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ASSESSMENT OF GROUNDWATER SAFE-YIELD

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Abstract: This paper presents a procedure for the estimation of safe-yield of a 3-layer aquifer groundwater system. First, based on the Darcy's Law, a set of equation is derived to describe the groundwater flow regime. The estimation of parameters for these equations is based upon the observations on water well elevation and natural recharge. These equations then serve as building blocks for a linear programming model, which maximizes the groundwater extraction while subject to minimum water table elevation requirement for the prevention of saline intrusion. The model is compact, transparent and flexible to handle different patterns of natural recharge and well field. Uncertainty in system modeling and parameters estimation is explicitly dealt with by gray linear programming.

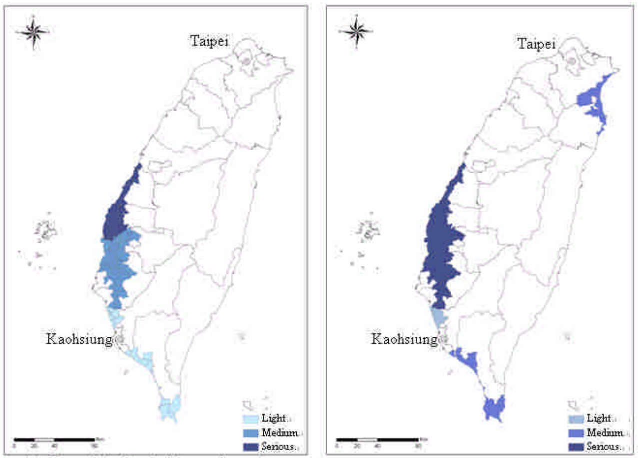
***Keywords*:** groundwater, well field, gray linear programming, safe-yield, land subsidence

1 Introduction

Accelerating demand on water resources for municipal, agricultural and industrial use caused by rapid economic development and increase in living standard has placed a serious stress on the national water supply in Taiwan. The increased usage of groundwater has caused excessive drawdown and land subsidence in many coastal areas [1]. In Taiwan, this problem has arisen from overly optimistic estimates of natural recharge and from a disregard for aquifer limitation. If the present trend continues, a total exhaustion of the water resources and massive loss of land by inundation is inevitable. Because its urgency in many parts of world, groundwater problem have been studied quite extensively by many researchers. Typically, these studies attempt to use a large-scale comprehensive model for a quantitative evaluation of groundwater and surface resources [2]. As to parameter estimation, a multitude of approaches have been proposed [3, 4], but many of these methods are still difficult for less-astute users to apply. Among prominent applications, Psilovikos [5] has proposed a mix integer linear programming to optimize the groundwater allocation; Garg and Ali [6] have presented an optimization model to develop the optimal pumping policy for the Lower Indus Basin. In these and many other applications the uncertainty in physical and economic parameters' value renders model solutions doubtful. To remedy the uncertainty problem, a host of analytical methods have been proposed, including the methods based on fuzzy set, intervals or stochastic theory [7]. In 1984, the grey systems theory, based on interval analysis [8], was developed by Dang [9] to join the rank. Grey system theory treats parameters as grey numbers that will take a value in a range when probabilistic distribution of parameters can not be identified. The optimistic and pessimistic results are assessed in the evaluation process with a set of upper and lower bound values for the input parameters [10]. This paper proposes a relatively simple grey linear model to make incisive use of available data for the assessment of groundwater availability.

The Problem of Over Pumping in Taiwan

One immediate consequence of groundwater extraction is land subsidence (Fig. 1). Table 1 shows the most critical areas affected by groundwater pumping in past decades. The severest impact occurs in an area of 19 km^2 in Pingtung County with 3.2-meter subsidence, followed by 2.1-meter subsidence observed in an area of 384 km^2 in Yuan-lin County. In Chang-hwa County, there is an area of 408 km^2 with 2.02-meter subsidence. Another consequence of over drafting is evidenced in groundwater table lower than the mean sea level in many areas. To relate cause and effect, a linear model based on Darcy's law applies to model the groundwater flow system of concerned. To expound the modeling efforts in detail, we select a study area with groundwater severe over-withdrawal in Chou-Shui-Chee basin, Taiwan.



