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Noise Radiated by Vibrating Rectangular Plate

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Abstract: Vibrating plates produce noise, which is of high interest in many technical applications. In most cases such plate vibrations have relatively large amplitudes and only a nonlinear von Kármán type model can be used. The rectangular plate vibrations are determined both numerically and experimentally. The first aim is to identify the parameters of the real boundary conditions, which are elastically clamped. The control parameters are elastic bending stiffness of the four edges of the plate as functions of vibration amplitude. Influences of actuator amplitude and excitation position on the plate are presented. Using the optimized model, spatial sound directivity diagrams are determined, based on a numerical procedure. Experiments in the anechoic chamber are used to further improve the model. By identification of various parameters, a better control of plate noise radiation is achieved.

Key words: von Kármán plate, sound directivity diagram.

1 Introduction

The linear Love-Kirchhoff model for plate vibrations uses the small displacements hypothesis, which implies limitations for plate response to excitations of technical interest. Maximum plate displacements are in many cases of the same order as the plate thickness or even greater. The dynamic response depends on plate amplitudes and consequently, the von Kármán plate model is best adapted in this case. The lowest frequency and mode shape as function of maximum displacement are used in this paper in order to determine the radiated sound pressure of the plate.

2 Moderately Large Vibrations of A Plate

A thin rectangular plate of length *a* in the x direction, width *b* in the y direction and thickness *h* in the z direction is considered in this paper. The plate is assumed to be homogeneous of mass density *ρ*. The displacement field for the points belonging to the plate is assumed to be of von Kármán type, with *u* and *v* in-plane displacements and *w* as transverse displacement:

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) - z w_{,x} \\ v(x,y,z,t) &= v_0(x,y,t) - z w_{,y} \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \qquad (1)$$

in which the subscript $_0$ indicates the displacements for points belonging to the reference plane.
The von Kármán dynamic differential equations for free vibrations, are [(1)]:

$$\begin{aligned} L(w,F) &= D\nabla^4 w + \rho h w_{,tt} - \\ &- h\left(w_{,xx}F_{,yy} + w_{,yy}F_{,xx} - 2w_{,xy}F_{,xy}\right) = 0 \end{aligned} \qquad (2)$$

$$\nabla^4 F = E\left(w_{,xy}^2 - w_{,xx}w_{,yy}\right) \qquad (3)$$

in which F is the Airy stress function, E is the Young modulus and $D = \frac{Eh^3}{12\left(1-\nu^2\right)}$.

The four edges of the plate are considered elastically clamped in the transverse direction, so the boundary conditions are

$$\begin{aligned} & w=0;\ w_{,x}=\frac{D}{k}\left(w_{,xx}+\nu w_{,yy}\right) \\ & \text{at } x=-a/2 \text{ and } x=a/2 \\ & w=0;\ w_{,y}=\frac{D}{k}\left(w_{,yy}+\nu w_{,xx}\right) \\ & \text{at } y=-b/2 \text{ and } y=b/2 \end{aligned} \tag{4}$$

in which k is the plate edge bending stiffness. For the in-plane conditions, the normal stresses are zero and edges are uniformly displaced:

$$\begin{aligned} & F_{,xy}=0,\ u_0=const. \\ & \text{at } x=-a/2 \text{ and } x=a/2 \\ & F_{,xy}=0,\ v_0=const. \\ & \text{at } y=-b/2 \text{ and } y=b/2 \end{aligned} \tag{5}$$

Equations (2), (3) are solved using the Galerkin method, assuming the displacement function:

$$w=h\Phi(t)W(x,y) \tag{6}$$

in which $\Phi_{\max}=\frac{w_m}{h}$, with w_m the maximum transverse displacement of the plate.

The Airy stress function can be expressed as a Fourier series, using the boundary conditions and the resulting differential equation of transverse motion is

$$\begin{aligned} & \rho h^2\Phi_{,tt}+\frac{16\pi^4 Dh}{9b^4}\left(3+2\lambda^2+3\lambda^4\right)\Phi- \\ & -\frac{32\pi^4 Eh^4}{9a^2b^2}\left(d_{01}+d_{10}+d_{11}+\tfrac{1}{2}d_{12}+\right. \\ & \left.\tfrac{1}{2}d_{21}++d_{02}+d_{20}\right)\Phi^3=0 \end{aligned} \tag{7}$$

in which the parameters d_{ij} are obtained from the given boundary conditions.

