Proceedings of the 2006 IASME/WSEAS International Conference on Energy & Environmental Systems, Chalkida, Greece, May 8-10, 2006 (pp165-172)

A Partially Closed Loop Distributed Controller for Constrained Water Quality Control in Streams with Time-Delays

MOHAMED FAHIM HASSAN
Electrical Engineering Department,
Kuwait University
P.O.Box:5969,13060-Safat
KUWAIT

Abstract: This paper investigates water quality control problem in streams taking into consideration time delays and system constraints. After decomposing the system into N subsystems, a distributed controller is proposed to solve the problem. The proposed controller consists of two parts, the closed loop part and the open loop one. The closed loop part guarantees reliable system operation under failure conditions, whilst the open loop part insures both optimality and the satisfaction of system constraints. Application to the problem of controlling the concentrations of biochemical oxygen demand and dissolved oxygen in a five reach river system is given which also illustrates the effectiveness of the developed procedure.

Keywords: Water quality control, optimal control theory, constrained optimization problem, distributed systems, time delay systems.

1 Introduction

With the extension of civilization and development, major environmental problems have been created. Among these problems, is that associated with the increase of pollution levels in streams. In general, pollutions may arise due to chemical wastes, radio-active materials, heat, biochemical or biological activities taking place in the water body as a result of discharging industrial effluents, and/or sewage into rivers and/or algae growth and its associated wastes resulting from its life cycle. This, of course, leads to a disturbance in the ecological balance of the water system which may affect all aspects of life. What is essentially required in a river system is to maintain pollution levels within reasonable bounds to satisfy community needs, the ecological balance in the stream and water quality standards.

On the other hand, wastes are normally treated prior to discharge into streams. Increasing treatment levels beyond certain limits will dramatically increase the cost. It is, therefore, more realistic to handle such a problem as an optimization problem with constraints on the states and/or controls.

Moreover, since streams are characterized by their geographically distributed nature, time delays required for pollutants transportation through the river system become an important factor which cannot be neglected while solving this problem. As a result of these features imposed by the physical nature of this problem, we are facing an optimization problem with time delays and system constraints.

Time delay control systems, have been solved using different approaches. Robust control methodologies constitute a class of these techniques which have been used to design feedback controllers to guarantee system stability. A lot of work was reported in the literature on this subject, among them we quote the works [1-3]. Another class of approaches is based on using auxiliary variables to approximate time delay variables, then applying the well developed techniques to control the behavior of the extended system model [4,5]. The main drawbacks of this technique are the increased dimensionality of the model and time delay approximation may lead to unstable system.

On the other side, constrained linear quadratic control problem (LQP) have been solved using many techniques which are based on model predictive control (MPC) [6-8], and anti-windup class of approaches [9,10].

Recently, a new approach has been developed to solve continues time constrained linear optimization problem [11,12]. This approach has been extended to discrete time constrained linear quadratic problem [13,14], as well as constrained LQP with time delays [15-17].

In this paper, we consider the interconnected dynamical system of water quality control in streams with time delays and system constraints. Due to the feature of this problem, it is natural to think about distributed controller rather than centralized one. Therefore, a partially closed loop distributed controller is developed to solve this problem. The closed loop part of the controller is decentralized and can maintain system stability under structural perturbations between the local controllers. On the other hand, the open loop part guarantees the global optimal performance of the system as well as the satisfaction of system constraints. The developed approach is used to control the concentrations of the biochemical oxygen demand (BOD) and the dissolved oxygen (DO) in a five reach river system.

The rest of the paper is divided into the following. Section 2 is devoted to problem formulation. The developed partially closed loop distributed controller is presented in Section 3, whilst the proposed algorithm is demonstrated in Section 4. In Section 5, the BOD-DO control problem of a five reach river system is formulated and solved. Finally, the paper is concluded in Section 6.

2 Problem Formulation

Consider the following linear time invariant interconnected dynamical system S, with time delays and constraints on the states and control:

$$\dot{x} = A_o x + \sum_{p=1}^{\theta} A_p x(t-\tau_p) + \sum_{q=0}^{\gamma} B_q u(t-\tau_q) \qquad (1)$$

with: $x(t_o) = x_o$

$$\underline{x} \le x \le \overline{x} \qquad (2)$$

$$\underline{u} \le u \le \overline{u} \qquad (3)$$

where $x \in R^n$ is the state vector, $u \in R^m$ is the control vector, p *and* q are the delays on the state and input vectors, $A_p \in R^{n\times n}$; $p \in \{0,1,....,\theta\}$ and $B_q \in R^{n\times m}$; $q \in \{0,1,....,\gamma\}$ are the system matrices, θ *and* γ are known positive integers representing the number of delays in the state and input vectors respectively, $\underline{x}, \overline{x}$, $\underline{u}, \overline{u}$ are the lower and upper bounds of the state and control vectors (component by component).

