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The Effect of Geometric Imperfections on the Amplitude and the Phase Angle of the Non-Linear Dynamic Behavior of Thin Rectangular Plates Parametrically Excited

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Abstract: The paper reveals recent developments of the influence of the geometric imperfections on the amplitude and the phase angle of the non-linear vibrations of thin rectangular plates parametrically excited. In the region of principal parametric resonance, starting from the temporal non-linear differential equation that describes the oscillatory movement and using the second order approximation of the asymptotic method were computed the amplitude and the phase angle as functions of system parameters and geometric imperfections. By varying the intensity of the geometric imperfections was obtained their influence upon the amplitude and the phase angle for the stationary non-linear dynamic response.

Key words: plates, non-linear parametric vibrations

1 Nomenclature

A_1, A_2, B_1, B_2 = unknown functions in asymptotic expansion; C = viscous damping coefficient; D = flexural rigidity of plate; E = Young's modulus; M = coefficient of the non-linear term; $N_y(t)$ = external in-plane loading per unit width; N_{yo} = static in-plane loading per unit width; N_{yt} = amplitude of harmonic in-plane loading per unit width; N_{cr} = critical buckling load of the plate, defined as in [14] pp. 353; a = length of plate in x-direction; b = length of plate in y-direction; f(x,y,t) = Airy's stress function; h = plate thickness;
$s=\Lambda/2\Omega$=frequency parameter;
t=time; w(x,y,t)=lateral midsurface displacement in z-direction; $w_0(x,y)$ = initial geometric imperfection in z-direction; Δ = decrement of damping; $\Lambda(t)$ = instantaneous frequency of the external in-plane excitation, $\Lambda = d\theta/dt$; Ω_q = free vibration circular frequency of a rectangular plate loaded by a constant component of in-plane force; $\underline{\Omega_q}$ =free vibration circular frequency of a rectangular plate , with initial geometric imperfections , loaded by a constant component of in-plane force;
ε = small positive parameter in asymptotic expansion, $0<\varepsilon<<1$; $\theta(t)$ = total phase angle of harmonic excitation; μ = load parameter of the plate; ν = Poisson's ratio; ρ = mass density per unit volume of plate; τ = slowing time in asymptotic analysis; $\psi_p(t)$ = phase angle of the parametric vibration; ω_q = free vibration frequency of unloaded rectangular plate; $\Delta\Delta$ = double iterated Laplace operator in R^2; $\dot{(\ )}$= differentiation with respect to time; $(\ )_{,\xi}$ = partial differentiation with respect to ξ.

2 Introduction

Extensive efforts and considerable amount of research has been concentrated on the prediction of the non-linear dynamic behaviour of rectangular plates with small deviation from flatness called initial geometric imperfection. Excellent reviews on the subject can be found in articles written by Hui [2-8]. Studies of the effect of geometric imperfection on the small-amplitude vibration frequencies of simply supported rectangular plates have been done by Hui and Leissa [2], Ilanko and Dickinson [9] and Bugaru [1].

They found out that geometric imperfections of the order of the plate thickness may raised the vibration frequencies and may even cause the structures to exhibit soft-spring behaviour [7]. The survey of the literature reveals that the work on the subject has been devoted to the investigation of various types of shapes, loadings, and boundary conditions [11-13].

The present work covers an existing gap in our understanding of the parametric resonance of continuous systems and presents a rational analysis of the influence of geometric imperfections upon the amplitude and the phase angle for the stationary non-linear dynamic response.

3 Conceptual Model

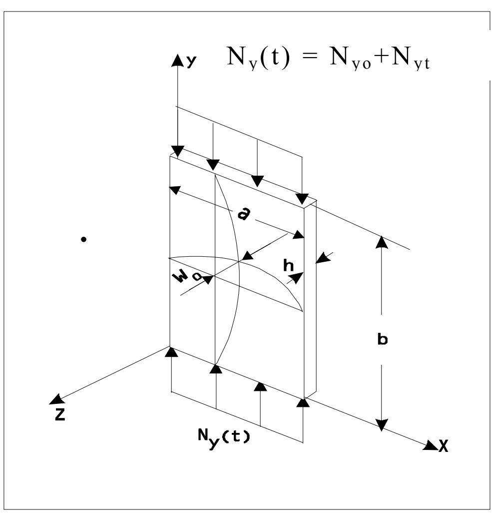


Fig. 1

The model under investigation is an imperfect rectangular plate simply supported along its edges and acted by periodic in-plane forces uniformly distributed along two opposite edges as shown in figure 1.