The Duffing equation (7) can be numerically integrated for initial conditions $\Phi(0)\in(0,3)$; $\dot{\Phi}(0)=0$ and the period of the free vibrations can be obtained from the time history of motion. The period T_0 of small vibrations in these conditions is taken as reference and the period ratio T/T_0 has been plotted in Fig. 1 as function of the relative maximum normal displacements w_m/h for a square plate having different boundary stiffness.

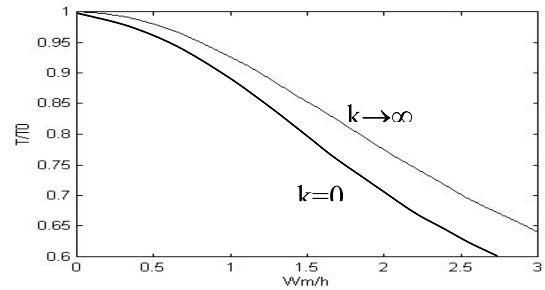

Fig.1: Period of plate vibrations vs. relative vibration amplitude for two boundary rigidities.

3 Noise Radiated By The Vibrating Plate

The vibrating plate is radiating sound in air of mass density ρ_0. The acoustic pressure in a point $P(R,\theta,\varphi)$ for z > 0, the domain in which the radiation takes place (Fig. 2), is given by the Rayleigh formula [2,3].

$$p(R,\theta,\varphi)=\frac{\rho_0 e^{ikR}}{2\pi R}\int_0^a\int_0^b \ddot{w}(x,y)e^{ik\sin\theta(x\cos\varphi+y\sin\varphi)}dydx$$

The far field (R>>a) is determined, thus the distance between a point P in space and any point of the plate is considered the same, denoted by R.

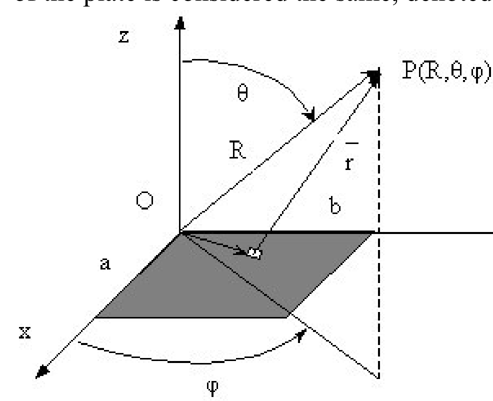

Fig. 2. Spherical coordinates used to determine the sound pressure and level.

The sound level can be numerically determined for an elastically clamped plate of steel, for which the material constants are E=2.1·10[11], ν=0.33. Plate dimensions are 0.98× 0.48×0.0006 mm and the plate is harmonically excited at 16.2 Hz, which is the natural frequency of this plate if it would be rigidly clamped and would have small displacements. is presented in fig. 3. Plate dimensions are 0.98× 0.48×0.0006 mm and the plate is harmonically excited at 16.2 Hz, which is the natural frequency of this plate if it would be rigidly clamped and would have small displacements, this for reference purposes(4).

Higher amplitude of the plate vibrations has a direct influence on the calculated noise level for the

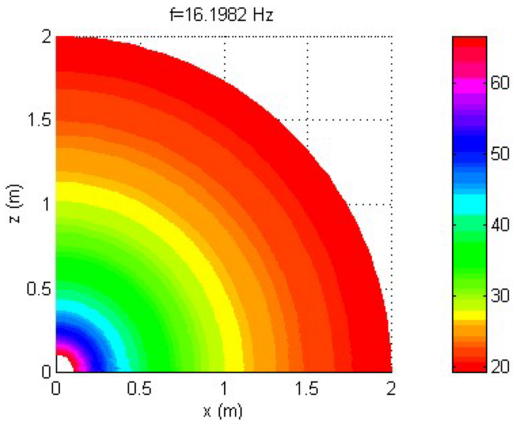

Fig. 3. Level of noise radiated in the far field by a clamped rectangular plate. Maximum amplitude 1 mm

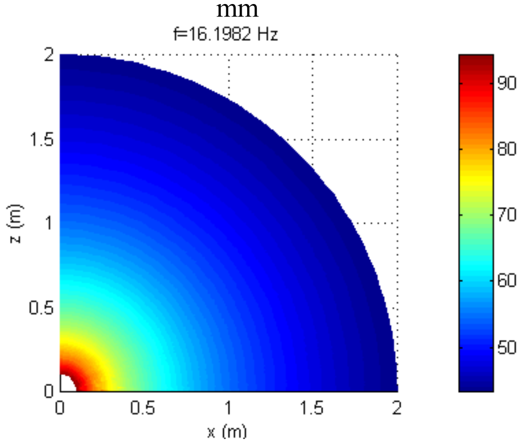

Fig. 4. Level of noise radiated in the far field by a clamped rectangular plate. Maximum amplitude 5 mm

rigidly clamped plate. In fig.4 is presented the noise level in the far field in this case, which is some 30 dB higher.

