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Influence of the Interface Forces to the Analysis of Beam Stiffened Plates

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Abstract: - In this paper the influence of the interface forces to the analysis of plates stiffened by arbitrarily placed nonintersecting beams of arbitrary cross section subjected to an arbitrary loading is presented. According to the proposed model, the stiffening beams are isolated from the plate by sections in the lower outer surface of the plate, taking into account the arising tractions in all directions at the fictitious interfaces. The aforementioned integrated tractions result in the loading of the beams as well as the additional loading of the plate. Their distribution is established by applying continuity conditions in all directions at the interfaces. The analysis of both the plate and the beams is accomplished on their deformed shape taking into account second-order effects. Six boundary value problems with respect to the plate transverse deflection, to the plate inplane displacement components, to the beam transverse deflections, to the beam axial deformation and to the beam nonuniform angle of twist are formulated and solved using the Analog Equation Method (AEM), a BEM based method employing a boundary integral equation approach. The solution of the aforementioned plate and beam problems, which are nonlinearly coupled, is achieved using iterative numerical methods. The adopted model describes better the actual response of the plate beams system and permits the evaluation of the shear forces at the interfaces in both directions, the knowledge of which is very important in the design of prefabricated ribbed plates. The evaluated lateral deflections of the plate - beams system are found to exhibit considerable discrepancy from those of other models, which neglect inplane and axial forces and deformations.

Key-Words: - Elastic stiffened plate, reinforced plate with beams, bending, nonuniform torsion, warping, ribbed plate, boundary element method, slab-and-beam structure

1 Introduction

Structural plate systems stiffened by beams are widely used in buildings, bridges, ships, aircrafts and machines. Stiffening of the plate is used to increase its load carrying capacity and to prevent buckling especially in case of in-plane loading. The extensive use of the aforementioned plate structures necessitates a rigorous analysis.

The behavior of stiffened plates under static loading has been studied for the past few decades. The behavior of the aforementioned structural systems was initially approximated by smearing – out the stiffness properties of the beams to get an equivalent orthotropic homogeneous slab of constant thickness [1-4]. This approximation may be applicable only when the stiffened plate satisfies two limitations. The first one is that ratios of spacing between two consecutive stiffeners to slab boundary dimensions are small enough to ensure approximate homogeneity of stiffness. The second limitation is that the ratio of stiffener rigidity to the slab rigidity must not become so large that the beam action is predominant.

Subsequently, in more refined approximations the adopted models for the analysis of the plate - beams system isolated the beams from the plate and employed numerical methods for the solution of the plate and the beams such as a semianalytical method [5], a methodology based on energy principles [6-7], the differential quadrature method [8], the finite strip or the finite element method [9-20], the boundary element method [21-29] or a combination of these methods [30-31]. In all these approximations the solution of the bending problem of stiffened plates is not general since either the analysis of the plate and the beams is performed on the undeformed shape ignoring second-order effects or the shear longitudinal or transverse forces at the interfaces have been neglected or the cross section of the stiffening beams is a symmetric one or the torsional and warping behavior of the stiffening beams has been neglected excluding in this way the placement of an eccentric stiffener. All these assumptions result in discrepancies from the actual response of the stiffened plate.

In this paper the influence of the interface forces to the analysis of plates stiffened by arbitrarily placed nonintersecting beams of arbitrary cross section subjected to an arbitrary loading is presented. The adopted structural model is a refined one of that proposed by Sapountzakis and Katsikadelis in [21]. According to this model, the stiffening beams are isolated from the plate by sections in the lower outer surface of the plate, taking into account the arising tractions in all directions at the fictitious interfaces. The aforementioned integrated tractions result in the loading of the beams as well as the additional loading of the plate. Their distribution is established applying continuity conditions in all directions at the interfaces. The analysis of both the plate and the beams is accomplished on their deformed shape taking into account second-order effects. Six boundary value problems with respect to the plate transverse deflection, to the plate inplane displacement components, to the beam transverse deflections, to the beam axial deformation and to the beam nonuniform angle of twist are formulated and solved using the Analog Equation Method (A.E.M.), a BEM based method employing a boundary integral equation approach [32]. The solution of the aforementioned plate and beam problems, which are nonlinearly coupled, is achieved using iterative numerical methods.

The essential features and novel aspects of the present formulation compared with previous ones are summarized as follows.

i. The stiffened plate is subjected to an arbitrary loading, while both the number and the placement of the nonintersecting stiffening beams are also arbitrary (eccentric beams are also included).
ii. The influence of the transverse traction component at plate-beams interfaces is taken into account. Thus, the adopted model permits the evaluation of the transverse inplane shear forces at the interfaces between the plate and the beams, the knowledge of which is very important in the design of prefabricated plate beams structures (estimation of shear connectors in the transverse direction).
iii. Displacement continuity conditions at the interfaces are applied along all three axes of the coordinate system, leading to the formulation of a system of equations involving two nonlinear functions, namely the longitudinal and transverse inplane shear forces at the interfaces.
iv. The cross section of the stiffening beams is an arbitrary one. Thereby, the eccentricities of both the centroid and the shear center axes with respect to the midline of the plate – beam interface are also included.
v. The nonuniform torsion in which the stiffening beams are subjected is taken into account by solving the corresponding problem and by comprehending the arising twisting and warping in the corresponding displacement continuity conditions.
vi. Terms arising from the internal variable axial loading of both the plate and the beams coming from the longitudinal and transverse inplane shear forces at the interfaces are taken into account.

The adopted model describes better the actual response of the plate beams system and permits the evaluation of the shear forces at the interfaces in both directions, the knowledge of which is very important in the design of prefabricated ribbed plates. The evaluated lateral deflections of the plate - beams system are found to exhibit considerable discrepancy from those of other models, which neglect inplane and axial forces and deformations.

