the initial fountain height to a smaller terminal value. This observed final height was related to the momentum and buoyancy fluxes at the source using dimensional arguments. The finding was that the terminal, steady state jet penetration normalized by the jet diameter, varied linearly with the initial densimetric Froude number, and it was verified experimentally. Several works have been completed since, regarding vertical negatively buoyant jets in a motionless uniform ambient. Turner's experiments have been verified numerically [1]. Temperature measurements [16] were performed in a heated air jet discharged downwards, while the stable stratification formed by a fountain has been studied [3], once it falls back and spreads along the floor of the dispersion chamber, re-entraining some of the already mixed heavier fluid. Recently, measurements have been performed [6, 20], of the penetration height of a fountain in a wide range of buoyant jets, for initial densimetric Froude numbers, from very low, plume-like to high, jet-like flows. The effects including cross flow and angle of injection on negatively buoyant jets has also been studied [11]. The evolution (transient character from startup to steady state) of positively and negatively buoyant jets, was investigated [14], using laser induced fluorescence imaging, and particle image velocimetry, to study the velocity field of a fountain near the top and at the bottom of the tank. Recently, some measurements of a vertical fountain out of a pipe have reported [4], regarding the maximum and terminal penetration heights in a uniform calm ambient.

We have conducted experiments to study the effect of reversing buoyancy on a turbulent buoyant jet, issuing from a source vertically into a uniform calm ambient fluid. We used circular nozzles of different diameters, and a variety of three different shape nozzles (orthogonal, square and triangular). Fresh water was discharged vertically downwards from a nozzle into a stagnant body of saltwater, or hot water into fresh cold water, and the vertical penetration depth of the jet was measured. The initial source Richardson numbers examined, were extended in the full range, from very low (jet-like flows) to about 1 (plumes). A set of experiments of hot water jets, discharged vertically downwards into fresh water ambient was carried out, to examine the effect of buoyancy deficiency in the mixing.

1.1 Dimensional arguments

Consider a jet of lighter fluid, issuing vertically downwards from a 'point' source, into a homogeneous calm ambient of higher density. We assume that the jet starts abruptly discharging constant volume flux of lighter fluid. During the early stages of the flow (transient period), the jet mixes with heavier ambient fluid, which applies an opposing (buoyancy) force to the jet fluid, that reaches a maximum depth Z_{max}. Then, it folds up, forming an upflow region, which surrounds the downflow region of the jet. The result is that the downflow jet mixes with the heavier upflow fluid, applying an opposing momentum, which results in a lower steady state terminal penetration depth Z. Full description of the mechanics of jets with reversing buoyancy has been presented by Turner [17].

The terminal, steady state vertical penetration depth Z of the jet, depends upon the variables characterizing the source conditions. Ignoring viscosity, a general functional relationship for the dependent variable Z is

$$Z = f(Q, M, B), \qquad (1)$$

where Q is the source volume (specific mass) flux, M and B are the specific momentum and buoyancy fluxes respectively, computed as

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$$Q = \frac{\pi D^2}{4} W$$
$$M = QW \qquad (2)$$
$$B = \frac{\rho_a - \rho_o}{\rho_o} gQ = g'_o Q .$$

In the equations above, W is the uniform jet velocity at the nozzle of diameter D, g is the gravitational acceleration, ρ_0 is the jet fluid density, and ρ_α is the ambient fluid density.

Two length scales based upon the initial kinematic buoyant jet characteristics can be defined as [7]

$$l_Q = \frac{Q}{M^{1/2}} \text{ and } l_M = \frac{M^{3/4}}{B^{1/2}} . \qquad (3)$$

The ratio l_Q / l_M is the initial buoyant jet Richardson number R_0

$$R_o = \frac{l_Q}{l_M} = \frac{QB^{1/2}}{M^{3/4}} = \left(\frac{\pi}{4}\right)^{1/4} \frac{\sqrt{g'_o D}}{W} , \qquad (4)$$

which is reversely proportional to the initial jet densimetric Froude number $F_0 = W/\sqrt{g_o' D}$.