It will be assumed in the rest of the paper that:

$$x(t-\tau_p) = 0 \quad \forall\, t-\tau_p < 0; \quad p \in \{0,1,....,\theta\}$$

$$u(t-\tau_q) = 0 \quad \forall\, t-\tau_q < 0; \quad q \in \{0,1,....,\gamma\}$$

Associated with the above system S, a performance index J to be minimized with respect to *x, u,* of the form:

$$\min_{x,u} J = \frac{1}{2}\int_{t_o}^{t_f} (\|x\|_Q^2 + \|u\|_R^2)dt \qquad (4)$$

where $Q > 0 \in R^{n\times n}$, $R > 0 \in R^{m\times m}$ are block diagonal weighting matrices, t_o and t_f are, respectively, the initial and final times which are assumed to be known.

The above system can be described as N-interconnected dynamical subsystems, S_1, S_2, …, S_N, the i[th] of which is given by:

$$\dot{x}_i = A_{o_{ii}} x_i + B_{o_{ii}} u_i + \sum_{p=0}^{\theta} \sum_{j=1,\, j\ne i}^{N} A_{p_{ij}} x_j(t-\tau_p) + \sum_{p=1}^{\theta} A_{p_{ii}} x_i(t-\tau_p) + \sum_{q=0}^{\gamma} \sum_{j=1,\, j\ne i}^{N} B_{q_{ij}} u_j(t-\tau_q) + \sum_{q=1}^{\gamma} B_{q_{ii}} u_i(t-\tau_q) \qquad (5)$$

with $x_i(t_o) = x_{o_i}$

$$\underline{x}_i \le x_i \le \overline{x}_i \qquad (6)$$

$$\underline{u}_i \le u_i \le \overline{u}_i \qquad (7)$$

where $n = \sum_{i=1}^{N} n_i$, $m = \sum_{i=1}^{N} m_i$, $x_i \in R^{n_i}$, $u_i \in R^{m_i}$ are the state and control variables of the i[th] subsystem; $A_{p_{ii}} \in R^{n_i\times n_i}$, $p \in \{0,1,...,\theta\}$, $B_{q_{ii}} \in R^{n_i\times m_i}$, $q \in \{0,1,...,\gamma\}$ are the block diagonal parts of the i[th] subsystem matrices, $A_{p_{ij}}$; $p \in \{0,1,...,\theta\}$, $B_{q_{ij}}$, $q \in \{0,1,...,\gamma\}$ are the off-diagonal blocks of the i[th] subsystem with appropriate dimensions.

Associated with each subsystem S_i , the corresponding performance index J_i, given by:

$$J_i = \frac{1}{2}\int_{t_o}^{t_f} (\|x_i\|_{Q_i}^2 + \|u_i\|_{R_i}^2)dt \qquad (8)$$

where $Q_i > 0 \in R^{n_i\times n_i}$, $R_i > 0 \in R^{m_i\times m_i}$ are the subsystem weighting matrices which are related to the original ones by the following:

$$Q = diag\{Q_1, Q_2, ..., Q_N\}$$

$$R = diag\{R_1, R_2, ..., R_N\}$$

and hence $J = \sum_{i=1}^{N} J_i \qquad (9)$

In (5), the third and fifth terms in the R.H.S. include the coupling with the other subsystems through the state and control trajectories $x_j(t-\tau_p), u_j(t-\tau_q)$ for $j \in \{0,1,..,i-1,i+1,..,N\}$, $p \in \{0,1,...,\theta\}$, $q \in \{0,1,...,\gamma\}$, whilst the forth and sixth terms include the delayed state and control vectors of the i[th] subsystem.

Using a procedure similar to that given in [11,12], the above optimization problem (5)-(8) can be reformulated as follows:

$$\min_{x_i, x_i^o, u_i, u_i^o} J_i = \frac{1}{2}\int_{t_o}^{t_f} (\|x_i\|^2_{Q_{i_1}} + \|x_i^o\|^2_{Q_{i_2}} + \|x_i - x_i^o\|^2_{Q_{i_3}} + \|u_i\|^2_{R_i} + \|u_i - u_i^o\|^2_{R_{i_1}})dt \tag{10}$$

subject to:

$$\dot{x}_i = A_{o_{ii}} x_i + B_{o_{ii}} u_i + \sum_{p=0}^{\theta} \sum_{j=1,\, j\neq i}^{N} A_{p_{ij}} x_j^o(t-\tau_p) + \sum_{p=1}^{\theta} A_{p_{ii}} x_i^o(t-\tau_p) + \sum_{q=0}^{\gamma} \sum_{j=1,\, j\neq i}^{N} B_{q_{ij}} u_j^o(t-\tau_q) + \sum_{q=1}^{\gamma} B_{q_{ii}} u_i^o(t-\tau_q) \tag{11}$$