(a) Cumulated subsidence in meters (b) Cumulated subsidence in area

Fig. 1. Extent of land subsidence in Taiwan

Table 1. Extent of land subsidence

County	Period	Cumulated subsidence (meters)	Area (km^2)
Chang-hwa	1985-2001	2.02	408
Yuan-lin	1975-2001	2.1	384
Chia-yi	1988-2001	1.24	173
Ping-tun	1972-2001	3.2	19

Source: Water Resources Agency, Ministry of Economics, Taiwan, Year 2001

Study Area

Fig. 2 portrays the study area; its east boundary is mountain ranges and high plains and west boundary is the Taiwan Strait. Both north and south sides are bounded by rivers. Groundwater and surface water flow is primarily in east-west direction. Fig. 3 is a vertical profile for the aquifer system in the study area. It shows that the aquifer split into three layers around mid-way towards the sea, with the middle and lower layers becoming confined aquifer eventually. Aquifer materials are pervious, with hydraulic conductivity in the order of 10^{-5} to 10^{-3} m/sec.

Model Description

Fig. 4 sketches out the three-layer aquifer system represented by Fig. 3. There are 9 computational cells in top layer and 5 cells in each of the other two layers. This schematization is to facilitate the data collection, computation and model application. In the figure, q_i denotes the well pumping in cell i, Q_i represents the natural recharge in cell i and h_i stands for water head in cell i. By Darcy Law, flow between adjacent cells can be expressed as: $(h_i - h_{i+1})/T_{i,i+1}$, in which $T_{i,i+1}$ is the friction coefficients. The hydraulics of the system can be described by 20 flow continuity equations, one at each cell plus one at the mountain range (boundary condition).

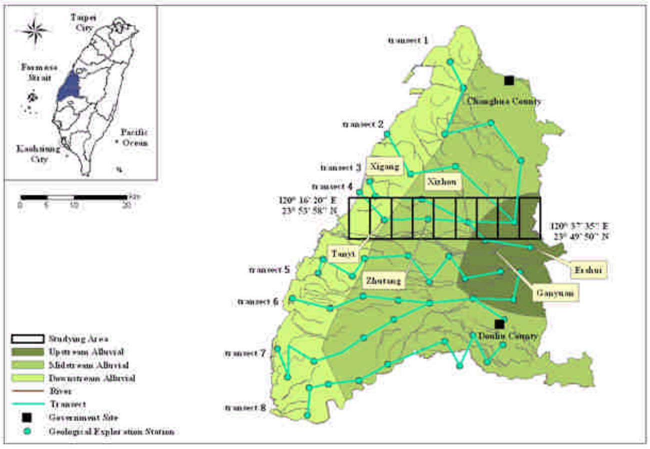


Fig. 2. Environmental system in study area

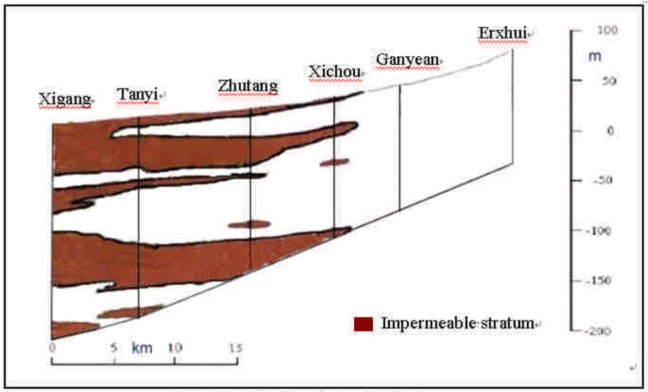


Fig. 3. A Profile of Aquifer System

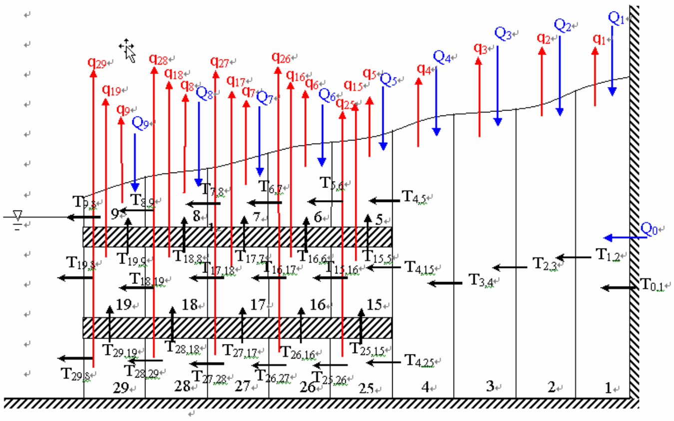


Fig. 4. Schematization of Aquifer System

$$\frac{h_0 - h_1}{T_{0,1}} = Q_0 \qquad (1)$$

$$\frac{h_{i-1} - h_i}{T_{i-1,i}} + Q_i - q_i - \frac{h_i - h_{i+1}}{T_{i,i+1}} = 0 \quad i = 1, 2, 3 \qquad (2)$$

