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THEORETICAL MODEL OF THE DYNAMIC INTERACTION BETWEEN WAGON TRAIN AND CONTINOUS RAIL

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Abstract: The paper reveals the vibrations of a wagon with two bogies having elastic links and dampers between the wagon box and the bogie and also between the wheels and the bogie. The entire system has two vertical planes of symmetry therefore the models has 15 d.o.f. . The railway is considered elastic solid in both planes horizontal and vertical that induces forced vibrations of the wagon. The vibrations are studied using the Language's equations and the Hamilton's principle. By this way it was found for the first time a mathematical approach for the wagon box forced vibrations in connection with the non-linear vibrations of the rail. In the mean time the authors found, by using the Hamilton's principle and the Dirac function, two partial derivatives equations which describe the forced non-linear vibrations at the rail in two planes namely: vertical and horizontal.

Key words: wagon vibrations, continuous rail

1 Introduction

With the continuos increase at the speed for the railway transport it is necessary to increase the comfort and the safty of the railway traffic. Because of these demands it was developed the research in the area of forced vibrations of the wagon train. These vibrations are due to the geometrical imperfections of the rails and produce, in the case of constant speed of the wagon, the following vibrating movements: vertical, rolling, pitching and gyration.

Also the rail is a deformable solid in both vertical and horizontal plane, having a distributed elasticity and damping in these planes [5]. In the mean time it was taking into account, for the vibrations of the rail, the effects of rotary inertia and shear stress.

The wagon is with one suspended stage, its box being elastic mounted into the bogies. Also the suspension and the anti-gyrations elastic elements have viscous dampers. The paper reveals the forced damped vibrations of the system wagon-rail in interaction.

2 Basic Model and Hypothesis

The wagon model (see figure 1) is composed of two rigid bogies 1 and 2, having the mass m_1=m_2, the wagon box 3 of mass m_3 and the wheels "left", "right" identical 4,5,6 and 7.

The bogies and the box at the wagon have two planes of symmetrie and the rails are simetric in geometrical rotations, being deformable in both horizontal and vertical planes. The rails have distributed damping c_y and c_z in the vertical and horizontal planes, the model of the rails being illustrated in figure 2. The model consists into clamped beams at the ends. The links between the bogies and the wagon box, together with the links between the wheels axes and the bogies are identical in vertical and horizontal planes, their mechanical characteristics being:

- stifness: k, k' for the system bogies-wagon and k_1, k'_1 for the system wheel axes-bogie;
- viscous damping coefficients: c, c' for the system bogies-wagon and c_1, c'_1 for the system wheel axes-bogie;
- mass moments of inertia: $J_1=J_2$ (axes c_{1z}, c_{2z}), $J'_1=J'_2$ (axes c_{1x}, c_{2x}), $J''_1=J''_2$ (axes c_{1y}, c_{2y}) for the bogies; J_3 (axe c_{3z}), J'_3 (axe c_{3x}), J''_3 (axe c_{3y}) for the wagon box, where c_1, c_2 and c_3 are the mass centers.

The model has 15 degree of freedom (d.o.f.) as it follows:

- the coordinates y_1, y_2, y_3 are the vertical displacements of the mass centers;
- $\varphi_1, \varphi_2, \varphi_3$ are the rotation due to the pitching movement;
- $\theta_1, \theta_2, \theta_3$ are the rotation due to the rolling movement;
- z_1, z_2, z_3 are the coordinates of the mass center for the gyration movement;
- ψ_1, ψ_2, ψ_3 are the rotation due to the gyration movement;

The rail has: the mass moments of inertia J'_{zo}, J'_{yo} ;I_x is the geometric moment of inertia; E is the Young's modulus and the distance between two clamped ends is $L_0=19.2m$.