2 Statement of the problem

Consider a thin plate of homogeneous, isotropic and linearly elastic material with modulus of elasticity E and Poisson ratio ν, having constant thickness h_p and occupying the two dimensional multiply connected region Ω of the x,y plane bounded by the piecewise smooth $K+1$ curves $\Gamma_0,\Gamma_1,...,\Gamma_{K-1},\Gamma_K$, as shown in Fig.1. The plate is stiffened by a set of $i=1,2,...,I$ arbitrarily placed nonintersecting beams of homogeneous, isotropic and linearly elastic material with modulus of elasticity E_b^i and Poisson ratio ν_b^i, which may have either internal or boundary point supports. For the shake of convenience the x axis is taken parallel to the beams. The stiffened plate is subjected to the lateral load $g=g(x,y)$. For the analysis of the aforementioned problem a global coordinate system Oxy for the analysis of the plate and local coordinate ones $O_i x_i y_i$ and $O_i \tilde{x}_i \tilde{y}_i$ corresponding to the centroid and shear center

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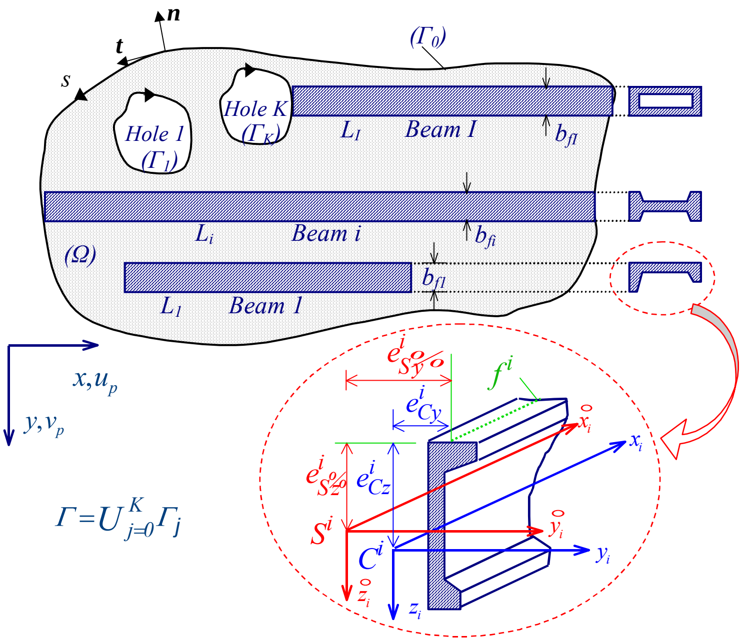


Fig.1. Two dimensional region Ω occupied by the plate.

axes of each beam are employed as shown in Fig.1.

The solution of the problem at hand is approached by a refined model of that proposed by Sapountzakis and Katsikadelis in [21]. According to this model, the stiffening beams are isolated from the plate by sections in the lower outer surface of the plate, taking into account the arising tractions at the fictitious interfaces (Fig.2). Integration of these tractions along the width of the i-th beam results in line forces per unit length, which are denoted by q_x^i, q_y^i and q_z^i encountering in this way the influence of the transverse component q_y, which in the aforementioned model [21] was ignored. The aforementioned integrated tractions result in the loading of the i-th beam as well as the additional loading of the plate. Their distribution is unknown and can be established by imposing displacement continuity conditions at the interfaces along x_i, y_i and z_i local axes following the procedure developed in this investigation.

The arising additional loading at the middle surface of the plate and the loading along the centroid and the shear center axes of each beam can be summarized as follows

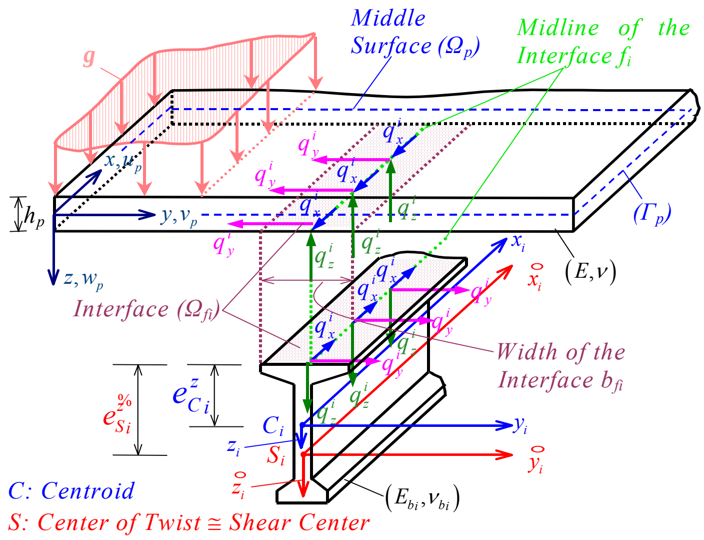


Fig.2 Thin elastic plate stiffened by beams (a) and isolation of the beams from the plate (b).

a. In the plate (at the trace of the midline of each (i-th) plate – beam interface)

(i) A lateral line load q^i_z at the interface.

(ii) A lateral line load $\partial m^i_{py} / \partial x$ due to the eccentricity of the component q^i_x from the middle surface of the plate. $m^i_{py} = q^i_x h_p / 2$ is the bending moment.

(iii) A lateral line load $\partial m^i_{px} / \partial y$ due to the eccentricity of the component q^i_y from the middle surface of the plate. $m^i_{px} = q^i_y h_p / 2$ is the bending moment.

(iv) An inplane line body force q^i_x at the middle surface of the plate.

(v) An inplane line body force q^i_y at the middle surface of the plate.

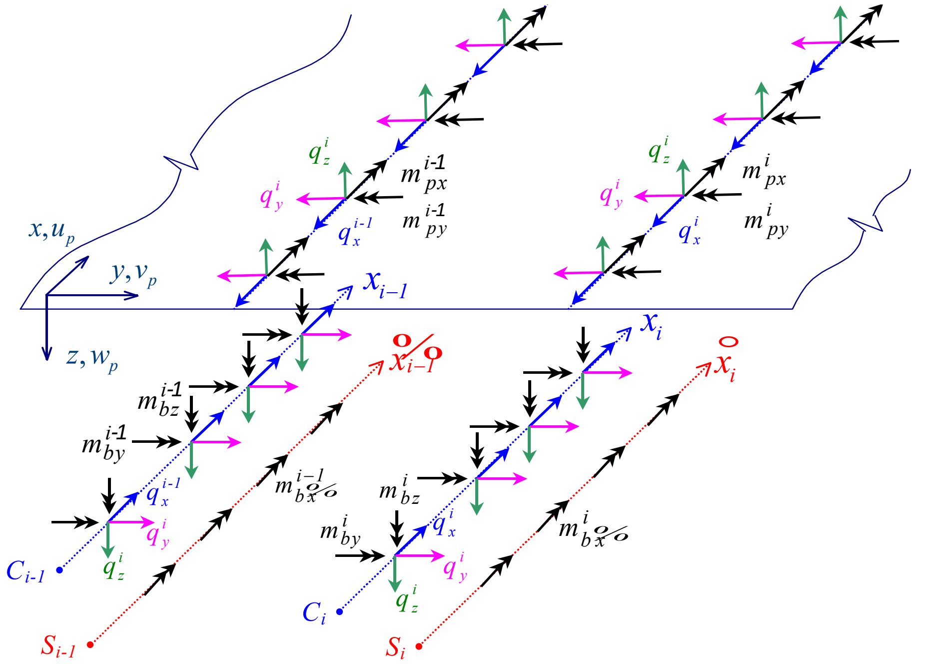


Fig.3. Structural model and directions of the additional loading of the plate and the beams.

b. In each beam

(i) A perpendicularly distributed line load q^i_z along the beam centroid axis.