$$\frac{h_3 - h_4}{T_{3,4}} + Q_4 - q_4 - \frac{h_4 - h_5}{T_{4,5}} - \frac{h_4 - h_{15}}{T_{4,15}} - \frac{h_4 - h_{25}}{T_{4,25}} = 0 \quad (3)$$

$$\frac{h_{i-1} - h_i}{T_{i-1,i}} + Q_i - q_i - \frac{h_i - h_{i+1}}{T_{i,i+1}} + \frac{h_{i+10} - h_i}{T_{i+10,i}} = 0 \quad i=5\sim8 \ (4)$$

$$\frac{h_8 - h_9}{T_{8,9}} + Q_9 - q_9 - \frac{h_9 - h_s}{T_{9,s}} + \frac{h_{19} - h_9}{T_{19,9}} = 0 \qquad (5)$$

$$\frac{h_4 - h_{15}}{T_{4,15}} - q_{15} - \frac{h_{15} - h_{16}}{T_{15,16}} + \frac{h_{25} - h_{15}}{T_{25,15}} - \frac{h_{15} - h_5}{T_{15,5}} = 0 \qquad (6)$$

$$\frac{h_{i-1} - h_i}{T_{i-1,i}} - q_i - \frac{h_i - h_{i+1}}{T_{i,i+1}} + \frac{h_{i+10} - h_i}{T_{i+10,i}} - \frac{h_i - h_{i-10}}{T_{i,i-10}} = 0$$
$$i = 16, 17, 18 \qquad (7)$$

$$\frac{h_{18} - h_{19}}{T_{18,19}} - q_{19} - \frac{h_{19} - h_s}{T_{19,s}} + \frac{h_{29} - h_{19}}{T_{29,19}} - \frac{h_{19} - h_9}{T_{19,9}} = 0 \quad (8)$$

$$\frac{h_4 - h_{25}}{T_{4,25}} - q_{25} - \frac{h_{25} - h_{26}}{T_{25,26}} - \frac{h_{25} - h_{15}}{T_{25,15}} = 0 \qquad (9)$$

$$\frac{h_{i-1} - h_i}{T_{i-1,i}} - q_i - \frac{h_i - h_{i+1}}{T_{i,i+1}} - \frac{h_i - h_{i-10}}{T_{i,i-10}} = 0$$
$$i = 26, 27, 28 \qquad (10)$$

$$\frac{h_{28} - h_{29}}{T_{28,29}} - q_{29} - \frac{h_{29} - h_s}{T_{29,s}} - \frac{h_{29} - h_{19}}{T_{29,19}} = 0 \qquad (11)$$

The value of h_s in Eq. (5) is zero for it represents the datum for h_i, i.e., mean sea level. If Q_i and $T_{i,i+1}$ are known, then there are more unknowns in q_i and h_i than the number of equations. This permits one to select h_{29} so that flow direction at the very downstream cell of lowest layer towards sea to prevent sea water intrusion. The main purpose of the model for the study area is to estimate the amount of natural recharge that can be extracted in conformity with a steady state flow condition allowing sufficient discharge to the sea to preclude damaging salinity intrusion.

To facilitate manipulation of the equations, the problem can be set up in a linear programming format. Let A be the total extraction and p_i be its spatial distribution, i.e.

$$q_i = p_i A \qquad (12)$$

$$\Sigma p_i = 1 \qquad (13)$$

The values of p_i reflect a particular extraction pattern, such as current condition (a grandfathering rule) and can be estimated in advance of solution. Hence, the problem becomes maximizing A subject to Eq. (1) through Eq. (13) and also subject to h_{29} taking on value large enough to direct flow toward sea. This format provides a convenient bookkeeping framework and computational aid for systematic investigation of the properties of the aquifer system.

Application of the Model

Parametization of the model requires the estimation of (1) natural recharge, Q_i, at each cell of the 9 cells in the top aquifer, (2) spatial distribution of extraction, p_i and (3) friction coefficients for flow through cell boundaries. The sample calculations that follow pertain to a strip of 8 km width along lat. 26.4°. The axis of the 19-cell model runs from the base of the Pa-qua Shan westward to the coast. The east-west length is 36 km in total, with each cell 4 km in length. The surface of each cell is 32 km^2. The estimates of natural recharge and extraction distribution are displayed in column 2 and column 3 of Table 2. The values of p_i correspond to the current extraction pattern. About 43%, 42% and 15% extraction are from the top, middle and lowest layer respectively. Column 4 exhibits the water table estimate representing the best of actual situation prior to rapid drawdown of groundwater. For this case study, these data are collated based on information for 1980 situation.