From equation (1), using equations (3) and (4), we may deduce two dimensionless terms, the terminal normalized penetration depth Z/l_M or Z/l_Q and the initial jet Richardson number R_o. Thus equation (1) may be written as

$$\frac{Z}{l_Q} = f(R_o) \text{ or } \frac{Z}{l_M} = f(R_o) . \qquad (5)$$

In case that the initial jet momentum flux M is large, if compared to buoyancy flux B (R_o is small, jet-like flow), ignoring the volumetric discharge Q in equation (1), equation (5) becomes

$$\frac{Z}{l_M} = C , \qquad (6)$$

C being a constant, independent of R_o.

Following the dimensional arguments made above, the maximum penetration depth Z_{max} can be scaled accordingly

$$\frac{Z_{max}}{l_Q} = f(R_o) \text{ or } \frac{Z_{max}}{l_M} = f(R_o) . \qquad (7)$$

In jet-like flows (small R_o), neglecting Q the ratio Z_{max}/l_M , must also be a constant.

Earlier investigators have used different representations of equation (6) to determine the constant of proportionality. In Fig. 1 below we have plotted data collected by earlier investigations, while in Table 1 we tabulate the constants calculated from available data [4, 6, 11, 14, 17, 20].

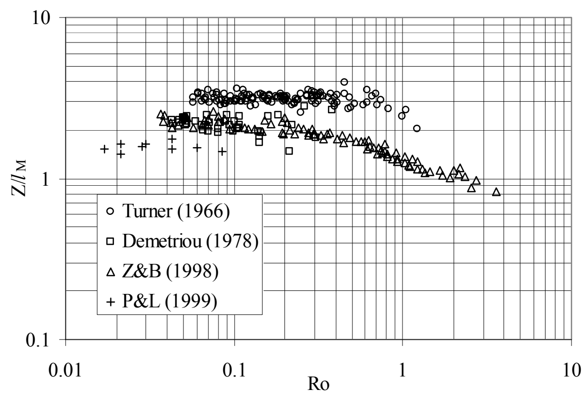

Fig. 1. Normalized terminal penetration depth Z/l_M versus the initial jet Richardson number R_o.

Table 1. Constant of proportionality C from earlier investigations.

Author	C
Turner [17]	3.17
Demetriou [6]	2.18
Zhang & Baddour [20]	2.11
Pantzlaff & Lueptow [14]	1.57
Bloomfield & Kerr [4]	1.70
Lindberg [11]	2.40

The constant C of data by Lindberg [11] was computed as the average value z^*/F_j given by the author, which is related to Z/l_M as

$$\frac{Z}{l_M} = \frac{1}{\pi^{1/4}} \frac{z^*}{F_j} ; \; F_j = \frac{W}{\sqrt{g'_o r}} , \qquad (8)$$

r = D/2 being the jet nozzle radius, presuming that the flows under investigation have initially been jet-like, due to high source Reynolds numbers reported by the author. From Table 1, we may note that the constant of proportionality is quite different in each investigation. The average value of C is 2.19, while the standard deviation is 0.57, which is quite high. This may be due to the fact that either the initial buoyant jet parameters have been miscalculated, or the experimental tanks that were quite different have had an effect on the flow.

Table 2. Dimensions of the dispersion tanks used in earlier experiments (m).

Author	Section	Depth
Turner	0.45X0.45	1.40
Demetriou	1.20X1.20	1.55
Zhang & Baddour	1X1	1
Pantzlaff & Lueptow	D=0.295	0.89
Lindberg	3.64X0.405	0.508
Bloomfield & Kerr	0.40X0.40	0.70
Present experiment	0.80X0.80	0.94

In the present investigation we will determine the constant of proportionality C, from experiments of two types of fountains, with or without preserving buoyancy. Also, we will try to report data regarding the maximum penetration (wherever it is possible), as well as the transient characteristics of the fountain, before it reaches steady state.