(ii) A transversely distributed line load q^i_y along the beam centroid axis.

(iii) An axially distributed line load q^i_x along the beam centroid axis.

(iv) A distributed bending moment m^i_{by} along y_i local beam centroid axis due to the eccentricity of the component q^i_x from the beam centroid axis. $m^i_{by} = q^i_x e^i_{Cz}$ is the bending moment.

(v) A distributed bending moment m^i_{bz} along z_i local beam centroid axis due to the eccentricity of the component q^i_x from the beam centroid axis. $m^i_{bz} = -q^i_x e^i_{Cy}$ is the bending moment.

(vi) A distributed twisting moment $m^i_{b\overset{o}{x}}$ along $\overset{o}{x}_i$ local beam shear center axis due to the eccentricity of the components q^i_y, q^i_z from the beam shear center axis. $m^i_{b\overset{o}{x}} = -q^i_y e^i_{S\overset{o}{z}} + q^i_z e^i_{S\overset{o}{y}}$ is the twisting moment.

The structural models and the aforementioned additional loading of the plate and the beams are shown in Fig.3.

On the base of the above considerations the response of the plate and of the beams may be described by the following boundary value problems.

a. For the plate.

The plate undergoes transverse deflection and inplane deformation. Thus, for the transverse deflection the equation of equilibrium employing the linearized second order theory can be written as

$$D\nabla^4 w_p - \left(N_x \frac{\partial^2 w_p}{\partial x^2} + 2N_{xy} \frac{\partial^2 w_p}{\partial x \partial y} + N_y \frac{\partial^2 w_p}{\partial y^2} \right) =$$

$$g - \sum_{i=1}^{I} \left(q_z^i + \frac{\partial m_{py}^i}{\partial x} - q_x^i \frac{\partial w_p}{\partial x} - q_y^i \frac{\partial w_p}{\partial y} \right) \delta\left(y - y_i \right)$$

$$\text{in } \Omega \qquad (1)$$

and the corresponding boundary conditions as

$$\alpha_{p1} w_p + \alpha_{p2} R_{pn} = \alpha_{p3} \qquad (2a)$$

$$\beta_{p1} \frac{\partial w_p}{\partial n} + \beta_{p2} M_{pn} = \beta_{p3} \; \Gamma \qquad (2b)$$

where $w_p = w_p(x,y)$ is the transverse deflection of the plate; $D = Eh_p^3 / 12(1-\nu^2)$ is its flexural rigidity; $N_x = N_x(x,y)$, $N_y = N_y(x,y)$, $N_{xy} = N_{xy}(x,y)$ are the membrane forces per unit length of the plate cross section; $\delta(y-y_i)$ is the Dirac's delta function in the y direction; M_{pn} and R_{pn} are the bending moment normal to the boundary and the effective reaction along it, respectively, which using intrinsic coordinates *n, s* [33] are given as

$$M_{pn} = -D\left[\nabla^2 w_p + (\nu - 1)\left(\frac{\partial^2 w_p}{\partial s^2} + \kappa \frac{\partial w_p}{\partial n} \right) \right] \qquad (3)$$

$$R_{pn} = -D\left[\frac{\partial}{\partial n}\nabla^2 w_p - (\nu - 1)\frac{\partial}{\partial s}\left(\frac{\partial^2 w_p}{\partial s \partial n} - \kappa \frac{\partial w_p}{\partial s} \right) \right]$$

$$+ N_n \frac{\partial w_p}{\partial n} + N_{nt} \frac{\partial w_p}{\partial s} \qquad (4)$$

in which $\kappa = \kappa(s)$ is the curvature of the boundary; $\partial / \partial s$ and $\partial / \partial n$ denote differentiation with respect to the arc length *s* of the boundary and the outward normal *n* to it, respectively. Finally, a_{pi}, β_{pi} $(i = 1,2,3)$ are functions specified on the boundary Γ.

The boundary conditions (2a,b) are the most general boundary conditions for the plate problem including also the elastic support. It is apparent that all types of the conventional boundary conditions (clamped, simply supported, free or guided edge) can be derived form these equations by specifying appropriately the functions a_{pi} and β_{pi} (e.g. for a clamped edge it is $a_{p1} = \beta_{p1} = 1$, $a_{p2} = a_{p3} = \beta_{p2} = \beta_{p3} = 0$).

Since linearized plate bending theory is considered, the components of the membrane forces N_x, N_y, N_{xy} are given as

$$N_x = C\left(\frac{\partial u_p}{\partial x} + \nu \frac{\partial v_p}{\partial y} \right) \qquad (5a)$$

$$N_y = C\left(\nu \frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} \right) \qquad (5b)$$

$$N_{xy} = C\frac{1-\nu}{2}\left(\frac{\partial u_p}{\partial y} + \frac{\partial v_p}{\partial x} \right) \qquad (5c)$$

where $C = Eh_p / \left(1 - \nu^2\right)$; $u_p = u_p(x,y)$, $v_p = v_p(x,y)$ are the displacement components of the middle surface of the plate arising from the line body forces q_x^i, q_y^i $(i=1,2,...I)$. These displacement components are established by solving independently the plane stress problem, which is described by the following boundary value problem (Navier's equations of equilibrium)

$$\nabla^2 u_p + \frac{1+\nu}{1-\nu}\frac{\partial}{\partial x}\left[\frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} \right] - \frac{1}{Gh_p}\sum_{i=1}^{I} q_x^i \delta \left(y - y_i \right) = 0 \qquad (6a)$$

$$\nabla^2 v_p + \frac{1+\nu}{1-\nu}\frac{\partial}{\partial y}\left[\frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} \right] - \frac{1}{Gh_p}\sum_{i=1}^{I} q_y^i \delta \left(y - y_i \right) = 0$$

$$\text{in } \Omega \qquad (6b)$$

Proceedings of the 2006 IASME/WSEAS International Conference on Continuum Mechanics, Chalkida, Greece, May 11-13, 2006 (pp151-165)

$$\gamma_{p1}u_{pn} + \gamma_{p2}N_n = \gamma_{p3}$$
$$\delta_{p1}u_{pt} + \delta_{p2}N_t = \delta_{p3} \tag{7a}$$

in which $G = E/2(1+\nu)$ is the shear modulus of the plate; N_n, N_t and u_{pn}, u_{pt} are the boundary membrane forces and displacements in the normal and tangential directions to the boundary, respectively; γ_{pi}, δ_{pi} $(i = 1,2,3)$ are functions specified on the boundary Γ.

b. For each beam.