Estimation of friction coefficients requires the analysis of well records, natural recharge and water table data. Table 3 contains the estimated information. The values in the table are the reciprocals of T that are required in Eq. (1) through Eq. (11). These values are calculated as the ratio of flux to water table drop across two adjacent cells, an inverse use of Darcy Law. Recharge (and its accumulation) is postulated as flux in the calculation. For example, the first value in column 2 of Table 3, 10.6, is obtained by dividing 37.0 (from column 2 of Table 2) by the difference between 54.3 and 50.8 (from column 4 of Table 2.).

Table 2. Recharge rate and pumping distribution

Cell(i)	Q_i (million m^3/yr)	P_i	Groundwater table prior to overdraft (m)
0	37.00	0	54.3
1	10.26	0.0094	50.8
2	14.38	0.0296	47.3
3	13.55	0.0972	45.1
4	13.80	0.0977	37.3
5	14.27	0.0328	31.7
6	8.60	0.0159	27.7
7	10.66	0.0293	20.7
8	8.06	0.0532	16.6
9	1.91	0.0634	6.70
15		0.0548	24.6
16		0.0279	20.6
17		0.0551	12.5
18		0.1232	8.0
19		0.1569	4.0
25		0.0219	28.4
26		0.0111	19.4
27		0.0213	13.2
28		0.0442	6.9
29		0.0550	3.0

Table 3. Friction coefficients for optimization model

Between cell(i,j)	$1/T_{i,j}$ (million m^3/yr/m)		
	Linear Programming (best estimate)	Lower Bound for Grey linear programming	Upper Bound for Grey linear programming
(0,1)	10.6	9.5	12.6
(1,2)	13.7	12.3	16.0
(2,3)	27.4	24.7	32.3
(3,4)	9.8	8.9	11.6
(4,5)	5.00	4.5	5.9
(5,6)	10.4	9.5	12.2
(6,7)	7.1	6.4	8.4
(7,8)	14.7	13.3	18.6
(8,9)	6.5	5.9	7.7
(4,15)	2.9	2.6	3.7
(15,16)	9.4	8.5	11.0
(16,17)	4.6	4.2	5.5
(17,18)	11.0	9.9	13.0
(18,19)	7.8	7.0	8.4
(4,25)	2.5	2.3	2.6
(25,26)	2.7	2.3	6.0
(26,27)	3.9	3.5	4.6
(27,28)	4.0	3.6	8.8
(28,29)	6.8	6.1	7.9

In the shore area adjacent to study area, the difference in height between high and low tides is about 5 meters. For solution, we constrain the head in the last cell of each layer to be at least 2.5 meters above mean sea level to prevent salinity incursion. The solution of the linear program by EXCEL's Solver realizes that a maximum of 24.37 million m^3/year can be extracted. This is net pumping, gross pumping minus return. This amounts to about 1/5 of natural recharge. For this solution the head constraint on cell 29 is binding with a value of -36.14 for the elasticity of the permissible extraction rate with respect to the head constraints; that is a one percent increase in head constraint of cell 29 (from 2.5 to 2.525) will result in a 3.6 percent decrease in the extraction rate. Constraints in cell 9 and cell 19 are not binding in the solution. For another situation with h_{29} no less than 2 meter, the total extraction rate is 42.44 million m^3/year, about 1/3 of natural recharge. The information of these solutions is abstracted in Table 4. Profiles of the heads in the aquifers with h_{29} large or equal to 2.5 meter are shown in Fig. 5.

