Proceedings of the 2006 IASME/WSEAS Int. Conf. on Water Resources, Hydraulics & Hydrology, Chalkida, Greece, May 11-13, 2006 (pp146-151)

Multi-Stage Stochastic Model for a Multipurpose Water Reservoir With Target-Priority Operation

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Abstract: The capacity planning and operation problem of a multi-purpose water reservoir has been extensively studied in the literature. Optimization techniques, especially linear programming, have been used quite successfully. Although most models are robust enough as planning tools, they fall short of providing an operational policy for the shorter term. In this paper, we consider the operation of a multi-purpose water reservoir problem. To eliminate the shortfalls of the classical models, we develop a multistage stochastic programming model that generates a release policy adapted to the uncertainty in the problem for shorter time periods. The model is designed to be flexible enough to be used as a planning or an operational tool. Alternative solution approaches and future research directives are discussed.

Key–Words: stochastic programming, decision rules, target-priority policy, multistage stochastic programming, linear programming, water resources, reservoir operation.

1 Introduction

Water resources management has been studied extensively in the literature. In the past 30 years, many optimization techniques have been developed to optimize the management and operation of water resources. The planning and operation of water reservoirs have a strategic importance due to the high construction costs and to the importance of making reliable operational decisions. This stems from the fact that most reservoirs are of the multi-purpose type. Although most reservoirs have one primary objective such as generating hydropower or mitigating flood hazards, they can also be used to satisfy downstream demand for water for irrigation, enhancing wildlife, store water for drought periods,...etc. In this literature review, we shall focus mainly on linear and stochastic programming. Turgeon (2005) finds the optimal daily operating policy of a reservoir subject to yearly probabilistic constraints on floods and shortages. He highlights the difficulty presented by the fact that inflows are stochastic. He then decomposes the original problem into two subproblems that are solved with a set of inflow scenarios. Lamond et al. (1995) propose exact and approximate optimal policies for a reservoir hydroelectric system, where the inflows in successive periods are random variables. The model they analyze is a discrete-time one. Wang et al. (2004) consider the short-term scheduling of large hydro-power systems for energy maximization. They formulate the problem as a nonlinear program with linear constraints and solve it using a direct search procedure. In Sreenivasan et al.(1996) and Edirisinghe et al.(2000), the authors present a chance-constrained (stochastic) model to take into account the uncertainty in meeting system operation requirements. Edirisinghe et al. added a target-priority characteristic to the model, whereby demand for downstream water targets is given priority. Turgeon(1981) developed stochastic dynamic programming (SDP) models for the optimization of weekly operating policies of multireservoir hydroelectric power systems. Dorfman(1962) and Dupacova(1980) applied the same idea to the problem of water resources management and planning. In this type of models, the decisions in the consequent periods may be represented by loss functions of not meeting some operational characteristics. Stochastic linear programs (LP) for Markov processes have been studied by Manne(1962) and Thomas and Watermeyer(1962). Loucks(1968) developed a stochastic LP for a single reservoir subject to random, serially correlated, net inflows that were described by a first order Markov chain and transition probabilities were estimated using historical inflows. Houck and Cohon(1978) also assumed a discrete Markov structure for the streamflows. Dantzig(1955) suggested an LP model which includes random variables. In his model, the activity levels are determined in the first stage, then a corrective action is followed in the second stage. This is known as *stochastic programming with recourse*.