Each beam undergoes transverse deflection with respect to z_i and y_i axes, axial deformation along x_i axis and nonuniform angle of twist along x_i axis. Thus, for the transverse deflection with respect to z_i axis the equation of equilibrium employing the linearized second order theory can be written as

$$E_b^i I_y^i \frac{\partial^4 w_b^i}{\partial x_i^4} - N_b^i \frac{\partial^2 w_b^i}{\partial x_i^2} = q_z^i - q_x^i \frac{\partial w_b^i}{\partial x_i} + \frac{\partial m_{by}^i}{\partial x_i}$$

in L_i, $i = 1,2,...,I$ (8)

$$a_{1i}^z w_b^i + a_{2i}^z R_z^i = a_{3i}^z$$
$$\beta_{1i}^z \theta_y^i + \beta_{2i}^z M_y^i = \beta_{3i}^z$$
at the beam ends $x_i = 0, L_i$ (9b)

where $w_b^i = w_b^i(x_i)$ is the transverse deflection of the i-th beam with respect to z_i axis; I_y^i is its moment of inertia with respect to y_i axis; $N_b^i = N_b^i(x_i)$ is the axial force at the x_i centroid axis; a_{ji}^z, β_{ji}^z $(j = 1,2,3)$ are coefficients specified at the boundary of the i-th beam; θ_y^i, R_z^i, M_y^i are the slope, the reaction and the bending moment at the i-th beam ends, respectively given as

$$\theta_y^i = -\frac{\partial w_b^i}{\partial x_i} \tag{10}$$

$$R_z^i = -E_b^i I_y^i \frac{\partial^3 w_b^i}{\partial x_i^3} + N_b^i \frac{\partial w_b^i}{\partial x_i} \tag{11}$$

$$M_y^i = -E_b^i I_y^i \frac{\partial^2 w_b^i}{\partial x_i^2} \tag{12}$$

on Γ (7b)

It is apparent that all types of the conventional boundary conditions (clamped, simply supported, free or guided edge) can be derived from eqns (9a,b) by specifying appropriately the coefficients a_{ji}^z, β_{ji}^z (e.g. for a simply supported end it is $a_{1i}^z = \beta_{2i}^z = 1$, $a_{2i}^z = a_{3i}^z = \beta_{1i}^z = \beta_{3i}^z = 0$).

Similarly, the $v_b^i = v_b^i(x_i)$ transverse deflection with respect to y_i axis must satisfy the following boundary value problem

$$E_b^i I_z^i \frac{\partial^4 v_b^i}{\partial x_i^4} - N_b^i \frac{\partial^2 v_b^i}{\partial x_i^2} = q_y^i - q_x^i \frac{\partial v_b^i}{\partial x_i} - \frac{\partial m_{bz}^i}{\partial x_i}$$
in

(13)

$$a_{1i}^y v_b^i + a_{2i}^y R_y^i = a_{3i}^y$$

(14a)

$$\beta_{1i}^y \theta_z^i + \beta_{2i}^y M_z^i = \beta_{3i}^y$$
at (9a) the beam ends $x_i = 0, L_i$ (14b)

where I_z^i is the moment of inertia of the i-th beam with respect to y_i axis; a_{ji}^y, β_{ji}^y $(j = 1,2,3)$ are coefficients specified at its boundary; θ_z^i, R_y^i, M_z^i are the slope, the reaction and the bending moment at the i-th beam ends, respectively given as

$$\theta_z^i = \frac{\partial v_b^i}{\partial x_i} \tag{15}$$

$$R_y^i = -E_b^i I_z^i \frac{\partial^3 v_b^i}{\partial x_i^3} - N_b^i \frac{\partial v_b^i}{\partial x_i} \tag{16}$$

$$M_z^i = E_b^i I_z^i \frac{\partial^2 v_b^i}{\partial x_i^2} \tag{17}$$

Since linearized beam bending theory is considered the axial deformation u_b^i of the beam arising from the arbitrarily distributed axial force

q_x^i $(i=1,2,...I)$ is described by solving independently the boundary value problem

$$E_b^i A_b^i \frac{\partial^2 u_b^i}{\partial x_i^2} = -q_x^i \qquad \text{in } L_i,\ i=1,2,...,I \tag{18}$$

$$a_{1i}^x u_b^i + a_{2i}^x N_b^i = a_{3i}^x \qquad \text{at the beam ends } x=0,\ L_i \tag{19}$$

where N_b^i is the axial reaction at the i-th beam ends given as

$$N_b^i = E_b^i A_b^i \frac{\partial u_b^i}{\partial x_i} \tag{20}$$

Finally, the nonuniform angle of twist with respect to $\bar{x}_i$ shear center axis has to satisfy the following boundary value problem [34]

$$E_b^i I_w^i \frac{\partial^4 \theta_{\bar{x}}^i}{\partial \bar{x}_i^4} - G_b^i I_{\bar{x}}^i \frac{\partial^2 \theta_{\bar{x}}^i}{\partial \bar{x}_i^2} = m_{b\bar{x}}^i \qquad \text{in } L_i,\ i=1,2,...,I \tag{21}$$

$$a_{1i}^{\bar{x}} \theta_{\bar{x}}^i + a_{2i}^{\bar{x}} M_{\bar{x}}^i = a_{3i}^{\bar{x}}$$

$$\beta_{1i}^{\bar{x}} \frac{\partial \theta_{\bar{x}}^i}{\partial \bar{x}_i} + \beta_{2i}^{\bar{x}} M_w^i = \beta_{3i}^{\bar{x}} \qquad \text{at the beam ends } x_i = 0,\ L_i \tag{22b}$$

where $\theta_{\bar{x}}^i = \theta_{\bar{x}}^i(\bar{x}_i)$ is the variable angle of twist of the i-th beam along the $\bar{x}_i$ shear center axis; $G_b^i = E_b^i / 2(1+\nu_b^i)$ is its shear modulus; I_w^i, $I_{\bar{x}}^i$ are the warping and torsion constants of the i-th beam cross section, respectively given as

$$I_w^i = \int_{A^i} \varphi_S^{P^2}\, dA^i \tag{23a}$$

$$I_{\bar{x}}^i = \int_{A^i} \left(\bar{y}_i^2 + \bar{z}_i^2 + \bar{y}_i \frac{\partial \varphi_S^P}{\partial \bar{z}_i} - \bar{z}_i \frac{\partial \varphi_S^P}{\partial \bar{y}_i} \right) dA^i \tag{23b}$$

with $\varphi_S^P(\bar{y}_i, \bar{z}_i)$ the primary warping function with respect to the shear center S of the A^i beam cross section; $a_{ji}^{\bar{x}}$, $\beta_{ji}^{\bar{x}}$ $(j=1,2,3)$ are coefficients specified at the boundary of the i-th beam; $\frac{\partial \theta_{\bar{x}}^i}{\partial \bar{x}_i}$ denotes the rate of change of the angle of twist and it can be regarded as the torsional curvature; $M_{\bar{x}}^i$ is the twisting moment and M_w^i is the warping moment due to the torsional curvature at the boundary of the i-th beam given as

$$M_{\bar{x}}^i = M_{\bar{x}}^{iP} + M_{\bar{x}}^{iS} \tag{24a}$$

$$M_w^i = -E_b^i I_w^i \frac{\partial^2 \theta_{\bar{x}}^i}{\partial \bar{x}_i^2} \tag{24b}$$