Table 4. Solution for various head constraint

Constraint (m)	A ($10^6 m^3$/year)	Elasticity
2.0	42.44	-36.14
2.5	24.37	-36.14

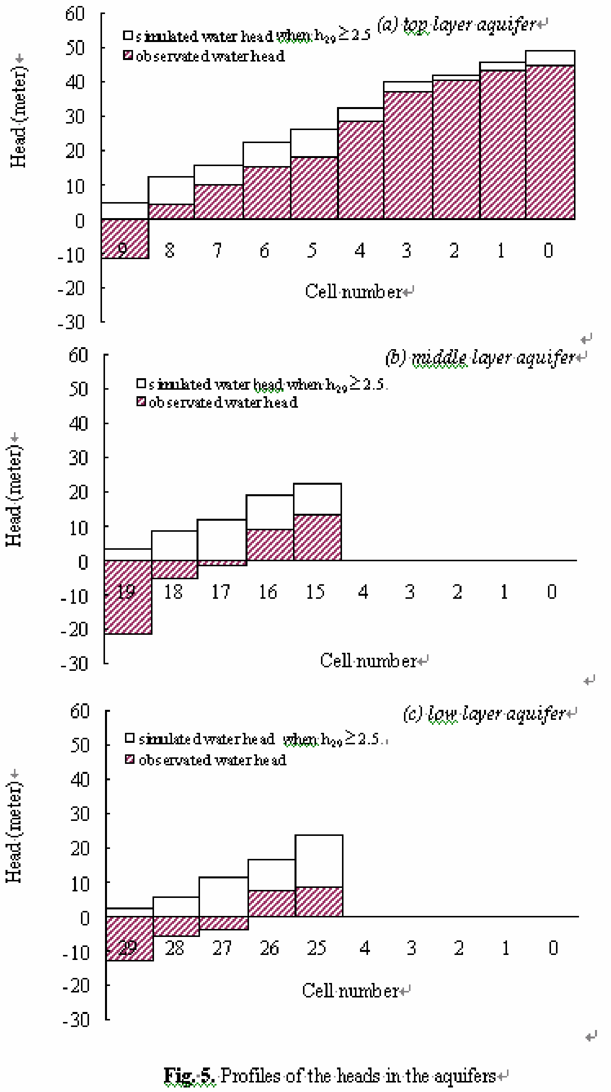


Fig. 5. Profiles of the heads in the aquifers

Uncertainty Analysis

Uncertainty inherent in parameters estimate will impinge upon the reliability and usefulness of modeling. This paper employs grey linear programming to cope with the uncertainty in estimating T_{ij} and Q_i. Considering these parameters as grey numbers, $\otimes T_{ij}$ and $\otimes Q_i$ can take values in intervals specified by Eq. (14) ~ (16).

$$\otimes T_{ij} = TL_{ij} + (1-\alpha)(TU_{ij} - TL_{ij}) \qquad (14)$$

$$\otimes Q_i = QL_i + \alpha(QU_i - QL_i) \qquad (15)$$

$$0 \le \alpha \le 1 \qquad (16)$$

In which $\otimes T_{ij}$ is the gray number of T_{ij}; $\otimes Q_i$ is the gray number of Q_i; TU_{ij} is the upper bound of $\otimes T_{ij}$; TL_{ij} is the lower bound of $\otimes T_{ij}$; QU_i is the upper bound of $\otimes Q_i$; QL_i is the lower bound of $\otimes Q_i$. The lower and upper bounds are estimated by referencing to data pertaining to the optimistic and pessimistic situations advanced by research reports relevant to the study area. With a value for α in line with Eq. (16), a linear programming will realize an optimal objective value, A_α. When α is equal to 0, the optimal objective value, A_{max}, is the most optimistic extraction rate. Eq. (17) is an expression of A_α in relation to A_{max}. The parameter, μ_α, provides information for the decision makers an apparatus to estimate the relative risk and achievable maximum objective value when selecting α.

$$\mu_\alpha = \frac{A_\alpha}{A_{max}} \qquad (17)$$

When using gray numbers in optimization model, the solution and elasticity of the permissible extraction rate with respect to the head constraint are described as grey numbers as shown in Table 5. The allocation of extracted water by grey linear programming is showed in Table 6. Column 2 of Table 6 presents the practical water extraction rate in study area. Comparing to reasonable extraction rate (column 3 and 4 in Table 6) estimated by this paper, the current extraction rate is too large to prevent sea water intrusion obviously. Table 7 describes the total extraction, A_α,.and μ_α by varying α. Figure 6 describes the relationship between α and a nonlinear relation. In this figure, the deterministic case solution, α is equal to 0.82, is shown as a black dot.