In a deterministic environment, Morel-Seytoux (1999) define an optimal daily operating policy for a system of rivers and associated reservoirs. He points in particular to the delicate interdependency in the optimization of the objectives and the hydraulic characteristics of the system. Martin (1995) develop a methodology using both optimization and simulation techniques to evaluate the ability of the hydropower plants to meet weather-related winter peak power requirements. He uses a linear programming procedure to determine the hourly generation schedule for the Lower Colorado River Authority of Texas. The simulation/optimization modeling of water resources has also been studied by Belaineh and Peralta (1999). The integrative approach is quite efficient for solving large complex problems. In this case, the authors integrate linear decision rules with detailed simulations of stream/aquifer system flows. Lamond and Sobel (1995) discuss the exact and approximate solutions of affine reservoir models where autocorrelated inflows are modeled with a linear autoregressive stochastic process. The important finding they report is the fact that a myopic policy, for the case where interbasin transfers are included, is optimal if the deterministic and stochastic portions of the inflow process are always non-negative. Yang et al. (1995) compare real-time reservoir operation techniques and confirm the value of simple optimization methods and the applicability of scenarios methods in real-time reservoir operation.

However, deterministic models have important limitations that Philbrick and Kitandis (1999) report. The authors contrast the control policies developed using deterministic optimization with policies using stochastic optimization of probabilistic inflows and conclude that the stochastic approach is more accurate. For a state-of-the-art review of the optimal operation of multireservoir systems, the reader is directed to refer to the Labadie (2004) review paper and the closure and discussion of that review by Labadie (2005) and Lund (2005). Other approaches to the management and operation of water reservoirs have been developed in the literature. For instance, fuzzy multi-stage stochastic programs such as in Maqsood et al. (2005), fuzzy-state stochastic dynamic programming such as in Mousavi et al. (2004), and neural networks such as in Chandramouli and Raman (2001). However, as mentioned above, the emphasis of this review is on the linear and stochastic programming approaches.

In this paper, we develop a multi-stage stochastic programming model for the reservoir problem involving multiple periods representing 12 months of operation. The main source of randomness in the reservoir is the monthly water inflow to the reservoir. The downstream demand for irrigation water is prescribed *a priori* and thus it is not random, see Edirisinghe et al (2000). During any month, the randomness of inflow will be modeled by a sample of discrete outcomes, generated randomly subject to the history of inflow up until that month. In the sequel, we will develop a *scenario tree* of potential future inflow patterns. With fairly dense scenario trees, such models tend to become exponentially large as the number of stages and periods increase, and thus the computational cost to solve them also increases exponentially. Therefore, it would be imperative to either use approximation techniques such as Edirisinghe (1999), and/or exploit the structure of the problem and devise decomposition techniques that render efficient solution of the multi-period reservoir model with scenario trees.

In section 2, the multistage stochastic programming model is developed. In section 3, solution alternatives are presented and discussed. Summary and future research remarks in section 4 conclude this exposition.

2 Multistage Stochastic Model

The reservoir manager must make a release decision before knowing what the inflows will be in the future. Therefore, the model we devise is *nonanticipative*, and it requires a "here-and-now" solution. The proposed model will minimize the deviations from the specified reservoir operation characteristics, such as the firm energy level and the dead storage level. Operational or recourse costs are imposed on the model so as to penalize the system operation that would tend to violate the specified system constraints. These will be discussed next.

2.1 System Constraints

In CCP models, the system constraints are specified as chance constraints, where constraint violations are allowed and controlled via probabilities. In our model, the degree of violation of a constraint is considered and controlled explicitly. First, the storage level at the beginning of month $(t+1)$, S_t, must be at least SD, the dead storage level, for energy to be generated. Therefore the deviation from SD, denoted by δ^{SD}, is modeled by the following equation:

$$S_t - SD = \delta_t^{SD}. \qquad (1)$$

Note that δ_t^{SD} is a random variable and $\delta_t^{SD} \geq 0$ indicates the satisfaction of the dead storage constraint in month t.