with $x_i(t_o) = x_{o_i}$

$$x_i = x_i^o \tag{12}$$

$$u_i = u_i^o \tag{13}$$

$$\underline{x}_i \le x_i^o \le \overline{x}_i \tag{14}$$

$$\underline{u}_i \le u_i \le \overline{u}_i \tag{15}$$

where $Q_i = Q_{i_1} + Q_{i_2}$, $Q_{i_3} > 0 \in R^{n_i \times n_i}$, $x_j^o \in R^{n_j}, j\varepsilon\{1,2,..,N\}$ are coordinating vectors to be used to decouple the subsystems, satisfy state constraints and handle time delay variables, whilst $u_j^o \in R^{m_j}, j\varepsilon\{1,2,..,N\}$ are another introduced coordinating vectors to decouple the subsystems and deal with time delay variables, $\|x_i - x_i^o\|^2_{Q_{i_3}}$, $\|u_i - u_i^o\|^2_{R_{i_1}}$ are convexifying terms which may be added to speed up the convergence rate of the algorithm.

Remarks:

1. At the end of convergence, i.e. $x^o \to x$ and $u^o \to u$, the convexifying terms will not contribute to the value of the cost function.
2. In cases where $Q_i \ge 0$, it is necessary to add the term $\|x_i - x_i^o\|^2_{Q_{i_3}}$ to avoid matrix singularity as will be shown in the next section.

3 The Distributed Controller:

To do that, let us write the Hamiltonian of the interconnected system:

$$H(x, x^o, u, u^o, \lambda, \pi, \beta) = \sum_{i=1}^{N} H_i(x_i, x_i^o, u_i, u_i^o, \lambda_i, \pi_i, \beta_i) = \sum_{i=1}^{N} \{ \frac{1}{2}\|x_i\|^2_{Q_{i_1}} + \frac{1}{2}\|x_i^o\|^2_{Q_{i_2}} + \frac{1}{2}\|x_i - x_i^o\|^2_{Q_{i_3}} + \frac{1}{2}\|u_i\|^2_{R_i} + \frac{1}{2}\|u_i - u_i^o\|^2_{R_{i_1}}$$
$$+ \lambda_i^T [A_{o_{ii}} x_i + B_{o_{ii}} u_i + \sum_{p=0}^{\theta} \sum_{j=1, j\neq i}^{N} A_{p_{ij}} x_j^o(t-\tau_p) + \sum_{p=1}^{\theta} A_{p_{ii}} x_i^o(t-\tau_p) + \sum_{q=0}^{\gamma} \sum_{j=1, j\neq i}^{N} B_{q_{ij}} u_j^o(t-\tau_q) + \sum_{q=1}^{\gamma} B_{q_{ii}} u_i^o(t-\tau_q)] + \pi_i^T (x_i - x_i^o) + \beta_i^T (u_i - u_i^o) \} \tag{16}$$

Relaxing system constraints for the moment, the necessary conditions of the optimality lead to:

$\frac{\partial H}{\partial u_i} = 0$, which gives:

$$u = (R_i + R_{i_1})^{-1}[R_{i_1} u_i^o - B_{o_{ii}}^T \lambda_i - \beta_i] \tag{17}$$

$\frac{\partial H}{\partial \lambda_i} = \dot{x}_i$, accordingly:

$$\dot{x}_i = A_{o_{ii}} x_i + B_{o_{ii}} u_i + \sum_{p=0}^{\theta} \sum_{j=1, j\neq i}^{N} A_{p_{ij}} x_j^o(t-\tau_p) + \sum_{p=1}^{\theta} A_{p_{ii}} x_i^o(t-\tau_p) + \sum_{q=0}^{\gamma} \sum_{j=1, j\neq i}^{N} B_{q_{ij}} u_j^o(t-\tau_q) + \sum_{q=1}^{\gamma} B_{q_{ii}} u_i^o(t-\tau_q) \tag{18}$$

with $x_i(t_o) = x_{o_i}$

$\frac{\partial H}{\partial x_i} = -\dot{\lambda}_i$, which leads to:

$$\dot{\lambda}_i = -Q_{i_1} x_i - Q_{i_3}(x_i - x_i^o) - A_{o_{ii}}^T \lambda_i - \pi_i \tag{19}$$

with $\lambda_i(t_f) = 0$

$\frac{\partial H}{\partial \beta_i} = 0$, which implies:

$$u_i = u_i^o \tag{20}$$

$\frac{\partial H}{\partial u_i^o} = 0$, which gives:

$$\beta_i = -R_{i_1}(u_i - u_i^o) + \sum_{q=0}^{\gamma} \sum_{j=1, j\neq i}^{N} B_{q_{ji}}^T \lambda_j(t+\tau_q)$$
$$+\sum_{q=1}^{\gamma} B_{q_{ii}}^T \lambda_i(t+\tau_q) \qquad (21)$$

$\frac{\partial H}{\partial x_i^o} = 0$, which implies:

$$x_i^o = (Q_{i_2} + Q_{i_3})^{-1}[Q_{i_3} x_i - \sum_{p=0}^{\theta} \sum_{j=1, j\neq i}^{N} A_{p_{ji}}^T \lambda_j(t+\tau_p)$$
$$- \sum_{p=1}^{\theta} A_{p_{ii}}^T \lambda_i(t+\tau_p) + \pi_i] \qquad (22)$$

Finally: $\frac{\partial H}{\partial \pi_i} = x_i - x_i^o$

which leads to the updating algorithm for π_i given by:

$$\pi_i^{\nu+1} = \pi_i^{\nu} + \gamma_i^{\nu} g_i^{\nu} \qquad (23)$$

where ν is the iteration number, g_i^ν can be specified according to the selected algorithm (steepest descent, conjugate gradient,…etc), γ_i^ν has to be positive to insure maximization w.r.t. the dual variable π_i.