Higher rigidity of the clamped boundaries has as effects an increased frequency and noise level, provided that the same maximum displacements can be obtained. This requires sensibly higher energy of the actuator. If other external perturbations such as non-uniform boundary conditions along all edges can be avoided, this procedure can be used to control the noise level by variations of boundary rigidities.

4 Experimental Data

To verify the theoretical computations were carried out experiments in the anechoic chamber at the Department of Mechanics in University POLITEHNICA of Bucharest.

The system acquisition data is composed of:

a. Microphone Bruel&Kjaer 4189-L-1 of ½" with the range 5Hz-20kHz;
b. Pulse system Bruel&Kjaer 3560D with 6 channels (see fig. 5);
c. Lap-Top HP with specialized soft packages in order to realize the acoustic spectral analysis (see fig. 5).

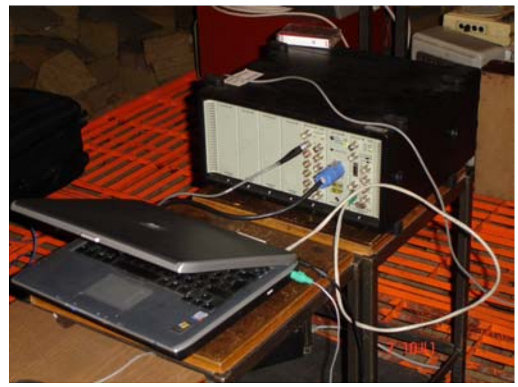

Fig. 5. Pulse system Bruel&Kjaer 3560D and Lap-Top HP.

To excite the plate considered previous it was used:

a. electro-magnetic hammer RFT 101(see fig. 6);
b. signal generator with amplifier Bruel&Kjaer 2706-1027 (see fig. 7).

Fig. 6. Electro-magnetic hammer RFT 101.

Fig. 7. Signal generator with amplifier Bruel&Kjaer 2706-1027

The experiments were carried out in view of testing the practical possibilities of generating infra-sounds in the range 6 Hz-16 Hz[10]. The results of spectral analysis of the sound generated by a rectangular plate harmonically excited, with the dimensions 0.98× 0.48×0.0006 mm (see fig. 8), are presented in the figures 9-14.

Fig. 8. Measurement system

As can be seen from figure 8 the microphone was positioned at 0.1-0.2 m distance from the plate.
Analyzing the data presented in figures 9-14 one can conclude:

a. the acoustic level increases from 57.4 dB at 6 Hz excitation frequency to 98.5 dB at 16 Hz;
b. excepting the peaks obtained at the excitation frequency *f*, supra-harmonics as $2f$ and $3f$ manifest its self;
c. there exist excitation frequency where the supra-harmonics have grater peaks as the fundamental excitation frequency such as: 6 Hz, 8 Hz, 10Hz;
d. the previous aspect it does not manifest starting from 12 Hz excitation frequency;
e. 90 % of the spectral density of acoustic energy it is obtained till the third supra-harmonic;
f. there is a close agreement between the theoretical data and the experimental one at 16 Hz excitation frequency (see fig. 14).

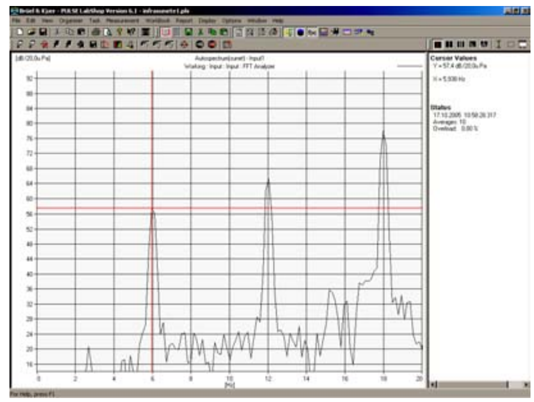

Fig. 9. Sound spectral analyze at the excitation frequency 6 Hz. Maximum amplitude 5 mm.