In eqn (24a) $M_{\bar{x}}^{iP}$ is the primary twisting moment resulting from primary shear stress distribution and $M_{\bar{x}}^{iS}$ is the secondary twisting moment resulting from secondary shear stress distribution due to warping given as [34]

$$M_{\bar{x}}^{iP} = G_b^i I_{\bar{x}}^i \frac{\partial \theta_{\bar{x}}^i}{\partial \bar{x}_i} \tag{25a}$$

$$M_{\bar{x}}^{iS} = -E_b^i I_w^i \frac{\partial^3 \theta_{\bar{x}}^i}{\partial \bar{x}_i^3} \tag{25b}$$

(22a)

The boundary conditions (22a,b) are the most general linear torsional boundary conditions for the beam problem including also the elastic support. It is apparent that all types of the conventional torsional boundary conditions (clamped, simply supported, free or guided edge) can be derived form these equations by specifying appropriately the coefficients $a_{ji}^{\bar{x}}$, $\beta_{ji}^{\bar{x}}$ $(j=1,2,3)$ (e.g. for a clamped edge it is $a_{1i}^{\bar{x}} = \beta_{1i}^{\bar{x}} = 1$, $a_{2i}^{\bar{x}} = a_{3i}^{\bar{x}} = \beta_{2i}^{\bar{x}} = \beta_{3i}^{\bar{x}} = 0$).

Eqns. (1), (6a), (6b), (8), (13), (18), (21) constitute a set of seven coupled partial differential equations including ten unknowns, namely w_p, u_p, v_p, w_b^i, v_b^i, u_b^i, $\theta_{\bar{x}}^i$, q_x^i, q_y^i, q_z^i. Three additional equations are required, which result from the displacement continuity conditions in the direction of x_i, y_i and z_i local axes at the midline of each (i-th) plate – beam interface. These conditions can be expressed as

$w_p - w_b^i = e^i_{S\tilde{y}}\theta^i_{\tilde{x}}$ in the direction of z_i local axis (26)

$$u_p - u_b^i = \frac{h_p}{2}\frac{\partial w_p}{\partial x} - e^i_{Cz}\frac{\partial w_b^i}{\partial x_i} + \left(\phi_S^P\right)_{f_i}\theta^i_{\tilde{x}}$$ in the direction of x_i local axis (27)

$$v_p - v_b^i = -\frac{h_p}{2}\frac{\partial w_p}{\partial y} - e^i_{S\tilde{z}}\theta^i_{\tilde{x}}$$ in the direction of y_i local axis (28)

where $\left(\phi_S^P\right)_{f_i}$ is the value of the primary warping function with respect to the shear center S of the beam cross section at the midline of the f_i (i-th) interface.

In all the aforementioned equations the values of all the eccentricities e^i_{Cz}, e^i_{Cy}, $e^i_{S\tilde{z}}$, $e^i_{S\tilde{y}}$ and of the primary warping function $\varphi_S^P(\tilde{y}_i, \tilde{z}_i)$ should be set having the appropriate algebraic sign corresponding to the local beam axes.

It is worth here noting that the coupling of the aforementioned equations is nonlinear due to the terms including the unknown q_x^i and q_y^i interface forces.

3 Numerical Solution

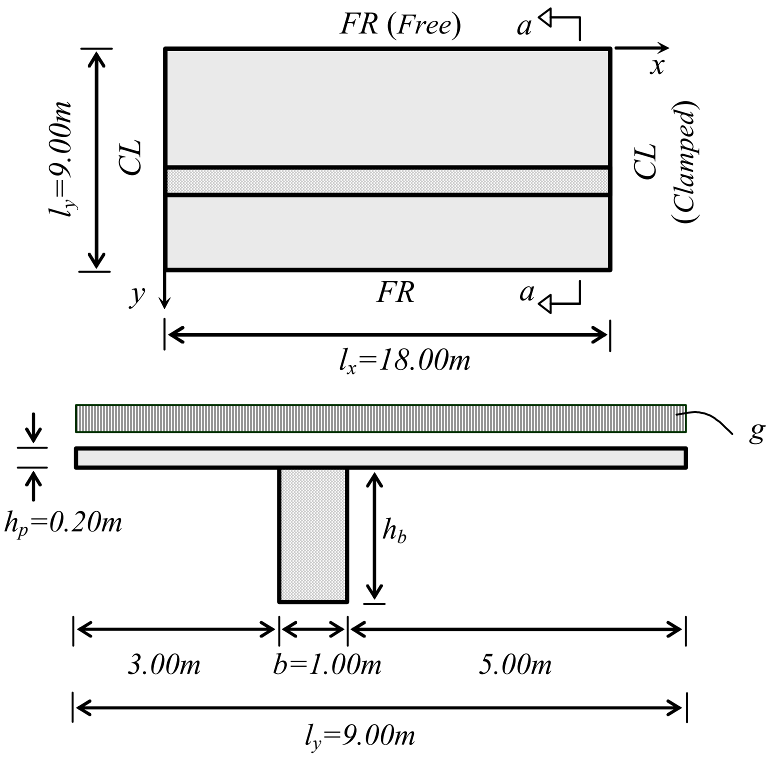
The numerical solution of the boundary value problems described by eqns (1-2a,b), (6a,b-7a,b), (8-9a,b), (13-14a,b), (18-19) and (21-22a,b) is accomplished employing the Analog Equation Method [32].

4 Numerical examples

On the basis of the analytical procedures presented in the previous sections, a computer program has been written in order to demonstrate the influence of the interface forces to the analysis of beam stiffened plates. In all the examples treated $E = E_b^i = 3.00E7$, $\nu = \nu_b^i = 0.20$, while the numerical results have been obtained using 180 constant boundary elements and 162 constant domain rectangular cells.