2 Experimental

Experiments have been carried out at the Hydromechanics and Environmental Engineering Laboratory, of the University of Thessaly. The experimental apparatus was a 94 cm deep tank made of 1.25cm thick glass, with square cross-section 80cmX80cm. The tank bottom was transparent, made of 1.5cm thick Lucite. A jet of lighter fluid was positioned near the free surface at the centre of the tank cross-section, pointing downwards. The jet plenum consisted of a 4cm i.d. PVC 25cm long tube, part of which was filled with sponge, followed by a 6cm long honeycomb section, to straighten the flow and destruct possible large scales which might affect the jet initial conditions. A flexible supply tube was mounted at its entrance section, and the jet nozzles could be mounted at its exit section. Four round nozzles of 0.5, 1, 1.5 and 2cm in diameter have been used, along with three different shape nozzles, a square, a triangular with equal sides and an orthogonal with a 2:1 side ratio, all of them with rounded corners. The jet nozzle elevation was set around 10cm below the free surface.

Two sets of experiments have been conducted. One where the initial jet specific buoyancy flux was conserved, and one where it decayed. For the first set of experiments, the tank was filled with saltwater at the desired density, while the jet fluid was fresh, tap water. A constant head tank was used as the jet supply, in order to obtain the desired steady discharge of light fluid throughout the experiment. The tank fluid was stirred to become homogeneous, and allowed to settle for about 20 minutes prior to a test. The temperatures of the jet freshwater and tank saltwater were practically identical, close to the room temperature, to minimize thermal effects on the flow. The density difference between jet and ambient fluid, the jet diameter and discharge were chosen properly, in order to obtain the desired initial jet Richardson number R_o. Throughout a test, the fresh water jet discharge was controlled by a precision vane, and monitored with a system of three rotameters. In the first set of experiments, all different nozzle shapes have been used for comparison.

For the second set of experiments, we have filled the tank with fresh water, while the jet fluid was hot water. The hot water supply was an electric water heater with a recirculation pump used to make the jet fluid homogeneous. The water heater was pressurized before each test, and the jet inlet tube was properly insulated. The jet fluid was bypassed and disposed, until a temperature sensor at the jet plenum entrance showed constant temperature for the jet fluid. Then the test started. The desired initial jet Richardson number for each test was obtained by varying the jet temperature (density), diameter and discharge appropriately. The jet flow rate was controlled by the same system of rotameters, used for the first set of experiments. In the second set of experiments, only round nozzles have been used.

The fountain flow was made visible using the shadowgraph technique. Two square grids with size 5cmX5cm were drawn on the two opposite side glasses of the tank. A slide projector that was used for illumination of the flow, projected the front side grid and the fountain shadowgraph onto the opposite side, that was covered with thin white paper. Thus, the two grids and the fountain shadowgraph could give us the desired information, regarding the depth of penetration of the fountain.

The jet flow field image was recorded via a digital video camera at a rate of 25fps. This procedure allowed for the fountain heights to be measured on each frame, and an average value of the fluctuating terminal height to be estimated over a period of time.

2.1 Description of the experiments

A great number of experiments were implemented in order to investigate the full range of initial jet Richardson numbers R_o. In the fresh water – salt water experiments we used all different types of nozzles, besides the circular ones, while in the hot – cold water experiments we have only investigated jets out of round nozzles. In Tables 3 and 4 below, we summarize the range of jet initial parameters of the experiments for preserving and non preserving buoyancy flux respectively.

3 Results and discussion

The finite volume of the tank does not affect the transient character of the buoyant jet, because the jet is far from the walls of the tank, and it disperses as if it were in an unbounded medium. Flow visualization of the transient character of a negatively buoyant jet is shown in Fig.2. The jet initially penetrates downward into the dense liquid Fig. 2(a). Then it reaches the maximum penetration depth Z_{max}, Fig. 2 (b) as the velocity vanishes. The flow reverses in the upward-moving annular region Fig. 2(c), and decreasingly it becomes stable at the terminal penetration depth Z, Fig. 2(d). The terminal depth of jet penetration undergoes a slight oscillation. These upward velocities can never become large enough, to cut off completely the downward flow from the nozzle.

Table 3. Summary of initial parameters in fresh water – salt water (buoyancy preserving) experiments.