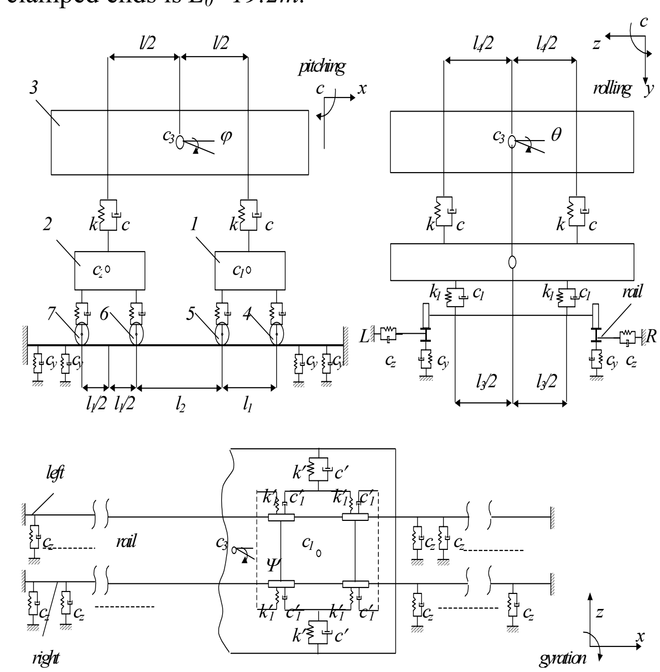

Figure 1

The geometrical imperfection of the rail into vertical plane y_s are due to the static and dynamic loads [1], [2], [3] as well as the imperfections into the horizontal plane z_s. It was considered that the wheels are in permanent contact with the rail.

The adapted model is a mechanical system with holonomic and scleronomic links, on which is acting elastic forces, viscous damping forces and perturbating forces generated by the imperfections and the deformations of the rail .It is considered that all the wheels are in permanent contact with the rails only inside the length L_0.

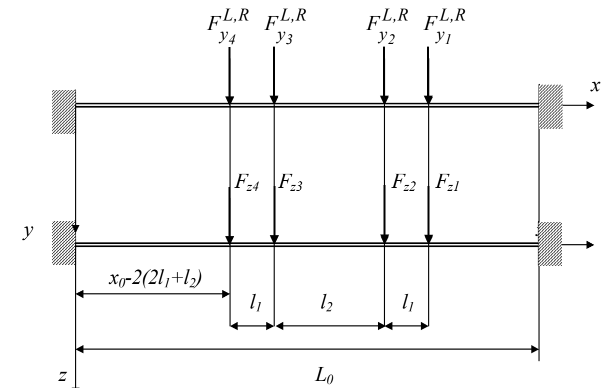

Figure 2.

3 The Equations of Vibrating Movement of The Wagon

Using the Lagrange's equations, the compact matrix equation for the small vibrations is

$$[M]\{\ddot{q}\}+[D]\{\dot{q}\}+[K]\{q\}=\{F\}, \tag{1}$$

by denoting:

- the vector of generalized coordinates

$$\{q\}^T=\{y_1 y_2 y_3 z_1 z_2 z_3 \theta_1 \theta_2 \theta_3 \psi_1 \psi_2 \psi_3 \varphi_1 \varphi_2 \varphi_3\} \tag{2}$$

- the inertia matrix

$$[M]=\begin{bmatrix} [m][o][o][o][o] \\ [o][m][o][o][o] \\ [o][o][J][o][o] \\ [o][o][o][J'][o] \\ [o][o][o][o][J''] \end{bmatrix} \tag{3}$$

where

$$[m]=\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_1 & 0 \\ 0 & 0 & m_3 \end{bmatrix};\ [J]=\begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_1 & 0 \\ 0 & 0 & J_3 \end{bmatrix} \tag{4}$$

and similar $[J'], [J'']$,

- the damping matrix

$$[M]=\begin{bmatrix}[D_y] & [o] & [o] & [o] & [D_{y\varphi}]\\ [o] & [D_z] & [o] & [D_{z\psi}] & [o]\\ [o] & [o] & [D_\theta] & [o] & [o]\\ [o] & [D_{z\psi}]^T & [o] & [D_\psi] & [o]\\ [D_{y\varphi}]^T & [o] & [o] & [o] & [D_\varphi]\end{bmatrix} \qquad (5)$$