Example 1

A rectangular plate with dimensions $a \times b = 18.0 \times 9.0\ m$ subjected to a uniform load $g = 10kN/m^2$ and stiffened by a $1.0 \times 1.0m$ rectangular beam ($I^1_{\tilde{x}}$=1.40574E-01m^4, I^1_w=1.34405E-04m^6, $e^1_{S\tilde{z}}$=-0.5m) eccentrically placed with respect to the center line of the plate (Fig.4) has been studied. The plate is clamped along its small edges, while the other two edges are free according to both its transverse and inplane boundary conditions, while the beam is also clamped at its edges according to its transverse, axial and torsional boundary conditions. In Fig.5 the contour lines of the deflections of the stiffened plate are presented as compared with those obtained from a BEM [27] solution in which the inplane forces and deformations are ignored ($q_x^i = q_y^i = 0$, $q_z^i \neq 0$). Moreover, in Fig.6 the corresponding deflections obtained from FEM solutions [35] using either 8-noded quadrilateral shell finite elements (parabolic elements) or 10-noded tetrahedron solid finite elements (parabolic elements) are presented for comparison reasons. The discrepancy in the results arising from the ignorance of the inplane interface forces is obvious. In the last three (BEM and FEM) solutions the analysis is linear since the equilibrium equations are referred to the undeformed state of the stiffened plate contrary to the proposed method in which the equilibrium equations are written in the deformed state. The discrepancy of the results of the solutions ignoring plate membrane and beam axial behavior demonstrates the influence and necessitates the inclusion of the interface forces in the analysis of a stiffened plate.



(a) Plan view

(b) Section *a-a*

Fig.4. Plan view (a) and section a-a (b) of the stiffened plate of Example 1.

Example 2

A rectangular plate with dimensions $a \times b = 18.0 \times 9.0\ m$ subjected to an eccentric uniformly distributed load $g = 25kN / m^2$ and stiffened by three identical I-section beams (I_{x}^{1}=6.51930E-03m^4, I_w^1 =9.42741E-04m^6, e_{S2}^{i}=-2.10048E-01m) has been studied (Fig.7). The plate is transversely simply supported and inplane clamped along its small edges, while the other two edges are free according to both its transverse and inplane boundary conditions, while the beams are also transversely simply supported and axially and torsionally clamped at their edges.

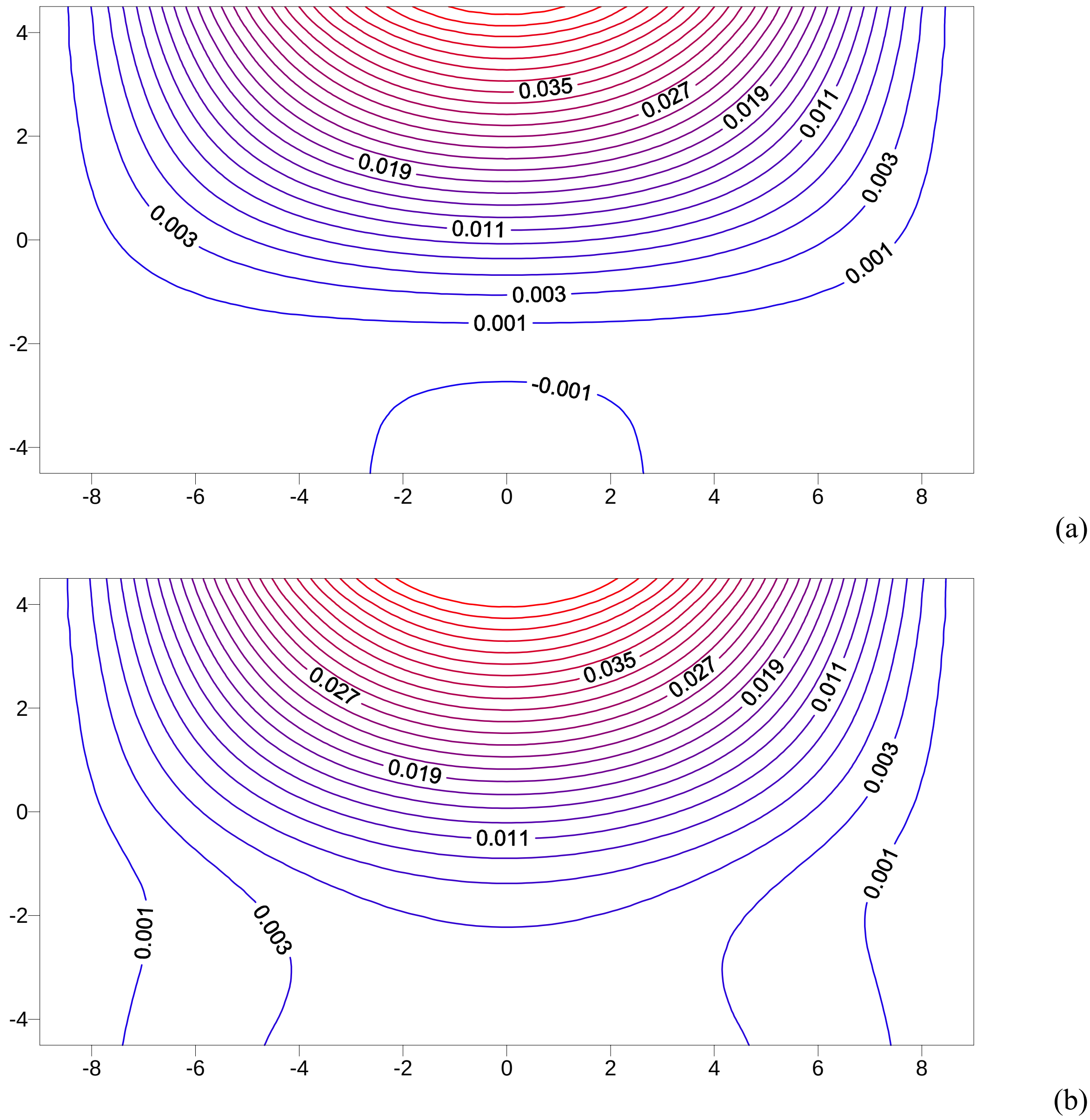
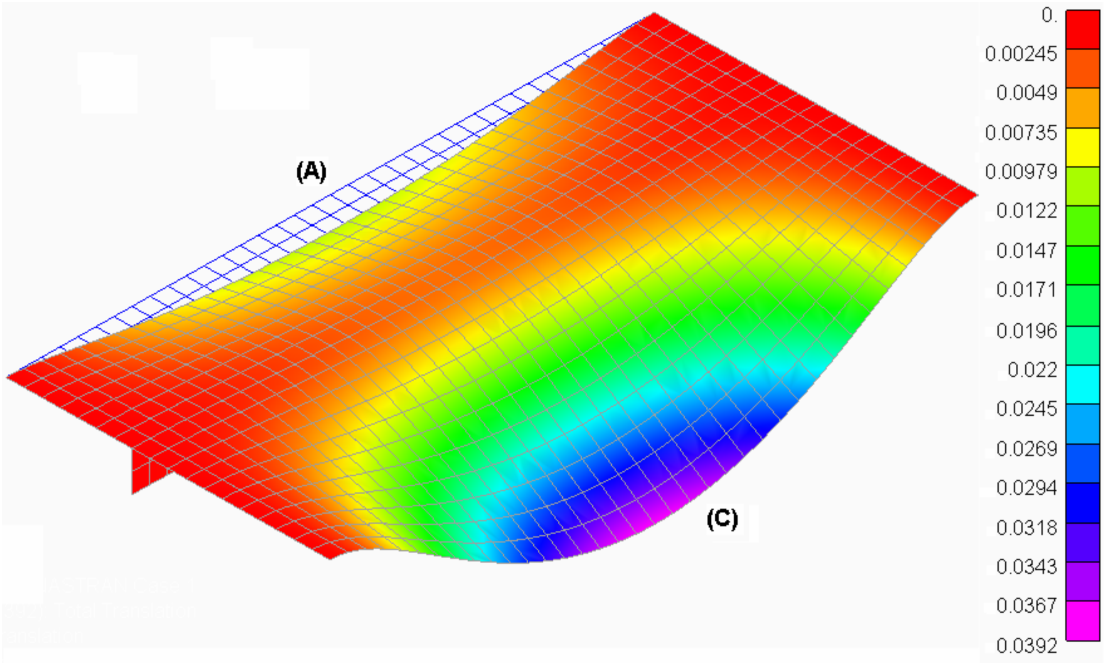