Round nozzles

Number of runs	46
Diameters used (cm)	0.5-1.0-1.5-2.0
Ambient density ρ_a (gr/cm^3)	1.007 - 1.021
Jet density ρ_o (gr/cm^3)	0.995 - 0.997
Reynolds number Re	770 - 5840
Richardson number R_o	0.0464 - 0.973

Orthogonal nozzle

Number of runs	15
Hyd. radius (cm)	0.277
Ambient density ρ_a (gr/cm^3)	1.006 - 1.023
Jet density ρ_o (gr/cm^3)	0.995 - 0.996
Reynolds number Re	745 - 4540
Richardson number R_o	0.071 - 0.632

Square nozzle

Number of runs	12
Hyd. radius (cm)	0.265
Ambient density ρ_a (gr/cm^3)	1.006 - 1.023
Jet density ρ_o (gr/cm^3)	0.995 - 0.996
Reynolds number Re	900 - 5500
Richardson number R_o	0.052 - 0.469

Triangular nozzle

Number of runs	11
Hyd. radius (cm)	0.256
Ambient density ρ_a (gr/cm^3)	1.006 - 1.023
Jet density ρ_o (gr/cm^3)	0.995 - 0.996
Reynolds number Re	900 - 5440
Richardson number R_o	0.054 - 0.455

Table 4. Summary of initial parameters in hot water – cold water (non preserving buoyancy) experiments.

Round nozzles

Number of runs	23
Diameters used (cm)	0.5-1.0-1.5-2.0
Ambient density ρ_a (gr/cm^3)	0.9965 – 0.9973
Jet density ρ_o (gr/cm^3)	0.9757 - 0.9866
Reynolds number Re	1030 - 15370
Richardson number R_o	0.019 - 0.797

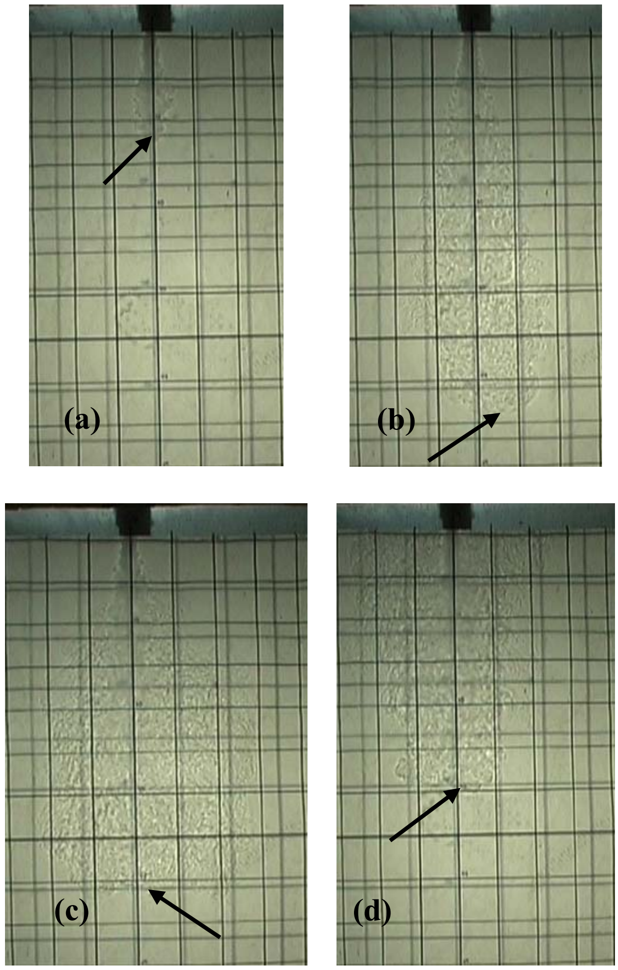

Fig. 2. Evolution of penetration depth of a buoyant jet (Ro=0.025). (a) Starting jet, (b) maximum penetration depth, (c) reversing flow, and (d) terminal penetration depth. Grid spacing is 5cm.

When the buoyant jet is plume-like, at higher initial Richardson numbers, the penetration depth is quite limited, and mixing with ambient fluid is quite marginal. In Fig. 3 we can observe the evolution of such a flow. The maximum penetration depth is about 7.5cm, while the terminal penetration depth stabilizes at around 4.5cm from the nozzle.