Table 5. Solutions for various head constraints under uncertainty

Head constraint(m)	$A(10^6 m^3/year)$ Upper limit (A_{max})	Lower limit (A_{min})	Elasticity Upper Bound	Lower Bound
2.0	51.57	12.38	-32.17	-43.75
2.5	35.48	0	-32.17	-43.75

Table 6. Practical and estimated extracted water in different head constraints

Cell i	Extracted water (million m^3/year) Current extraction rate	Head constraint (m) $h_{29} \ge 2.0$ m	$h_{29} \ge 2.5$ m
1	9.17	0.51~0.33	0.33~0.08
2	28.83	1.61~1.04	1.04~0.57
3	94.64	5.29~3.43	3.43~1.86
4	95.08	5.31~3.45	3.45~1.87
5	31.91	1.78~1.16	1.16~0.63
6	15.47	0.86~0.56	0.56~0.30
7	28.52	1.59~1.03	1.03~0.56
8	51.83	2.89~1.88	1.88~1.02
9	61.73	3.45~2.24	2.24~1.21
15	53.36	2.98~1.93	1.93~1.05
16	27.13	1.52~0.98	0.98~0.53
17	53.62	3.00~1.95	1.95~1.05
18	119.94	6.70~4.35	4.35~2.35
19	152.73	8.54~5.54	5.54~3.00
25	21.32	1.19~0.77	0.77~0.42
26	10.81	0.60~0.39	0.39~0.21
27	20.72	1.16~0.75	0.75~0.41
28	43.05	2.40~1.56	1.56~0.84
29	53.55	2.99~1.94	1.94~1.05
Summation	973.41	54.4~35.3	35.3~19.1

Table 7. The achieved total maximum extraction under various α

α	A_α	$\mu_\alpha = \frac{A_\alpha}{A_{max}}$ (%)[a]
0	51.57	100.00
0.1	49.30	95.60
0.2	44.30	85.91
0.3	39.76	77.10
0.4	35.52	68.88
0.5	31.47	61.04
0.6	27.57	53.47
0.7	23.75	46.06
0.8	19.97	38.73
0.9	17.61	34.14
1.0	12.38	24.00

a: h_{29} is limited to be large or equal to 2.5 m

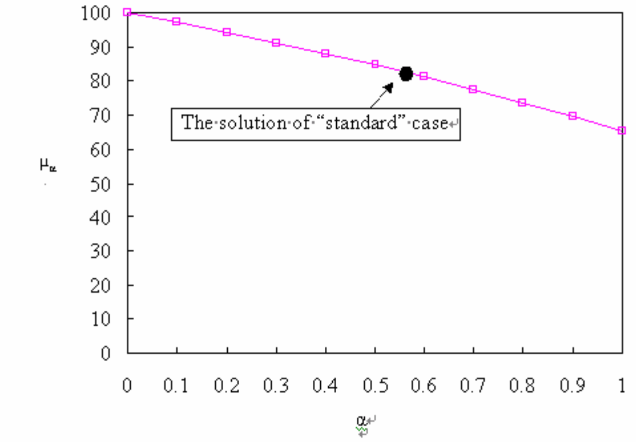


Fig. 6. Relationship between α and μ_α

Conclusions

Implementation of the model to the Chou-shui Chee basin, Taiwan, results that the steady state safe-yield is far less than current extraction rate. It is estimated that at most only about 1/3 of natural recharge can be extracted, with the remaining 2/3 discharged continuously to the sea to prevent seawater encroachment. This paper uses EXCEL as a tool to integrated groundwater management system and grey system theory for the purpose of establishing an intuitive decision model to decide the total maximum extraction when exact estimation for parameters is unavailable. From the results, decision makers may make a more flexible plan as long as the extraction in each cell don’t exceed the grey number obtained in table 6. Uncertainties exist in anywhere. In large scale system, a fixed determinate parameter usually can’t reflect real environmental conditions. Integrating grey system theory into simulation or optimization modeling seems to be an efficient approach to overcome the problems about parameter estimations for modeling.

Notation

The following symbols are used in this paper:

A = the total extraction

A_{max} = the most optimistic extraction rate

A_α = total extraction at a given α

h_i = water head in cell i

h_s = sea level and defined as zero for it represents the datum for h_i

p_i = particular extraction pattern for cell i

q_i = well pumping in cell i

Q_i = natural recharge in cell i

QL_i = lower bound of $\otimes Q_i$

QU_i = upper bound of $\otimes Q$

$T_{i,i+1}$ = friction coefficients between cell i and cell i+1

TU_{ij} = upper bound of $\otimes T_{ij}$

TL_{ij} = lower bound of $\otimes T_{ij}$

$\otimes T_{ij}$ = gray number of T_{ij}

$\otimes Q_i$ = gray number of Q_i

α = coefficient for changing grey number to a determine value

μ_α = ratio of A_α to the most optimistic extraction in grey linear model

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