From the developed necessary conditions of optimality, one can notice that:

1- $\sum_{i=1}^{N} \lambda_i^T(t)[\sum_{p=0}^{\theta} \sum_{j=1, j\neq i}^{N} A_{p_{ij}}^T x_j^o(t-\tau_p)]$

$= \sum_{i=1}^{N} \{ \sum_{p=0}^{\theta} \sum_{j=1, j\neq i}^{N} \lambda_j^T(t+\tau_p) A_{p_{ji}}^T x_i^o(t)$

This is also applicable to other similar terms.

2- From (8),(10), it is clear that if $Q_i \geq 0$, then $Q_{i_2} \geq 0$, and hence, it is necessary to add the convexifying term with $Q_{i_2} \geq 0$ in order to insure the existence of the matrix $(Q_{i_2} + Q_{i_3})^{-1}$.

To avoid the solution of the TPBVP resulting from (17)-(19), let:

$$\lambda_i = K_i x_i + s_i \qquad (24)$$

By substituting from (17) into (18), replacing λ_i by the expression given in (24), and under the assumption that $t_f >> T$, where T is the settling time of the system, we get the following decentralized algebraic Riccati equation to be solved for each subsystem:

$$A_{o_{ii}}^T K_i + K_i A_{o_{ii}} - K_i B_{o_{ii}} (R_i + R_{i_1})^{-1} B_{o_{ii}}^T K_i$$
$$+ (Q_{i_1} + Q_{i_3}) = 0 \qquad (25)$$

The differential equation for s_i is given by:

$$\dot{s}_i = (-A_{o_{ii}}^T + K_i B_{o_{ii}} (R_i + R_{i_1})^{-1} B_o^T K_i) s_i + Q_{i_3} x_i^o$$
$$-K_i(\sum_{p=0}^{\theta} \sum_{j=1, j\neq i}^{N} A_{p_{ij}}^T x_j^o(t-\tau_p)) - K_i(\sum_{p=1}^{\theta} A_{p_{ii}} x_i^o(t-\tau_p))$$
$$-K_i(\sum_{q=0}^{\gamma} \sum_{j=1, j\neq i}^{N} B_{q_{ij}} u_j^o(t-\tau_p)) - K_i(\sum_{q=1}^{\gamma} B_{q_{ii}} u_i^o(t-\tau_p))$$
$$-K_i B_{o_{ii}} (R_i + R_{i_1})^{-1} R_{i_1} u_i^o - K_i B_{o_{ii}} (R_i + R_{i_1})^{-1} \beta_i - \pi_i \qquad (26)$$

with $s_i(t_f) = 0$

Using (24) into (17), we get:

$$u_i = -(R_i + R_{i_1})^{-1} B_{o_{ii}}^T K_i x_i - (R_i + R_{i_1})^{-1} B_{o_{ii}}^T s_i$$
$$+ (R_i + R_{i_1})^{-1} R_{i_1} u_i^o - (R_i + R_{i_1})^{-1} \beta_i \qquad (27)$$

The first term in the RHS of (27) gives the closed loop decentralized component of the distributed controller whilst the remaining terms gives the open loop part which will be used to satisfy system constraints. Now by activating input constraints and after imbedding (27) into (7), we get the following equivalent expression for (7) :

$$\underline{v}_i(t) \leq v_i(t) \leq \overline{v}_i(t) \qquad (28)$$

where: $v_i(t) = -(R_i + R_{i_1})^{-1} B_{o_{ii}}^T s_i \qquad (29)$

$$\overline{v}_i(t) = \overline{u}_i + (R_i + R_{i_1})^{-1}[B_{o_{ii}}^T K_i x_i - R_{i_1} u_i^o + \beta_i] \qquad (30)$$

$$\underline{v}_i(t) = \underline{u}_i + (R_i + R_{i_1})^{-1}[B_{o_{ii}}^T K_i x_i - R_{i_1} u_i^o + \beta_i] \qquad (31)$$

Follow [11,12,14,18], the control signal which minimizes the Hamiltonian and satisfies (7) or equivalently (28) is given by:

$$v_i(t) = \begin{cases} \underline{v}_i(t) & if \quad v_i(t) < \underline{v}_i(t) \\ v_i(t) & \underline{v}_i(t) \leq v_i(t) \leq \overline{v}_i(t) \\ \overline{v}_i(t) & v_i(t) > \overline{v}_i(t) \end{cases} \qquad (32)$$