The evolution of the terminal penetration depth of round buoyant jets is depicted in Fig.4. It is evident that after the abrupt start up of a jet, in a short time it reaches the maximum penetration depth Z_m. Then the flow reverses and folds back up, while the penetration depth is reduced, and subsequently it oscillates around the terminal penetration depth Z. The density difference in all experiments was 10.5gr/l, and jet diameter 1.5cm. Smaller initial jet Richardson numbers R_o gave bigger penetration depths, since the jet initial momentum was quite high. This transient behaviour has also been observed in earlier investigations [14, 17]. Following [14], we plot the normalized penetration depth $z(t)/l_M$ as a function of the normalized time tB/M in Fig. 5. All the data plotted correspond to round jets, with different diameters and initial conditions, so that R_o's are different. From Fig. 5 we can observe the following: (i) Buoyant jets at all R_o reach Z_{max}, then the penetration depth is reduced, and oscillates around the terminal penetration depth Z. (ii) The greater the initial jet R_o, the lower the penetration depth is. High R_o's seem to give a normalized Z/l_M around 1.3.

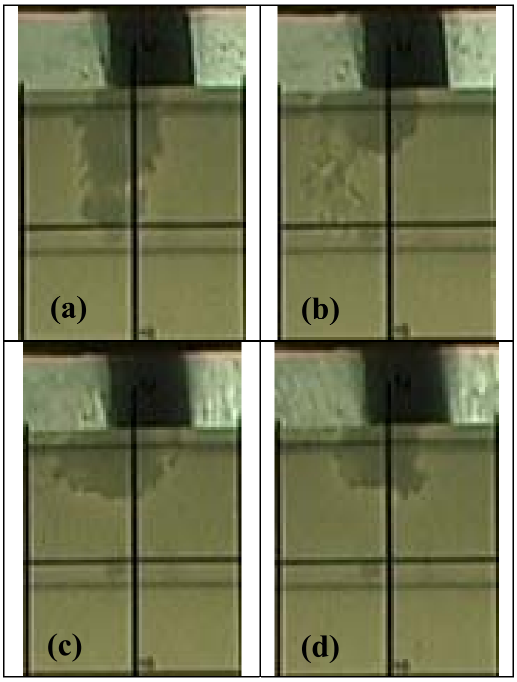

Fig. 3. Evolution of penetration depth of a plume (R_o=0.295). (a) maximum penetration depth, (b), (c) reversing flow, and (d) terminal penetration depth. Grid spacing is 5cm. Lighter grid lines are the projections of the front glass panel grid.

3.1 Terminal penetration depth

The normalized terminal penetration depth of round (buoyancy preserving) buoyant jets is plotted in Fig. 6 as a function of the initial R_o. For R_o < 0.2-0.3, the normalized penetration depths Z/l_M and Z_{max}/l_M seem to take constant values around 2.0 and 3.0 respectively, result which is congruent with dimensional analysis. For R_o > 0.3, Z/l_M is drastically reduced. Therefore, for round vertical fountains we may state that the terminal penetration depth oscillates around the length $2l_M$, while $Z_{max} \cong 3l_M$. The experimental values obtained in the present investigation are quite lower from those of [17], close to those reported by [6, 20] and higher from those measured by [4, 14]. We believe that a value of constant C around 2 is reasonable, because the size of the tanks in the present investigation and those by [6, 20] are about the same, and they are quite large, with small boundary effect on the dynamics of the negatively buoyant jet. In the same plot we have plotted the experimental data normalized as Z/l_Q. We can observe a change in slope for R_o>0.3, indicating faster reduction of Z with R_o.

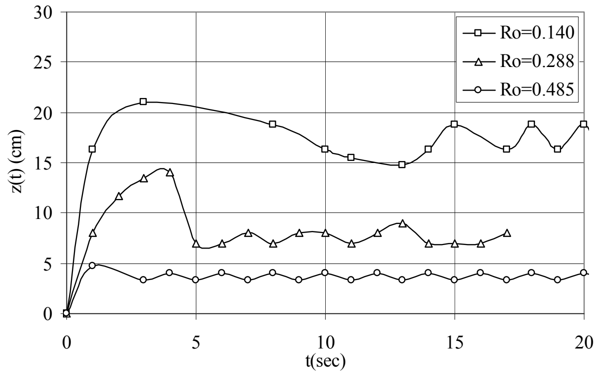

Fig.4. Temporal evolution of penetration depth for negatively buoyant jets, D=1.5cm, at different R_o.