Fig.5. Contour lines of the deflectionw_p (m) for $g = 10kPa$ (a) taking into account and (b) ignoring the inplane interface forces of the stiffened plate of example 1.

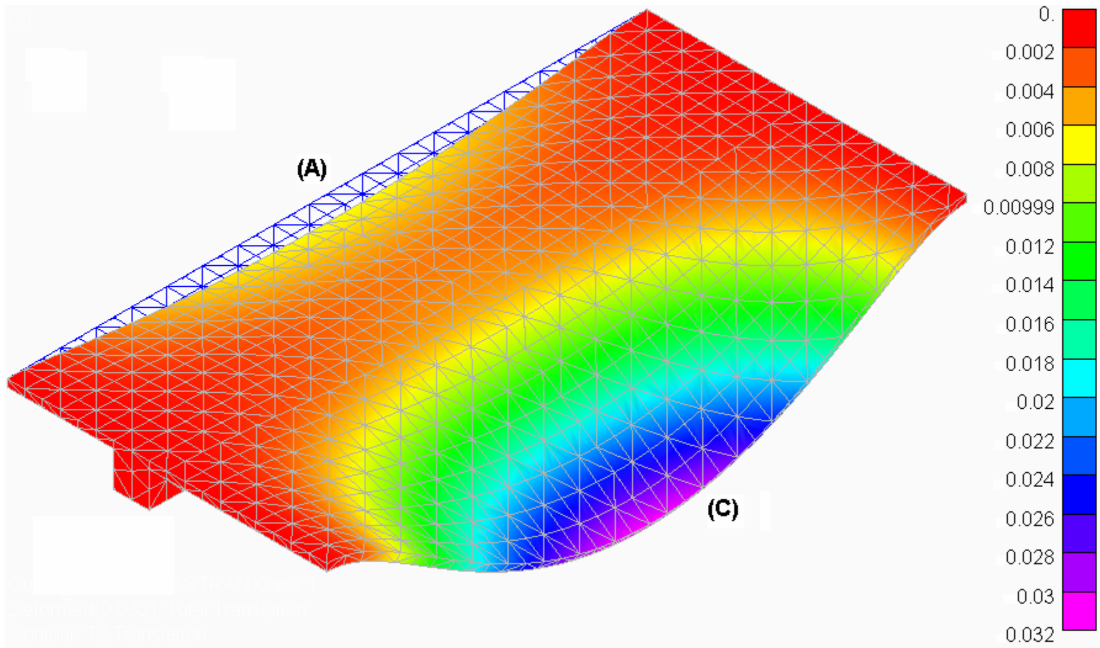
In Fig.8 the contour lines of the deflections of the stiffened plate are presented as compared with those obtained from a BEM [27] solution in which the inplane forces and deformations are ignored ($q_x^i = q_y^i = 0$, $q_z^i \neq 0$). The discrepancy of the results of the solution ignoring plate membrane and beam axial behavior demonstrates their influence and necessitates once again their inclusion in the analysis of a stiffened plate.

5 Concluding remarks

The influence of the interface forces to the analysis of plates stiffened by arbitrarily placed nonintersecting beams of arbitrary cross section subjected to an arbitrary loading is presented. A realistic model has been adopted, which contrary to other approaches, takes into account the arising tractions in all directions at the fictitious interfaces. The main conclusions that can be drawn from this investigation are