It is assumed that the plate is of uniform thickness, "stress free", elastic, homogeneous and isotropic and also the plate thickness and the resulting displacements are small compared with the wavelength of lateral vibration in order to be able to use thin plate theory. Consequently, since thin plate theory is used in the analysis, the loading frequencies over which lateral vibrations occur are considerably below the natural frequencies of longitudinal vibrations and in-plane inertia forces can be neglected.

4 Basic Equations

The plate theory used in this analysis may be considered as the dynamic analogue of the Von Karman large-deflection theory and is derived in terms of Airy's stress function, the lateral displacement and the initial geometric imperfection. The differential equations governing the non-linear flexural vibrations of the plate are:

$$\Delta\Delta f=E[((w+w_0)_{,xy})^2-(w_{0,xy})^2-(w+w_0)_{,xx}(w+w_0)_{,yy}+ +w_{0,xx}\cdot w_{o,yy}] \quad (1)$$
$$\Delta\Delta w = h/D[f_{,yy}\cdot(w+w_0)_{,xx} - 2\cdot f_{,xy}(w+w_0)_{,xy} + +f_{,xx}(w+w_0)_{,yy} - \rho\cdot w_{,tt}]$$

where $D = Eh^3/12(1-\nu^2)$.

The boundary stress conditions (in-plane movable edges) are expressed as:

$$f_{,YY} = 0 \text{ and } f_{,XY} = 0 \text{ along } x = 0,a \quad (2)$$
$$f_{,XX} = -N_Y(t) \text{ and } f_{,XY} = 0 \text{ along } y = 0,b$$

The boundary supporting conditions are expressed as:

$$w = w_{,xx} + \nu\cdot w_{,yy} = 0 \quad \text{along } x = 0,a \quad (3)$$
$$w = w_{,yy} + \nu\cdot w_{,XX} = 0 \quad \text{along } y = 0,b$$

The problem consists in determining the functions f and w, for a given function w_0, which satisfy the governing equations (1) together with the boundary conditions (2) and (3).

5 Method Of Solution

Applying the Kantorovich's method to the governing equations (3) as in [1], introducing linear damping and taking one term in the expansion for the lateral displacement, the system is reduced to the following differential equation of motion:

$$\ddot{w} + 2\cdot C\cdot\dot{w} + \bar{\Omega}^2[1 - 2\cdot\mu\cdot(\Omega/\bar{\Omega})^2\cos[\theta(t)]]\cdot w - 2\cdot\mu\cdot\cos[\theta(t)]\Omega^2(w_0+d) + Mw^3 + 3M w^2\cdot(w_0+d) = 0, \quad (4)$$

where d is the amplitude of the static deformation of the plate and

$$\mu_q = N_{yt} / [2(N_{cr} - N_{yo})]. \quad (5)$$

This is a second-order non-linear differential equation with periodic coefficients, which may be considered as an extension of the standard Mathieu-Hill's equation.

6 Solution of the Temporal Equation of Motion

Mathematical techniques for solving such problems are limited and approximate methods are generally used. The method of asymptotic expansion in powers of a small parameter ε, developed by Mitropolskii [10], is a most effective tool for studying non-linear vibrating systems with slowly varying parameters. Assuming that the viscous damping and the nonlinearity are small and the instantaneous frequency of excitation and the load parameter vary slow with the time the equation (5) can be written, by denoting W=w and Θ=θ, in the following asymptotic form:

$$\ddot{W}+\bar{\Omega}^2 \cdot W = \varepsilon[2\mu \cdot \cos[\Theta(T)] \cdot \Omega^2 (W+\dot{W}_0+d) - 2 \cdot C \cdot W - MW^3-3MW^2(W_0+d) \qquad (6)$$

where $\tau = \varepsilon t$ is the "slowing" time. For the second order of approximation in ε, we seek a solution for the equation (6) in the following form:

$$W=W_p(\tau)\cos[((1/2)\Theta+\psi_p)]+\varepsilon\, u(\tau,W_p, \Theta,(1/2)\Theta+\psi_p), \qquad (7)$$

where W_p, ψ_p are functions of time defined by the system of differential equations:

$$\begin{aligned} dW_p/dt &= \varepsilon A_1(\tau , W_p , \psi_p) + \varepsilon^2 A_2(\tau , W_p , \psi_p) \\ d\psi_p/dt &= \Omega - (1/2)\Lambda + \varepsilon B_1(\tau , W_p , \psi_p) + \\ &+\varepsilon^2 B_2 (\tau , W_p , \psi_p) \end{aligned} \qquad (8)$$

and $d\Theta(t)/dt = \Lambda(t)$. Functions u, A_1, A_2, B_1, B_2 are selected in such a way that the W , given by (7), will represent a solution of the equation (6) , after replacing W_p and ψ_p by the functions defined in the system (8).