Since x_i^o is obtained from the set of algebraic equations (22), it can be treated in a similar way as the control signal. Therefore, x_i^o is calculated as follows:

$$x_i^o(t) = \begin{cases} \underline{x}_i & f_i(x,K,s,\pi) < \underline{x}_i \\ f_i(x,K,s,\pi) & \underline{x}_i \leq f_i(x,K,s,\pi) \leq \overline{x}_i \\ \overline{x}_i & f_i(x,K,s,\pi) > \overline{x}_i \end{cases} \qquad (33)$$

where:

$$f_i(x,K,s,\pi)=(Q_2+Q_3)^{-1}\{Q_3 x_i-\sum_{p=0}^{\theta}\sum_{j=1,j\neq i}^{N} A_{p_{ji}}^T[K_j x_j(t+\tau_p)$$
$$+s_j(t+\tau_p)]-\sum_{p=1}^{\theta} A_{p_{ii}}^T[K_i x_i(t+\tau_p)+s_i(t+\tau_p)]+\pi_i\} \qquad (34)$$

Remarks:

1. The closed loop part of the controller is calculated based on completely decentralized information.
2. The calculation of the open loop part of the controller, the coordinating vector x^o and the Lagrange multiplier associated with the decoupling vector u^o, necessitates the transfer of the information concerning x, x^o, s between the subsystems. However, the necessary calculations are performed in a completely decentralized framework.

4 The Algorithm:

Based on the above necessary conditions of optimality, the following algorithm is proposed:

Closed loop controller: Calculate K_i for i=1,2,..,N using (25), hence the closed loop system matrix $(A_{o_{ii}} - B_{o_{ii}}(R_i + R_{i_1})^{-1} B_{o_{ii}} K_i)$ for each subsystem.

Open loop component:

Initialization: Initialize the vectors $\pi_i^\nu, v_i^\nu, x_i^{o^\nu}, s_i^\nu$, u_i^o, β_i; put the iteration number υ=1.

Step(1): If υ=1, go to ***step (4)***

Else: calculate the error:

$$error = \sqrt{\int_{t_o}^{t_f} (\|x^\nu - x^{o\nu}\|^2 + \|u^\nu - u^{o\nu}\|^2)dt}$$

Test:

If: error < ε, where ε is a pre-specified small constant, record the trajectories and exit.

Else: update $u_i^{o^{\nu+1}}$ using (20) with $u_i^{o^{\nu-1}}$; $\gamma^{\nu+1}$ (using heuristic approach, linear search, ..etc), then the vector $\pi^{\nu+1}$; put $\nu = \nu + 1$.

Step(2): Update $\beta_i^\nu, x_i^{o^\nu}$ using (21), (33), (34) with $s^{\nu-1}, x^{\nu-1}, \pi^\nu$, hence s_i^ν by backward integration of (26) with $s_i(t_f)$=0 while incorporating $\pi_i^\nu, \beta_i^\nu, x_i^{o^\nu}, u_i^{o^\nu}$.

Step(3): Calculate v_i^ν using (29), (32) with $x_i^{\nu-1}, u_i^{o^\nu}, \beta_i^\nu$.

Step(4): Calculate x_i^ν by forward integration of (18) with $x_i(t_o) = x_{o_i}$ after replacing u_i by (27) while setting the second term equals to $v_i^\nu(t)$, then go to ***step (1).***

Having developed the decentralized controller for linear interconnected dynamical system with time delays and constraints, it will be used to solve the water quality control problem in streams.

5 BOD-DO Water Quality Control Model

The biochemical oxygen demand (BOD) and the dissolved oxygen (DO) water quality control model in streams can be described by the following differential equation [18]:

$$\dot{z} = -k_1 z - k_3 z \qquad (35)$$

$$\dot{q} = k_2(q^s - q) - k_1 z - \frac{k_4}{A_x dx} \qquad (36)$$

where *z, q,* are respectively the concentration of BOD (mg/l), and DO (mg/l), k_1 is the rate of decay of BOD, k_3 is the rate of loss of BOD due to settling, q^s is the concentration of DO at saturation level, k_2 is the re-aerations rate, ($k_4/A_x dx$) is the removal of DO due to bottom sludge requirement.