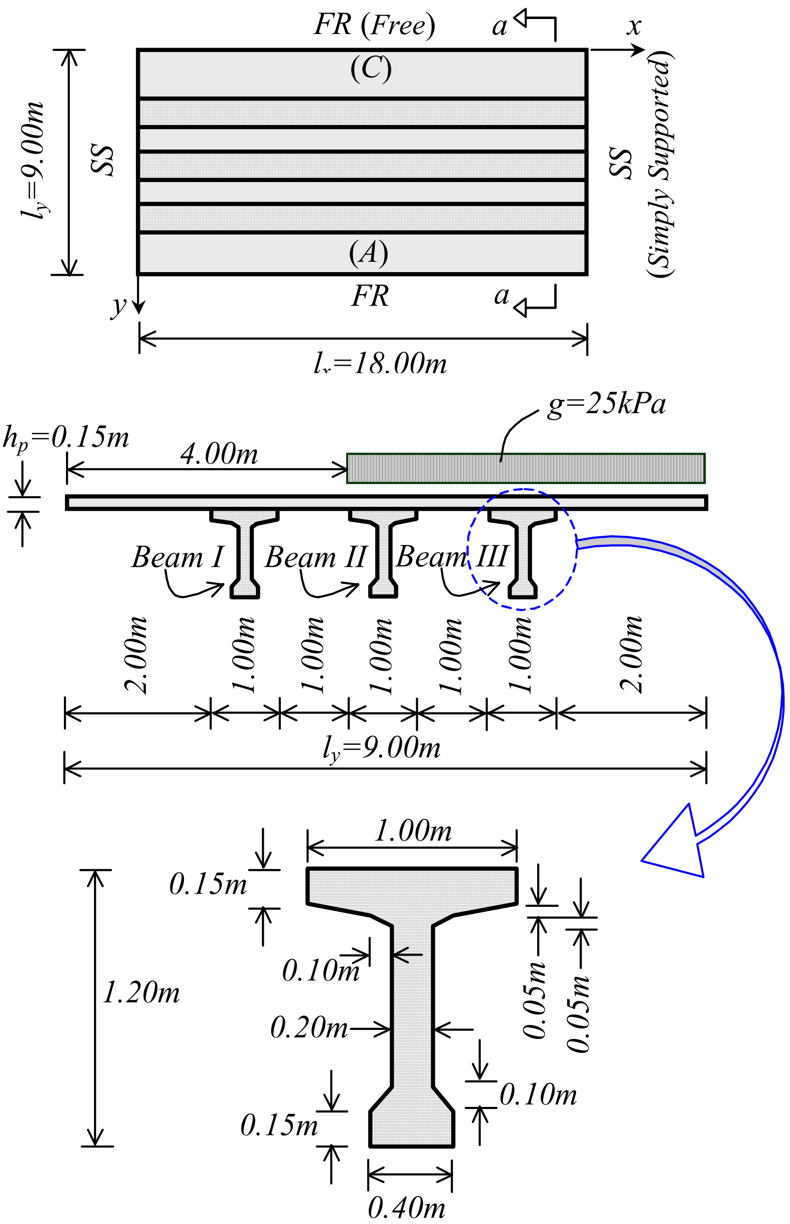


(a)



(b)

Fig. 6. Deflections w_p (m) for $g = 10kPa$ using (a) 720 8-noded (Parabolic) quadrilateral shell finite elements and (b) 4243 10-noded (Parabolic) tetrahedron solid finite elements of the stiffened plate of example 1.



(a) Plan view

(b) Section *a-a*

Fig.7. Plan view (a) and section a-a (b) of the stiffened plate of Example 2.

a. The proposed model permits the study of a stiffened plate subjected to an arbitrary loading, while both the number and the placement of the nonintersecting stiffening beams are also arbitrary (eccentric beams are also included).
b. The proposed model permits the evaluation of both the longitudinal and the transverse inplane shear forces at the interfaces between the plate and the beams, the knowledge of which is very important in the design of prefabricated plate beams structures (estimation of shear connectors in both directions).

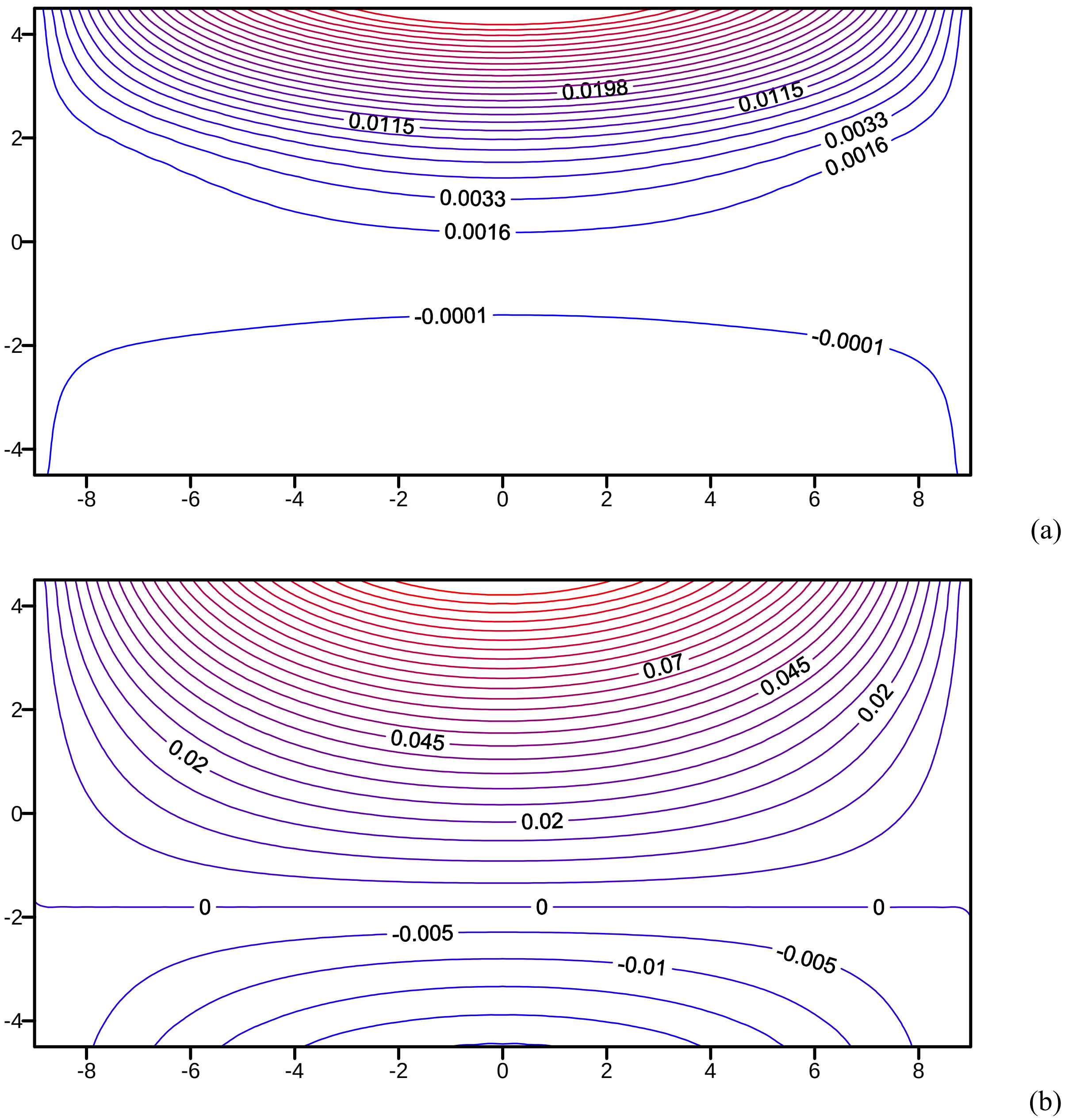


Fig.8. Contour lines of the deflection w_p (m) (a) taking into account and (b) ignoring the inplane interface forces of the stiffened plate of example 2.

c. The proposed model can handle stiffening beams of arbitrary cross section. Thereby, the eccentricities of both the centroid and the shear center axes with respect to the midline of the plate – beam interface are also included.
d. The nonuniform torsion in which the stiffening beams are subjected is taken into account by solving the corresponding problem and by comprehending the arising twisting and warping in the corresponding displacement continuity conditions.
e. The evaluated lateral deflections of the plate - beams system are found to exhibit considerable discrepancy from those of other models, which neglect inplane and axial forces and deformations.

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