Following the general scheme of constructing asymptotic solutions and performing numerous transformations and manipulations, we can finally arrive at a system of equations describing the nonstationary response of the discretized system. By integrating this system of equations, amplitude W_p and phase angle ψ_p can be obtained as functions of time. The solution W of the equation (6) is

$$\begin{aligned} W(t) &= W_p \cos\left((1/2)\Theta + \psi_p\right) - \\ &- [(\mu\Omega^2)/(\Lambda(\Lambda + 2\bar{\Omega}))]W_p \cos\left((3/2)\Theta + \psi_p\right) + \\ &+ [M/(32\bar{\Omega}^2)]W_p^3 \cos\left((3/2)\Theta + 3\psi_p\right) - \\ &- [2\mu\Omega^2/(\Lambda^2 - \bar{\Omega}^2)](W_o + d)\cos\Theta - \\ &- [3M/(2\bar{\Omega}^2)](W_o + d)W_p^2 + \\ &+ [M/(2\bar{\Omega}^2)](W_o + d)W_p^2 \cos\left(\Theta + 2\psi_p\right) \end{aligned} \qquad (9)$$

The solution (9) was computed for the region of principal parametric resonance. The parametric resonance occurs when the excitation frequency is approximately equal to twice the natural frequency and can be written as:

$$\Lambda \cong 2\,\bar{\Omega} \qquad (10)$$

Analysing relation (9), the paper reveals, for the first time, new terms not yet mentioned by the researchers in the field.

7 Stationary Response

The stationary response given by the amplitude W_p and the phase angle ψ_p, associated with the assumed spatial forms of vibration of our system, may be computed as a special case of the nonstationary motion in the resonant regime described by the systems of equations (8) and (9). As mentioned by Ostiguy and Nguyen [12, 13] the solution for simply-supported plates indicates the presence of principal parametric resonances, the possibility of internal resonances and the occurrence of simultaneous resonances but precludes the possibility of combination resonances. As can be seen in relation (9), the authors founded for the first time, with analytical tools, the influence of the geometric imperfections in the regions of forced, sub-harmonic and supra-harmonic parametric resonances. In this way was found theoretical the presence of internal resonances and the occurrence of simultaneous resonances already mentioned experimentally by Ostiguy and Nguyen.

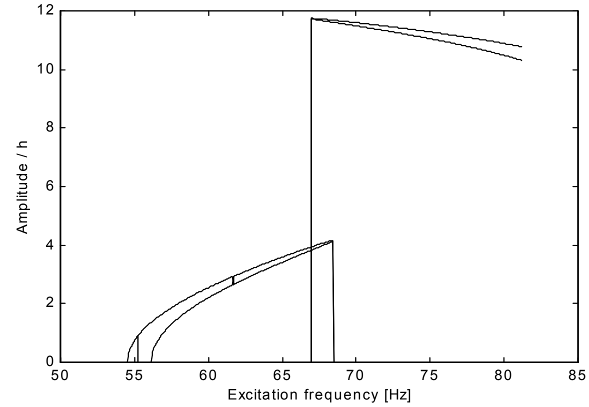
Stationary principal parametric response, associated with various spatial forms of vibration, are given by the system (8) setting $\varepsilon = 1$, $dW_p/dt = 0$, $d\psi_p/dt = 0$ and eliminating ψ_p from this system of equations. Thus the stationary amplitude W_p can be obtained as function of external excitation frequency and represents the solution of the following equation

$$\sum_{i=1}^{7} \beta_i W_p^{12-2(i-1)} = 0 \qquad (11)$$

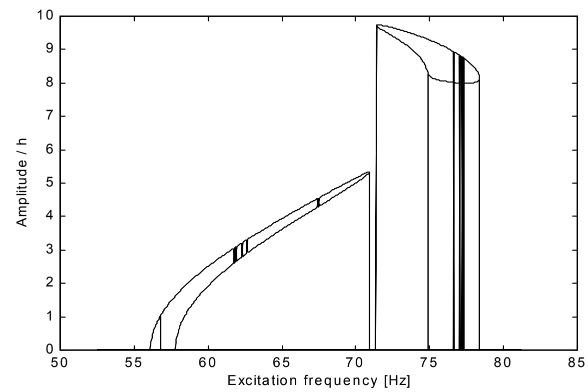
As mentioned by Ostiguy and Evan-Iwanowski [11] the base width of the stationary parametric response is the only region in which vibrations may normally initiate. The phase angle of the stationary parametric response can be obtained from the same system (8) setting $dW_p/dt = 0$, $d\psi_p/dt = 0$ and eliminating the amplitude W_p. By this way was obtained the stationary phase angle in the region of principal parametric resonance from the following equation:

$$\begin{aligned}\psi_p = &\frac{1}{2}\arcsin\{[C-\frac{3}{8}(CM/\overline{\Omega}^2)W_p^2]/\\ &/[(\frac{1}{32})[(\mu\Omega^2 M(\Lambda^2+2\Lambda\overline{\Omega}+72\overline{\Omega}^2))\\ &/(\Lambda(\Lambda+2\overline{\Omega})(4\overline{\Omega}-\Lambda)\overline{\Omega}^2]W_p^2-\\ &-[(6\mu\Omega^2 M)/(\Lambda(\Lambda^2-\overline{\Omega}^2))](W_o+d)^2-\\ &-\frac{\mu\Omega^2}{\Lambda}]\}\end{aligned} \qquad (12)$$

Equations (11) and (12) make it possible to compute the stationary response of the plate at the principal parametric resonance by taking into account the geometrical imperfections of the plate.



a. w_o / h = 0.1



b. w_o / h = 0.6

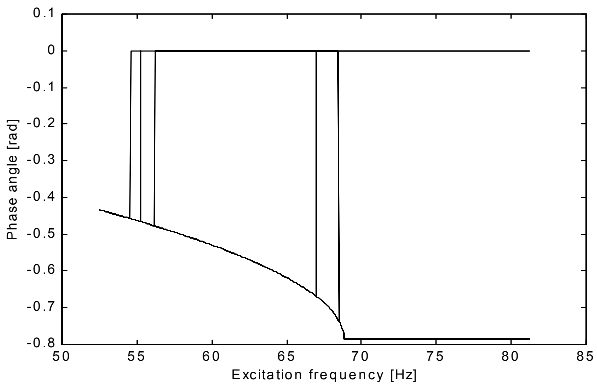
Fig. 2

8 Results and Discussions

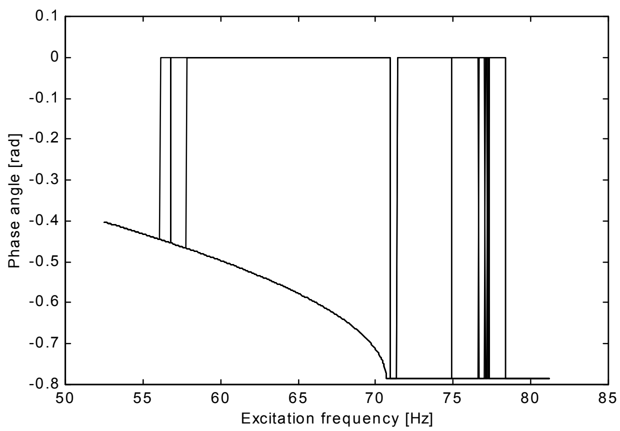
For the computer programs developed to obtain the numerical results the authors used the soft packages MATLAB.

In order to get more insight into various aspects of the problem and to highlight the influence of the initial geometric imperfections on the non-linear dynamic response of rectangular plates, numerical evaluation of the solution were performed for a wide variety of cases. The results shown in figures 2 and 3 are typical of those obtained.

For Δ=0.12 were founded the amplitude and the phase angle of the vibrations for the plate subjected to parametric excitation having moderate imperfections (w_o/h=0.1) and large ones (w_o/h=0.6).By regarding the above-mentioned figures we can conclude that by increasing the imperfections appears the phenomena of simultaneous resonances mentioned by Nguyen [13]. This phenomena manifests itself by multiple salts and the effect of "soft spring" in the area of [65,85] Hz. This was determined for the first time theoretical while Nguyen discovered it experimentally. Also from figure 3 we see that in the area of simultaneous resonances the phase angle is constant and in the mean time all over the area is negative therefore the non-linear dynamic response of the plate is in advance with regard to the excitation.



a. w_o / h = 0.1



b. w_o / h = 0.6

Fig. 3

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