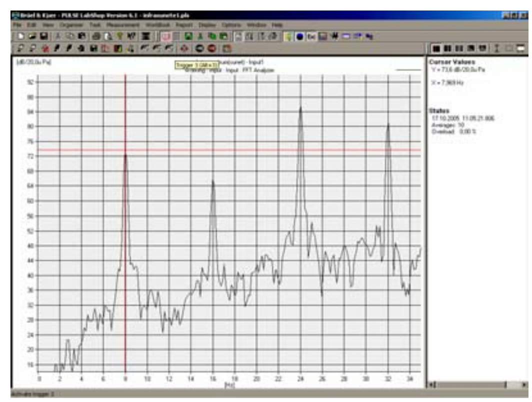

Fig. 10. Sound spectral analyze at the excitation frequency 8 Hz. Maximum amplitude 5 mm.

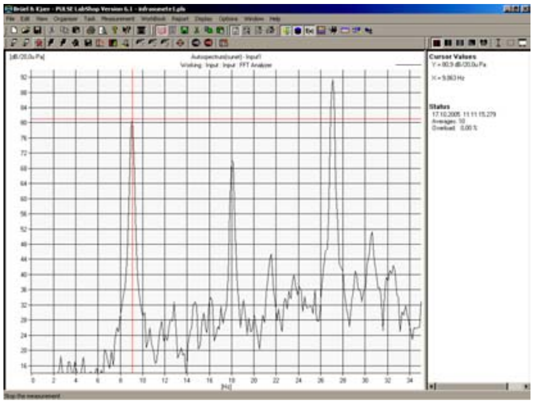

Fig. 11. Sound spectral analyze at the excitation frequency 10 Hz. Maximum amplitude 5 mm.

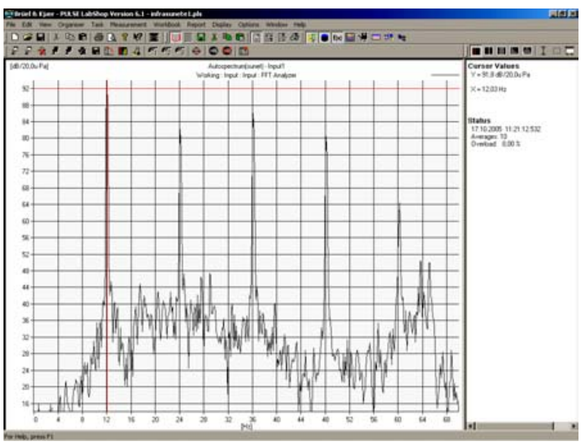

Fig. 12. Sound spectral analyze at the excitation frequency 12 Hz. Maximum amplitude 5 mm.

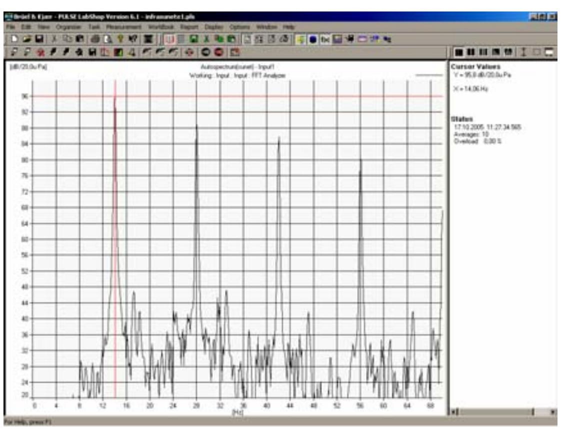

Fig. 13. Sound spectral analyze at the excitation frequency 14 Hz. Maximum amplitude 5 mm.

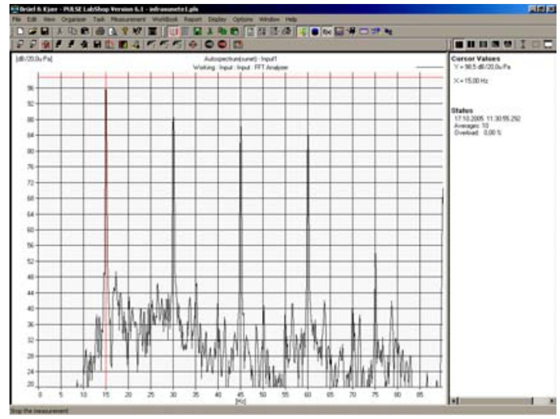

Fig. 14. Sound spectral analyze at the excitation frequency 16 Hz. Maximum amplitude 5 mm.

5 Conclusions

The numerical procedure yields solutions depending on plate vibrations amplitudes and clamping rigidity, proving that the plate model used is adequate for plate radiation investigations. These results are currently under experimental tests and preliminary experiments are promising.

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