Defining a reach of a river as a section of the water body receiving one major controlled effluent to be discharged from a sewage or industrial water treatment facility. Assuming that perfect mixing takes place in each reach. Let Q_{Ei} be the flow rate of effluent in the i^th^ reach, Q_i is the stream flow rate, V_i is the water volume in the i^th^ reach, u_i is the concentration of BOD in effluent to be discharged in the i^th^ reach, y_i ,y_{i-1} are the vectors of concentration of the water quality constituents in the i^th^ and (i-1)^th^ reaches and y_{i-1} affects the i^th^ reach through a distributed delay model [19]. Therefore, the model describing the concentration of BOD and DO in the i^th^ reach is given by:

$$\dot{z}_i = -(k_1 + k_3)z_i + \frac{Q_i}{V_i}\sum_{i=1}^{j} a_i z_{i-1}(t-\tau_j)$$
$$-\frac{Q_i + Q_{Ei}}{V_i} z_i + \frac{Q_{Ei}}{V_i} u_i \qquad (37)$$

$$\dot{q}_i = k_1 z_i - k_2 q_i + \frac{Q_i}{V_i}\sum_{i=1}^{j} a_i q_{i-1}(t-\tau_j)$$
$$-\frac{Q_i + Q_{Ei}}{V_i} q_i + k_2 q^s \qquad (38)$$

where *j* is the number of delays.

Under steady state conditions, the BOD concentration in the effluent to be discharged in the i^th^ reach is treated to a desired level u_i^* which maintains the concentration of BOD and DO in the water body at the desired values z_i^*, q_i^*. When abnormal conditions take place, what is required from the river system is to maintain pollution levels within reasonable values as close as possible to the desired levels and at the same time guarantee that these levels do not exceed pre-specified upper limits to satisfy both community needs and the ecological balance in the river system. On the other hand, the effluent to be discharged from a sewage treatment facilities can be controlled either by discharging constant flow rate while using some variable treatment of sewage effluent, or by maintaining fixed level of treatment while controlling effluent flow rate. Choosing the first control methodology, lower bounds of the concentration of BOD in the effluent discharge has to be specified, otherwise the cost of effluent treatment will be very high. Therefore, the optimization problem associated with a river system can be described as follows:

$$\min J = \frac{1}{2}\sum_{i=1}^{N}\int_{t_o}^{t_f} (z_i - z_i^*)^2_{Q_{i_{11}}} + (q_i - q_i^*)^2_{Q_{i_{22}}} + R_i(u_i - u_i^*)^2\, dt \quad (39)$$

subject to (37), (38) and

$$z_i \le \bar{z}_i \quad (40)$$

$$u_i \ge \underline{u}_i \quad (41)$$

where N is the number of reaches.

Let $x = y - y^*$, $\bar{x}_i = \bar{y}_i - y_i^*$, $\Delta_i u = u_i - u_i^*$, $\underline{\Delta u}_i = \underline{u}_i - u_i^*$, $\overline{\Delta u_i} = \overline{u_i} - u_i^*$ then the above optimization problem can be written in the form described by (10)-(15) in which the state and control variables represent the deviation from steady state values.

6 Simulation Results

For simulation purposes, a five reach river system is considered. Typical data for this system is as follows:

$$k_1 = 0.32, k_2 = 0.2, k_3 = 0.0, \frac{Q_i}{V_i} = 0.9, \frac{Q_{Ei}}{V_i} = 0.1, q^s = 10mg/l$$

$$\frac{k_4}{V_1} = 0.1, number\ delayes = 3, a_1 = 0.15, a_2 = 0.7, a_3 = 0.15,$$

$$\tau_1 = 0, \tau_2 = 0.5, \tau_3 = 1.0, Q_{i_{11}} = 2.0, Q_{i_{22}} = 1.0, R_i = 1, i = 1,2,..,5.$$

The above system is decomposed into five subsystems. For each one, the closed loop part of the controller is firstly calculated. Then the developed algorithm is implemented to calculate the open loop part of the controller. For illustration purposes, it is assumed that the system is at steady state. Then an impulsive disturbance took place in the first reach which can be represented by the following initial conditions, indicating the deviation from steady state values of the system:

$x(0)$=[6.0 -2.0 0 0 0 0 0 0 0 0]

As a result, pollutants will propagate along the different reaches of the river, and consequently, will disturb the equilibrium balance of the system. The control strategy, and hence, the deviation of the constituents concentration from their steady state values which optimize the cost function under no constraints are shown in Figs.1-3.

Now, it is assumed that the system has the following set of constraints which must not be violated in order to satisfy what is required from the river system: $x_3 \le 1.0$, $x_5 \le 0.3$, $x_7 \le 0.1$, $\Delta u_i \ge -6.0\ \forall i \varepsilon \{1,2,..,5\}$

Figs.4-6 show the optimal solution obtained for this case which also satisfied imposed system constraints.

7 Conclusion

In this paper, water quality control in streams with time-delays and system constraints is studied. A distributed control structure is developed to solve the problem. The proposed controller consists of two parts; the closed loop part and the open loop one. Under normal operating conditions, the two parts of the controller insure optimal system performance and the satisfaction of system constraints. Through the proper choice of he weighting matrices Q_1, Q_3, the closed loop component of the controller can insure system reliability under any disturbances which may take place in the communication network. All necessary calculations can be done in a completely decentralized framework which allows parallel processing through computer networks, and hence a great saving in execution time. Simulation results show the applicability of the proposed technique to solve geographically distributed water quality control problems.