The reservoir is also used to mitigate flood hazards during high inflow seasons. The deviation from maintaining a specified flood reserve, V_t in month t, is given by the equation:

$$S_t - (K - V_t) = \delta_t^F. \qquad (2)$$

where K is the reservoir size. In order to ensure continued operation of the reservoir in subsequent years provided that the inflow distribution remains unchanged, we require the terminal storage, S_T, be close in value to the initial storage, S_0. The deviation of S_T from the initial storage level S_0 is given by the equation:

$$S_T - S_0 = \delta_T^{S_0}. \qquad (3)$$

In Edirisinghe et al. (2000), modeling the target priority directly with a linear constraint was not possible since the releases were considered to be deterministic. A *surrogate* constraint was used to ensure demand for water was satisfied with a certain probability. In the present model, releases, R_t, are not constrained to be deterministic, i.e. releases conform to a nonanticipative policy. Therefore, the deviation of meeting water targets, T_t, are given by:

$$R_t - T_t = \delta_t^D. \qquad (4)$$

Note that flood reserve constraint violations correspond to $\delta_t^F > 0$, and water target constraint violations correspond to $\delta_t^D < 0$. However, violation of the overyear storage requirement indicates $\delta_T^{S_0} \neq 0$. Since the operation of a water reservoir is a continuous process in time, the ending storage and the beginning storage are related by the continuity equation,

$$S_t = S_{t-1} + I_t - R_t, \qquad (5)$$

assuming no other loss of water is possible, where I_t is the inflow realized in period t. In the next section, we consider the case of modeling the energy generation under the stochastic programming approach.

2.2 Energy generation

The firm energy level, defined as the minimum guaranteed energy generated throughout a planning horizon, was maximized subject to the system constraints and that the target priority in the release policy is satisfied. In order to maintain the target priority nature and for computational convenience, a Δ_0 release policy was considered in the CCP model. However, in the present model, such a restriction is not needed and the releases are random functions that depend on the history of inflow realizations. In order to maximize the firm energy level, a certain firm energy level is specified to the model and the deviation of $\min(EG_t, t = 1, \ldots, T)$, is accounted for and minimized as will be explained next .

2.2.1 Energy generation constraint

The energy generated is a function of the release and the average water head on the turbines. Given a transition of the system form S_{t-1} to S_t, the energy generated at period t can be represented by the following

$$EG_t = \omega R_t[\frac{e}{2}(S_t + S_{t-1}) + f]. \qquad (6)$$

where e and f are constants based on a typical operating range of the reservoir, and ω is a dimensional constant that reflects turbine efficiency. Note that the release R_t will not contribute toward energy generation if both S_t and S_{t-1} are below the dead storage level. In order to compute the exact value of the energy generation, we define the variables x_t, y_t, h_t, and z_t as follows

$$x_t = \begin{cases} 1 & \text{if } S_t \geq \text{SD}, \\ 0 & \text{otherwise} \end{cases} \qquad (7)$$

$$y_t = \begin{cases} 0 & \text{if } S_t \geq \text{SD}, \\ -\delta_t^{SD} & \text{otherwise} \end{cases} \qquad (8)$$

$$h_t = \begin{cases} \delta_t^{SD} & \text{if } S_t \geq \text{SD}, \\ 0 & \text{otherwise} \end{cases} \qquad (9)$$

$$z_t = \begin{cases} 1 & \text{if } x_{t-1}\text{=1 OR } x_t\text{=1}, \\ 0 & \text{otherwise} \end{cases} \qquad (10)$$

Note that $(S_t + S_{t-1})$ may be restated as:

$$S_t + S_{t-1} = 2SD + \delta_t^{SD} + \delta_{t-1}^{SD}. \qquad (11)$$

For the case when both the beginning and ending storages in month t are above the dead storage,

$$S_t + S_{t-1} = 2SD + h_t + h_{t-1} \qquad (12)$$

holds. In general, however, $\frac{1}{2}(2SD + h_t + h_{t-1})$ represent the average "effective storage" available for hydropower generation. We also want to determine the *effective release*, defined as the released amount of water that contributes toward energy generation. This can be done by subtracting the amount of water below SD from the release R_t as follows

$$R_{eff} = R_t - y_t - y_{t-1} \qquad (13)$$

The energy generation function can therefore be written as follows

$$EG_t = \omega z_t(R_t - y_t - y_{t-1})[\frac{e}{2}(2SD + h_t + h_{t-1}) + f] \qquad (14)$$