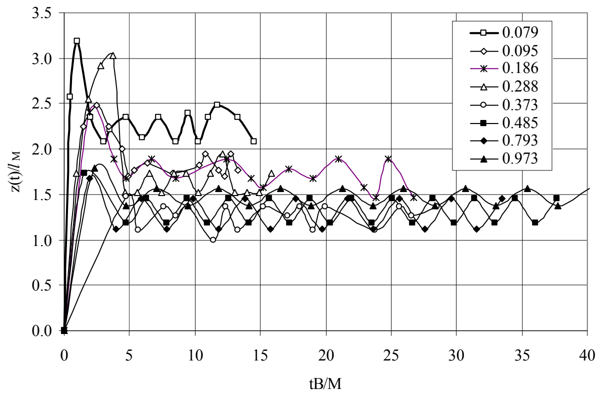

Fig.5 Evolution of the normalized penetration depth of round jets as a function of dimensionless time for various R_o.

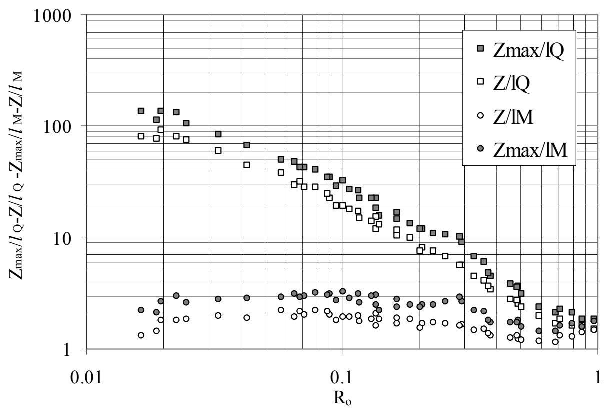

Fig.6. Normalized maximum and terminal penetration depths for round jets, versus initial Richardson number.

The results regarding buoyancy preserving non circular jets are plotted in Fig. 7, where we have plotted Z/l_M and Z_{max}/l_M versus R_o, for jets with orthogonal, square and triangular sections. One may note that negatively buoyant jets with triangular and square nozzle sections, penetrate around two characteristic lengths ($2l_M$) into the ambient, while the maximum penetration is slightly below $3l_M$. The jets with an orthogonal 2:1 nozzle section penetrate at a shorter depth, meaning that they entrain more ambient fluid than the circular, square or triangular ones. Buoyant jets from the same nozzle in a linear density gradient have shown similar behaviour, since their lateral spreading elevation was at lower levels, if compared to that for round jets [10]. Elliptical pure jets with 2:1 axis ratio [8] have also been found to entrain ambient fluid at higher rates, if compared to round ones. This result might be a suggestion towards the direction of enhancing mixing by buoyant jets, using more efficient nozzle shapes. From Fig. 7, it is evident that penetration depth is reduced, when R_o>0.3.

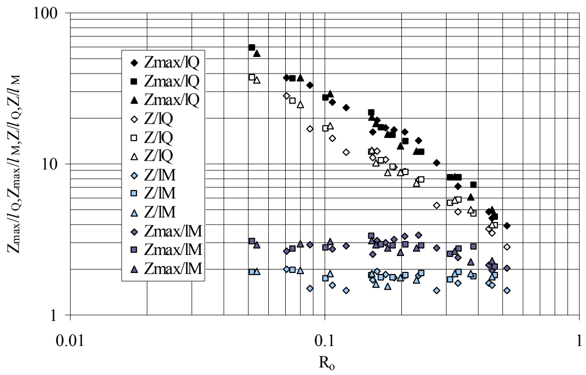

Fig. 7. Normalized maximum and terminal penetration depths of non circular nozzles, ◊-orthogonal, □-square, and Δ-triangular nozzle.