Acknowledgment

This work has supported by Kuwait University, Research Grant No. [EE02/04].

References

[1] Chen,S.-H. and J.H. Chou, Stability robustness of linear discrete singular time-delay systems with structured parameter uncertainties, *Control Theory and Applications, IEE Proceedings,* vol.150, Issue 3,2003, pp.295-302.

[2] Shugian, Z., L. Zhenbo and C. Zhaolin, Delay-dependent robust resilient H_∞ , control for

uncertain singular time delay systems, *Proc. 0f 44th IEEE Conf on Decision and Control*, 2005, pp.1373-1378.
[3] Moheimani, S.O.R. and I.R. Petersen, Optimal quadratic guaranteed cost control of a class of uncertain time-delay systems, *Control Theory and Applications, IEE Proceedings,* vol.144, no.2, 1997, pp.183-188.
[4] Mahmoud, M. S., M. F. Hassan, M. G. Darwish, *Large scale control systems: Theories and Techniques*, Marcel Dekker Inc., 1985.
[5] Singh, M. G. and A. Titli, *Systems decomposition, optimization and control*, Pergamon press. 1978.
[6] Bemporad, A., F. Borrelli and M. Morari, Min-max control of constrained uncertain discrete-time linear systems, *IEEE Trans. AC*, vol 48, 2003, pp.1600-1608.
[7] Bemporad, A., M. Morari, V. Dua and E. N. Pistikopoulous, The explicit linear quadratic regulator for constrained systems, *Automatica*, vol. 38, 2002, pp. 3-20.
[8] Bemporad, A., F. Borrelli and M. Morari, The explicit solution of constrained LP-based receding horizon control, presented at *Decision and Control, Proceedings of the 39" IEEE Conference,* vol.1, 2000, pp.632-637.
[9] Mulder ,O. F., M. V. Kothare and M. Morari, Multivariable anti-windup controller synthesis iterative linear Matrix inequality, *Automatica*, vol.37, no.9, 2001, pp.1 407-1416.
[10] Teel, A. and N. Kapoor, The L2 anti-windup problems, its definition and solution, *European Control Conference*, Brussels, 1997, Belgium.
[11] Hassan, M. F., Solution of linear quadratic constraint problem via coordinating approach, *International Journal of Innovative Computing, Information and Control (IJICIC)* , vol.2, no.3,2006.
[12] Hassan, M. F. and N.B. Almutairi, New algorithm for complex optimization problem with inequality constraints, *Journal of Autosoft Intelligent Automation and Soft Computing*, vol.12, no..,2,2006, pp. 151-172.
[13] Hassan, M. F., Large scale discrete LQP with system constraints, *International Journal of Automatic Control and system Engineering* vol.5, issue 4, 2005, pp 13-26.
[14] Hassan, M. F. and A. K. Ali, A coordinating approach for the solution of discrete time servo-mechanism problem with constraints, *presented for publication.*
[15] Hassan, M. F., A developed algorithm for solving constrained linear quadratic problems with time delays, 4th *WSEAS/IASME International Conference on : system science and simulation engineering, Tenerife, Canary Island, Spain,* 16-18 December, 2005, pp 71-76.
[16] Hassan, M. F., Convergence analysis of a coordinating approach for solving constrained linear quadratic problems with time delay, *WSEAS Transactions on systems,* Issue 3,5, March 2006, pp 450-457.
[17] Hassan, M. F. and E. Boukas, Multilevel Technique for large scale LQR with time-delays and system constraints, *Journal of Nonlinear Dynamics and Systems Theory.* (To appear)
[18] Kirk, D.E., *Optimal control theory: An introduction,* Englewood Cliffs, N.J., Prentice Hall, 1970.
[19] Hassan, M. F., Water quality simulation and control in streams, *Encyclopedia of environmental control technology,* vol. 3, *Waste Water Treatment Technology,* Golf Publishing Company, 1989, pp. 77-138.