Observe now that only the amount of water released that contributes toward energy generation is taken into account. Likewise, the average water head on the turbines is not over-estimated by taking the simple average of S_{t-1} and S_t as in the CCP model. Now to ensure a given firm energy level, say η, any deviation from η, which we shall represent by δ^{EG}, is penalized. Therefore, the energy generation constraint violation can be written as follows:

$$\eta - EG_t = \delta_t^{EG}. \tag{15}$$

2.3 Multistage stochastic model

2.3.1 Rewriting the constraints

As we mentioned in the introduction, the energy authority has to make *here-and-now* decisions regarding the releases. This implies that the release in the first period, R_1, is independent of the inflow. The same holds true for the target violation variable δ_1^D since the target satisfaction depends only on the release. However, the storage at the end of period 1 depends on the inflows due to the continuity equation (5). This dependence implies that δ_1^{SD}, δ_1^F, and δ_1^{EG} also depend on the realization of the random event, i.e. which inflow occurred. Note, however, that the release in period 2 depends on the ending storage of period 1, S_1, and does hence depend on the inflow in period 1, I_1. To reflect these dependencies on which of the inflows was manifested, let $\mathcal{H}_{t-1} := I_1, \ldots, I_{t-1}$ be the history of inflows up to a period t. The constraints (5),(4),(2), (1), and(3), are written as follows

$$S_{t,\mathcal{H}_{t-1}} = S_{t-1,\mathcal{H}_{t-1}} + I_{t,\mathcal{H}_{t-1}} - R_{t,\mathcal{H}_{t-1}} \quad ; t = 1, \ldots, T \tag{16}$$

$$R_{t,\mathcal{H}_{t-1}} - T_{t,\mathcal{H}_{t-1}} = \delta^D_{t,\mathcal{H}_{t-1}}; \quad t = 1, \ldots, T \tag{17}$$

$$S_{t,\mathcal{H}_{t-1}} - (K - V_t) = \delta^F_{t,\mathcal{H}_{t-1}}; \quad t = 1, \ldots, T \tag{18}$$

$$S_{t,\mathcal{H}_{t-1}} - SD = \delta^{SD}_{t,\mathcal{H}_{t-1}}; \quad t = 1, \ldots, T \tag{19}$$

$$S_{T,\mathcal{H}_{T-1}} - S_0 = \delta^{S_0}_{T,\mathcal{H}_{T-1}}; \quad t = T \tag{20}$$

The energy generation constraint is more cumbersome since we want to consider a firm-energy level, which is the same over a specified time horizon. Therefore, the penalty δ_t^{EG} is taken as the deviation of the firm energy level from the specified energy level across a scenario. This delicate dependence can be easily represented by

$$\eta - EG_{t,\mathcal{H}_{t-1}} = \delta^{EG}_{T,\mathcal{H}_{T-1}}; \quad t = 1, \ldots, T \tag{21}$$

2.3.2 The objective function

In the objective function, we want to minimize the penalty from operating the reservoir. The penalty being the cost of deviating from the reservoir operating characteristics. So it can be represented by the following

$$\min \quad F(\delta^{EG}_{t,\mathcal{H}_{t-1}}, \delta^{S_0}_{T,\mathcal{H}_{t-1}}, \delta^{D}_{t,\mathcal{H}_{t-1}}, \delta^{F}_{t,\mathcal{H}_{t-1}}, \delta^{SD}_{t,\mathcal{H}_{t-1}}); \quad t = 1, \ldots, T. \tag{22}$$