where

$$[D_y]=\begin{bmatrix}4c_1+2c & 0 & -2c\\ 0 & 4c_1+2c & -2c\\ -2c & -2c & 4c\end{bmatrix};$$

$$[D_z]=\begin{bmatrix}4c_1'+2c' & 0 & -2c\\ 0 & 4c_1'+2c' & -2c'\\ -2c' & -2c' & 4c'\end{bmatrix};$$

$$[D_\psi]=\begin{bmatrix}c_1'I_1^2 & 0 & 0\\ 0 & c_1'I_1^2 & 0\\ 0 & 0 & c_1'I_1^2\end{bmatrix}; \qquad (6)$$

$$[D_\theta]=\begin{bmatrix}I_3^2(c_1+c/2) & 0 & -cI_3I_4/2\\ 0 & I_3^2(c_1+c/2) & -cI_3I_4/2\\ -cI_3I_4/2 & -cI_3I_4/2 & -cI_4^2\end{bmatrix}$$

$$[D_\varphi]=\begin{bmatrix}c_1I_1^2 & 0 & 0\\ 0 & c_1I_1^2 & 0\\ 0 & 0 & c_1I_1^2\end{bmatrix};[D_{y\varphi}]=\begin{bmatrix}0 & 0 & -cI\\ 0 & 0 & -cI\\ 0 & 0 & 0\end{bmatrix}$$

$$;[D_{z\psi}]=\begin{bmatrix}0 & 0 & -c'I\\ 0 & 0 & -c'I\\ 0 & 0 & 0\end{bmatrix}$$

- the stifness matrix

$$[K]=\begin{bmatrix}[K_y] & [o] & [o] & [o] & [K_{y\varphi}]\\ [o] & [K_z] & [o] & [K_{z\psi}] & [o]\\ [o] & [o] & [K_\theta] & [o] & [o]\\ [o] & [K_{z\psi}]^T & [o] & [K_\psi] & [o]\\ [K_{y\varphi}]^T & [o] & [o] & [o] & [K_\varphi]\end{bmatrix} \qquad (7)$$

where $[K_y]$, $[K_z]$, $[K_\theta]$, $[K_\psi]$, $[K_\varphi]$, $[K_{y\varphi}]$, $[K_{z\psi}]$ are similar with $[D_y]$, $[D_z]$, $[D_\theta]$, $[K_\psi]$, $[D_\varphi]$, $[D_{y\varphi}]$, $[D_{z\psi}]$ if is replace c by k, c' by k', c_1 by k_1 and c_1' by k_1';

- the column matrix at the perturbating forces $\{F\}$ is

$$\{F\}=\left\{\{F_y\}^T\{F_z\}^T\{F_\theta\}^T\{F_\psi\}^T\{F_\varphi\}^T\right\}^T \qquad (8)$$

where

$$\{F_y\}=\left\{\sum_{i=4}^{5}\left[k_1\left(y_{si}^L+y_{si}^R\right)+c_1\left(\dot{y}_{si}^L+\dot{y}_{si}^R\right)\right],\right.$$
$$\left.\sum_{i=6}^{7}\left[k_1\left(y_{si}^L+y_{si}^R\right)+c_1\left(\dot{y}_{si}^L+\dot{y}_{si}^R\right)\right],0\right\}^T \qquad (9)$$

$$\{F_z\}=\left\{\sum_{i=4}^{5}\left[2k_1'(z_{si})+2c_1'(\dot{z}_{si})\right],\right.$$
$$\left.,\sum_{i=6}^{7}\left[2k_1'(z_{si})+2c_1'(\dot{z}_{si})\right],0\right\}^T \qquad (10)$$