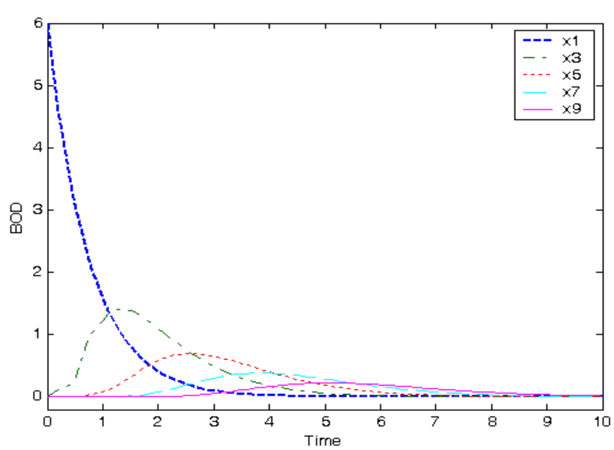

Figure (1) : BOD changes with no constraints

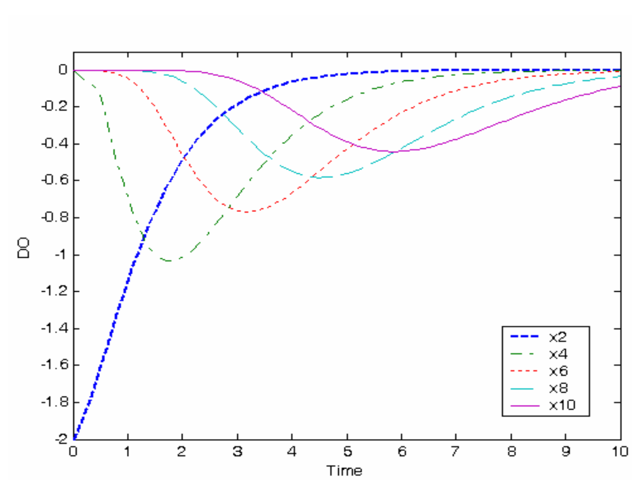

Figure (2) : DO changes with no constraints

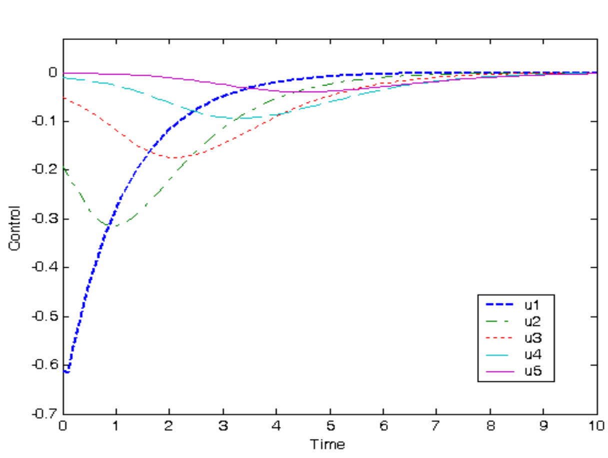

Figure (3) : control changes with no constraints

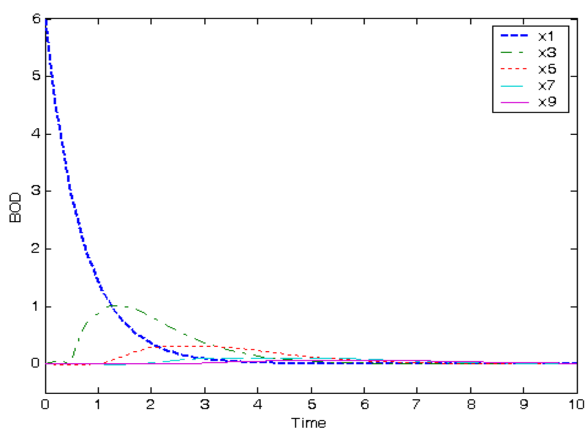

Figure (4) : BOD changes with constraints

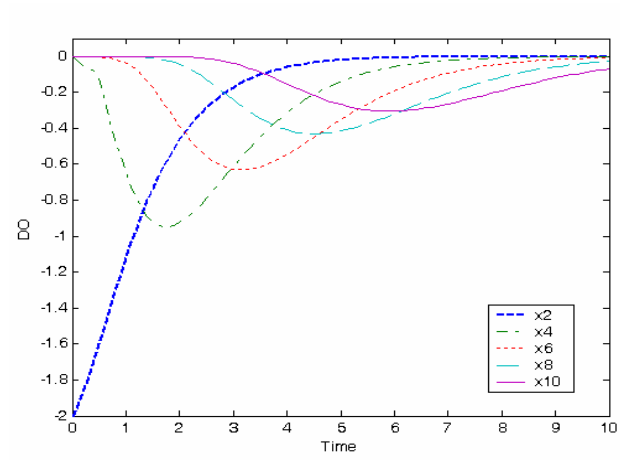

Figure (5) : DO changes with constraints

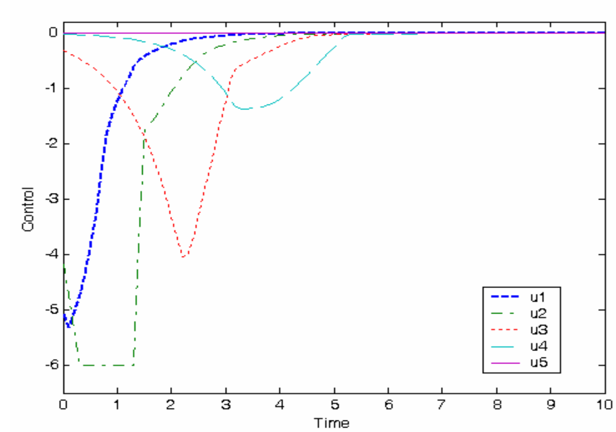

Figure (6) : control changes with constraints