Note, however, that δ_t^{SD} need not be considered explicitly in the objective function since it's impact is implicitly penalized in the energy generation function EG_t. The cost function in (22) is nothing but the sum of the expected cost of each variable in each scenario. The complete objective function can be written therefore as

$$\begin{aligned} \min \quad \lambda[& \sum_{t=1}^{T} \mathcal{P}^{\mathcal{H}_{t-1}} \mathcal{F}_{\delta^{EG}}(\delta^{EG}_{t,\mathcal{H}_{t-1}}) \\ & + \sum_{\varphi} \mathcal{P}^{\mathcal{H}_{T-1}} \mathcal{F}_{\delta^{S_0}}(\delta^{S_0}_{\mathcal{H}_{T-1}}) \\ & + \sum_{t=1}^{T} (\mathcal{P}^{\mathcal{H}_{t-1}} \mathcal{F}_{\delta^{D}}(\delta^{D}_{t,\mathcal{H}_{t-1}})) \\ & + \sum_{t=1}^{T} (\mathcal{P}^{\mathcal{H}_{t-1}} \mathcal{F}_{\delta^{F}}(\delta^{F}_{t,\mathcal{H}_{t-1}}))] \end{aligned} \tag{23}$$

where φ is the set of all possible scenarios.

2.3.3 Complete Multistage Stochastic Model

We have defined the variables x_t, y_t, h_t, and z_t earlier to define the energy generation constraint. However, we have introduced them as indicator function rather than constraints that can be included in the complete formulation of the model. To convert those to constraints, we proceed as follows. Let M denote a very large number.

$$M(x_{t,\mathcal{H}_{t-1}} - 1) \leq \delta^{SD}_{t,\mathcal{H}_{t-1}}; \quad t = 1, \ldots, T \tag{24}$$

$$-Mx_{t,\mathcal{H}_{t-1}} - y_{t,\mathcal{H}_{t-1}} \leq \delta^{D}_{t,\mathcal{H}_{t-1}}; \quad t = 1, \ldots, T \tag{25}$$

$$y_{t,\mathcal{H}_{t-1}} \geq 0; \quad t = 1, \ldots, T \tag{26}$$

$$z_{t,\mathcal{H}_{t-1}} \leq x_{t,\mathcal{H}_{t-1}} + x_{t-1,\mathcal{H}_{t-1}}; \quad t = 1, \ldots, T \tag{27}$$

$$0 \leq z_{t,\mathcal{H}_{t-1}} \leq 1; \quad t = 1, \ldots, T \tag{28}$$

$$h_{t,\mathcal{H}_{t-1}} = x_{t,\mathcal{H}_{t-1}} \delta^{SD}_{t,\mathcal{H}_{t-1}}; \quad t = 1, \ldots, T \tag{29}$$

With these definitions (constraints) in place, the multistage stochastic model is completely defined. The formulation is presented in its general form as the following MultiStage Stochastic Program ($MSSP_T$).

$$\min \lambda[\sum_{t=1}^{T} \mathcal{P}^{\mathcal{H}_{t-1}} \mathcal{F}_{\delta^{EG}}(\delta^{EG}_{t,\mathcal{H}_{t-1}})$$
$$+ \sum_{\varphi} \mathcal{P}^{\mathcal{H}_{T-1}} \mathcal{F}_{\delta^{S_0}}(\delta^{S_0}_{\mathcal{H}_{T-1}})$$
$$+ \sum_{t=1}^{T} (\mathcal{P}^{\mathcal{H}_{t-1}} \mathcal{F}_{\delta^{D}}(\delta^{D}_{t,\mathcal{H}_{t-1}}))$$
$$+ \sum_{t=1}^{T} (\mathcal{P}^{\mathcal{H}_{t-1}} \mathcal{F}_{\delta^{F}}(\delta^{F}_{t,\mathcal{H}_{t-1}}))]$$

s.t.