$$\{F_\theta\}=\left\{\sum_{i=4}^{5}\left[k_1I_3\left(y_{si}^R-y_{si}^L\right)/2+c_1I_3\left(\dot{y}_{si}^R-\dot{y}_{si}^L\right)/2\right],\right.$$
$$\left.\sum_{i=6}^{7}\left[k_1I_3\left(y_{si}^R-y_{si}^L\right)/2+c_1I_3\left(\dot{y}_{si}^R-\dot{y}_{si}^L\right)/2\right],0\right\}^T \qquad (11)$$

$$\{F_\psi\}=\left\{\sum_{i=4}^{5}\left[I_1k_1'\left((-1)^iz_{si}\right)+I_1c_1'\left((-1)^i\dot{z}_{si}\right)\right],\right.$$
$$\left.,\sum_{i=6}^{7}\left[I_1k_1'\left((-1)^iz_{si}\right)+I_1c_1'\left((-1)^i\dot{z}_{si}\right)\right],0\right\}^T \qquad (12)$$

$$\{F_\varphi\}=\left\{\sum_{i=4}^{5}\left[I_1k_1(-1)^i\left(y_{si}^L+y_{si}^R\right)+\right.\right.$$
$$\left.+I_1c_1(-1)^i\left(\dot{y}_{si}^L+\dot{y}_{si}^R\right)\right]/2,$$
$$\sum_{i=6}^{7}\left[I_1k_1(-1)^i\left(y_{si}^L+y_{si}^R\right)+\right.$$
$$\left.\left.+I_1c_1(-1)^i\left(\dot{y}_{si}^L+\dot{y}_{si}^R\right)\right]/2,0\right\}^T \qquad (13)$$

In the relations (9) ÷ (13) $y_{si}^{L,R}$, with $i=\overline{4,7}$, are the deformations of the rails “Left” and “Right”, in the vertical plane, for the contact points with the wheels. Also, z_{si} have the same meaning in the horizontal vibrations of the wagon. The $y_{si}^{L,R}$ and z_{si} deformations are functions of x and t and represent the excitations of the wagon which has a constant speed. To solve the system (1) it is necessary to compute the functions $y_{si}^{L,R}(x,t)$ and $z_{si}(x,t)$. These functions represent the static and dynamic deformations of the rail in both vertical and horizontal planes.

4 The Forced Vibrations Equations of The Rails

To study the forced vibrations of the rails it was considered that the wagon, throughout a rigid contact, is acting into the rails with dynamic and static forces. These forces represent a system $\{F_{yi}^{L,R}\}_{i=\overline{1,4}}$, $\{F_{zi}\}_{i=\overline{1,4}}$ which correspond to the contact with the wheels 4,5,6,7 "Left" and "Right" (see fig.2). The rail is considered to be a beam, having: the length L_0 , clamped at both ends, the mass per unit length m and being loaded by the above-mentioned forces.

Therefore, in vertical plane, taking into account the rotary inertia and the shear stress, by using the Hamilton's principle it was derived the partial derivatives equation of the forced vibrations of rails "Left", "Right":

$$
\begin{aligned}
& EI_x \frac{\partial^4 y_s^{L,R}}{\partial x^4} + \\
& + \left[m + (J'_z k_1 / K) \sum_{i=1}^{4} \delta(x - x_i) \right] \frac{\partial^2 y_s^{L,R}}{\partial t^2} - \\
& - (EI_x / K) \left[c_y + c_1 \sum_{i=1}^{4} \delta(x - x_i) \right] \frac{\partial^3 y_s^{L,R}}{\partial t \partial x^2} - \\
& - [J'_z + (mEI_x)/K] \frac{\partial^4 y_s^{L,R}}{\partial t^2 \partial x^2} + \\
& + (mJ'_z / K) \frac{\partial^4 y_s^{L,R}}{\partial t^4} + \\
& + (J'_z / K) \left[c_y + c_1 \sum_{i=1}^{4} \delta(x - x_i) \right] \frac{\partial^3 y_s^{L,R}}{\partial t^3} + \\
& + \left[c_y + c_1 \sum_{i=1}^{4} \delta(x - x_i) \right] \frac{\partial y_s^{L,R}}{\partial t} - \\
& - k_1 (EI_x / K) \frac{\partial^2 y_s^{L,R}}{\partial x^2} \sum_{i=1}^{4} \delta(x - x_i) + \\
& + k_1 y_s^{L,R} \sum_{i=1}^{4} \delta(x - x_i) = \frac{G_t}{8} \sum_{i=1}^{4} \delta(x - x_i)
\end{aligned} \tag{14}
$$