When we use vertical hot water round jets with reversing buoyancy, the initial jet buoyancy flux cannot be preserved. When the jet mixes with ambient fluid, its temperature, and subsequently the coefficient of thermal expansion of water is reduced, resulting in reduced local buoyancy flux. Therefore, the buoyancy force, opposing the jet movement, is reduced. One could expect a dramatic increment of coefficient C in equation 6, but it is not the case. Plotting the normalized penetration depth Z/l_M as a function of R_o, we observe that for R_o<0.3, C ≈ 2, from Fig. 8. This is not any different from the constant we obtained for round saltwater jets in fresh water. Thus, we can state that hot water negatively buoyant jets, behave as negatively buoyant jets which preserve buoyancy. This is probably due to the fact that the jet turns into a plume-like flow after a short distance from the nozzle, and the reversing buoyancy that has reduced the initial jet momentum drastically, takes over.

The maximum penetration depth in the case of hot water jets could not be obtained, since the jet plenum was initially filled with cold water, once we fed it with constant temperature hot water. After start up, when B = 0, there was a transient jet of increasing temperature, thus increasing B for a short time, during which l_M wasn't constant. Thus Z_{max} couldn't be obtained, while Z was obtained once the jet temperature became constant. It is evident from Fig. 8 that the constant C is reduced, when R_o>0.3.

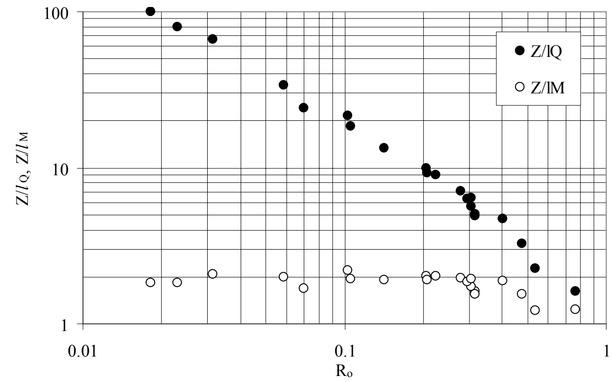

Fig. 8. Normalized terminal penetration depth versus initial Richardson number, in a heated jet.

The measurements of [20] were extended to R_o ≈ 3, initial Richardson number which is very high, meaning that the length scale l_M is a fraction of the jet nozzle diameter. Efficient mixing under these circumstances cannot be obtained since Z/l_M<1 (Fig. 1), meaning that jet penetration doesn't exceed one jet diameter.

4. Conclusion

Turbulent fountains of buoyancy preserving fresh water buoyant jets into salt water, and hot water jets into cold water have been investigated experimentally. The maximum Z_{max}/l_M and the steady state terminal normalized penetration Z/l_M for jet-like flows initially (R_o<0.1), were measured to be 3 and 2 respectively, regardless of the buoyancy preserving or non preserving type of the flow. The results are in agreement with those reported in earlier investigations [6, 20], regarding round jets. The same experimental constants were found for jets with square and equilateral triangular nozzles, with rounded corners. In the case of rectangular jet nozzles with 2:1 side ratio and rounded corners, the constants of proportionality have been found lower from the previous, meaning that this type of nozzle is more efficient as a mixing mechanism, result that is congruent with earlier experiments [8, 10].

We can apply the entrainment coefficients reported earlier [15] for jets (α_j=0.0545) and plumes (α_p=0.0875), accounting for the fact that they are functions of the local buoyant jet Richardson number [7], and the 1/e jet width ratio λ, varying from 1.2 in jets to 1.067 in plumes, to integrate the equations of motion [7]. Using exponential time-averaged velocity and excess density profiles, the constant C is found to be 1.78, result which is congruent only with [4] earlier data. The constant of proportionality C=2 can only be obtained numerically, upon reduction of the jet entrainment coefficient to α_j=0.04.

It would be of great interest to investigate the velocity field of negatively buoyant jets, by means of particle image velocimetry, in order to verify the entrainment and jet width constants to be used, for the numerical prediction of the terminal penetration depth and the average dilution, of turbulent vertical fountains in a homogeneous calm ambient.