$$S_{t,\mathcal{H}_{t-1}} = S_{t-1,\mathcal{H}_{t-1}} + I_{t,\mathcal{H}_{t-1}} - R_{t,\mathcal{H}_{t-1}}; t = 1, \ldots, T$$
$$R_{t,\mathcal{H}_{t-1}} - T_{t,\mathcal{H}_{t-1}} = \delta^{D}_{t,\mathcal{H}_{t-1}}; \quad t = 1, \ldots, T$$
$$S_{t,\mathcal{H}_{t-1}} - (K - V_t) = \delta^{F}_{t,\mathcal{H}_{t-1}}; \quad t = 1, \ldots, T$$
$$S_{t,\mathcal{H}_{t-1}} - SD = \delta^{SD}_{t,\mathcal{H}_{t-1}}; \quad t = 1, \ldots, T$$
$$S_{T,\mathcal{H}_{T-1}} - S_0 = \delta^{S_0}_{T,\mathcal{H}_{T-1}}; \quad t = T$$

$$M(x_{t,\mathcal{H}_{t-1}} - 1) \leq \delta^{SD}_{t,\mathcal{H}_{t-1}}; \quad t = 1, \ldots, T$$
$$-Mx_{t,\mathcal{H}_{t-1}} - y_{t,\mathcal{H}_{t-1}} \leq \delta^{D}_{t,\mathcal{H}_{t-1}}; \quad t = 1, \ldots, T$$
$$y_{t,\mathcal{H}_{t-1}} \geq 0; \quad t = 1, \ldots, T$$
$$z_{t,\mathcal{H}_{t-1}} \leq x_{t,\mathcal{H}_{t-1}} + x_{t-1,\mathcal{H}_{t-1}}; \quad t = 1, \ldots, T$$
$$0 \leq z_{t,\mathcal{H}_{t-1}} \leq 1; \quad t = 1, \ldots, T$$
$$h_{t,\mathcal{H}_{t-1}} = x_{t,\mathcal{H}_{t-1}} \delta^{SD}_{t,\mathcal{H}_{t-1}}; \quad t = 1, \ldots, T$$

$$\eta - EG_{t,\mathcal{H}_{t-1}} = \delta^{EG}_{T,\mathcal{H}_{T-1}}; \quad t = 1, \ldots, T$$

(30)

This is a general formulation in all aspects. The time horizon is left as a parameter, T, and so are the penalty cost functions. Now that the model is complete, a solution procedure needs to be devised. In the next section, we discuss the different alternatives to solve this model efficiently.

3 Solution Alternatives

The above formulated model suffers from the *curse of dimensionality*, whereby these models become very large as the number of periods and scenarios increase. Despite the tremendous computing power that is available nowadays, it would still be quite impractical to solve such a model using conventional, even upscale, linear programming solvers. With only 12 periods and 10 outcomes per period, the model will have to consider 10^{12} different scenarios. Stochastic dynamic programming (SDP) can be used as a tool to solve these models. It has been proposed and used in the literature. However, SDP models also suffer from the curse of dimensionality, and as the number of nodes increase, the solution cost increases exponentially. Hence, it would be imperative to look into the structure of such models, and attempt to devise solution algorithms that exploit the problem structure in order to render an efficient and practical solution. This venue has been taken by the author and such solution technique has been developed.

4 Conclusion

We have presented a multistage stochastic programming model to study the planning and operation problem for a single multi-purpose water reservoir. The stochastic programming models have the flexibility of accounting for inflow dependence to a much higher degree. This is due to the fact that these models actually consider the randomness in the inflows explicitly in the search for a solution through a scenario approach, whereas the CCP model only uses the marginal distributions. While the proposed model provides a good long-term operation tool, its focus is limited to monthly decision periods. An operational model would have to take into account a decision period much shorter than a month, and would need to have the flexibility of providing better solutions as random events unfold. An other alternative is the *rolling horizon* approach, where the model is re-solved at the end of each operational period, the model being revised with new observations of inflow data. These avenues would certainly be worth considering and should be the subject of future research.

Acknowledgements: Part of this research was supported by a SARIF grant from the University of Tennessee in the summer of 1999. The authors would also like to thank two anonymous referees for their constructive comments.

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