where $J_{z'} = J_{zo} / L_o$, δ is the Dirac's function, $k = (1.4 \div 2.8) \cdot A \cdot G$ [6], A is the cross-section area of the beam, G is the transversal modulus and G_t is the entire mass force of the wagon. It was considered that in both vertical and horizontal planes the external forces are into a plane witch contains the axe of the beam and each wheel equally supports the mass force of the wagon. Also, from the Hamilton's principle, the boundary conditions are

$$
y_s^{L,R}\Big|_{0,L_0} = 0; \frac{\partial y_s^{L,R}}{\partial x}\Bigg|_{0,L_0} = 0 \tag{15}
$$

In the same manner it was derived the partial derivatives equations of the forced vibrations of the rail into the horizontal plane, obtaining:

$$
\begin{aligned}
& EI_x \frac{\partial^4 z_s}{\partial x^4} + \left[m + (J'_y k'_1 / K) \sum_{i=1}^{4} \delta(x - x_i) \right] \frac{\partial^2 z_s}{\partial t^2} - \\
& - (EI_x / K) \left[c_z + c'_1 \sum_{i=1}^{4} \delta(x - x_i) \right] \cdot \\
& \cdot \frac{\partial^3 z_s}{\partial t \partial x^2} - [J'_y + (mEI_x)/K] \frac{\partial^4 z_s}{\partial t^2 \partial x^2} + \\
& + (mJ'_y / K) \frac{\partial^4 z_s}{\partial t^4} + (J'_y / K) \left[c_z + c'_1 \sum_{i=1}^{4} \delta(x - x_i) \right] \cdot \\
& \cdot \frac{\partial^3 z_s}{\partial t^3} + \left[c_z + c'_1 \sum_{i=1}^{4} \delta(x - x_i) \right] \frac{\partial z_s}{\partial t} - \\
& - k'_1 (EI_x / K) \frac{\partial^2 z_s}{\partial x^2} \sum_{i=1}^{4} \delta(x - x_i) + k'_1 z_s \sum_{i=1}^{4} \delta(x - x_i) = 0
\end{aligned} \tag{16}
$$

where $J'_y = J_{yo} / L_o$, and $k = (1.1 \div 1.6) \cdot A \cdot G$ [6]. The boundary conditions are:

$$
z_s\Big|_{0,L_0} = 0; \frac{\partial z_s}{\partial x}\Bigg|_{0,L_0} = 0 \tag{17}
$$

In equations (14) and (16) x_i represent the coordinates of the contact points between the wheels and the rail, their expressions being:

$$
x_1 = vt;\ x_2 = x_1 - I_1;\ x_3 = x_2 - I_2;\ x_4 = x_3 - I_1 \tag{18}
$$

The key of solving the system (1) is therefore the computation of the solutions of equations (14) and (16).

5 Conclusions

By taking into account that the rails are deformable bodies represents the main contribution of the paper. The equations (14) and (16) enable us the computation of the deformations of the rail which represent the excitations for the wagon.

Another contribution of the paper is that vibrations of the wagon are considered in a complex form of a mechanical system having 15 d.o.f. . By this way it is possible to check in the early stage of design the comfort and the saftey conditions for the railway traffic.

The system (1) could be solved by numerical methods such as Runge-Kutta's method, while the equations (14) and (16) could be solved by a mixed Kautorovich's method coupled with a perturbation method.

Therefore, the paper reveals a complex and usefull algorithm for the CAD stages.

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