References:

[1] Abraham, G. Jets with negative buoyancy in homogeneous fluids. *J. Hyd. Res.*Vol. 5, No. 4, 1967, pp. 235-248.

[2] Baines, W.D., & Chu, V.H. Jets and plumes. *Environmental hydraulics*, (Singh, V.P., & Hager, W.H., Editors), Water Science and Technology Library, pp. 7-61.

[3] Baines, W.D., Turner, J.S., and Campbell, I.H. Turbulent fountains in an open chamber. *J. Fluid Mech.,* Vol. 212, 1990, pp. 557-592.

[4] Bloomfield, L.J., & Kerr, R.C.,1999. A theoretical model of a turbulent fountain. *J. Fluid Mech.*, Vol. 389, 2000, pp. 197-216.

[5] Campbell, I.H., & Turner, J.S. Fountains in magma chambers. *J. Petrol*., Vol. 30, 1989, pp. 885-923.

[6] Demetriou, J.D., 1978. *Turbulent diffusion of vertical water jets with negative buoyancy*. (In Greek), Ph.D. Thesis, National Technical University of Athens, 527 pp.

[7] Fischer, H.B., List, E.J., Koh, R.C.Y., Imberger, J., and Brooks, N.H. *Mixing in inland and coastal waters.* Academic Press, 1979.

[8] Ho, C.M. & Gutmark. E., 1987. Vortex induction and entrainment in a small-aspect-ratio elliptic jet. *J. Fluid Mech.*, Vol. 179, pp. 383-405.

[9] Koh, R.C.Y., & Brooks, N.H. Fluid mechanics of waste-water disposal in the ocean. *Annu. Rev. Fluid Mech.*, Vol. 7, 1975, pp 187-211.

[10] Konstantinidou, K. & Papanicolaou, P.N. Vertical round and orthogonal buoyant jets in a linear density-stratified fluid. *XXX IAHR Congress*, 24-29 August 2003, Thessaloniki, Greece, Theme C, Vol. 1, pp. 293-300.

[11] Lindberg, W.R., 1994. Experiments on negatively buoyant jets, with and without cross-flow. NATO, Series E: Applied sciences, Vol. **255**. "*Recent Research Advances in the Fluid Mechanics of Turbulent Jets and Plumes*", P.A. Davies and M.J. Valente Neves Editors, Kluwer Academic Publishers, Dordrecht, 1994, pp. 131-145.

[12] List, E.J. Mechanics of turbulent buoyant jets and plumes. From '*Turbulent buoyant jets and plumes*', W. Rodi Ed., Pergamon, 1982, pp. 1-68.

[13] McClimans, T. & Eidnes, G. Forcing nutrients to the upper layer of a fjord by a buoyant plume. *Proc. 5th Intl. Symp. Stratified Flows*, Vancouver, Canada, 2000, pp. 199-204.

[14] Pantzlaff, L., & Lueptow, R.M. Transient positively and negatively buoyant turbulent round jets. *Exp. Fluids*, Vol. 27, 1999, 117-125.

[15] Papanicolaou, P.N., and List, E.J. Investigations of round vertical turbulent buoyant jets. *J. Fluid Mech*., Vol. 195, 1988, pp. 341-391.

[16] Seban, R.A., Behnia, M.M., and Abreau, K.E. Temperatures in a heated jet discharged downwards. *Int. J. Heat Mass Trans*., Vol. 21, 1978, pp. 1453-1458.

[17] Turner, J.S. Jets and plumes with negative or reversing buoyancy. *J. Fluid Mech*., Vol. 26, 1966, pp. 779-792.

[18] Turner, J.S., & Campbell, I.H. Convection and mixing in magma chambers. *Earth Sci. Rev*., Vol. 23, 1986, pp. 255-352.

[19] Woods, A.W. & Caulfield, C.P. A laboratory study of explosive volcanic eruptions. *J. Geophys. Res*. Vol. 97, 1992, pp. 6699-6712.

[20] Zhang, H., & Baddour, R.E. Maximum penetration of vertical round dense jets at small and large Froude numbers. Technical Note No. 12147, *ASCE, J. Hyd. Eng*., Vol. 124, No. 5, 1998, 550-553.