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High Speed Couette – Poiseuille Flow Stability in Reverse Flow Conditions

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Abstract: - The linear stability of reverse high speed-viscous plane Couette – Poiseuille flow is investigated numerically. The conservation equations along with Sutherland's viscosity law are studied using a second order finite difference scheme. Basic velocity and temperature distributions are perturbed by a small amplitude normal-mode disturbance. Small amplitude disturbance equations are solved numerically using a global method to find all the eigenvalues at finite Reynolds numbers. The results indicate that instabilities occur, although the corresponding growth rates are often small. The aim of the study is to see the effect of the reverse flow on the stability compared to the direct flow. In the combined plane Couette – Poiseuille flow, the new mode, Mode 0, which seems to be a member of even modes such as Mode II, is the most unstable mode.

Key-Words: - Couette flow, Poiseuille flow, High Speed flow , Viscous flow, Flow Stability, Eigenvalue, Reverse flow

1 Introduction

In an internal combustion engine, the processes leading to the formation of pollutants are complex and therefore experimental techniques are of widespread use in engine development. The oil flow through the piston-cylinder system has numerous flow passages and local volumes where engine oil can flow through and accumulate ending up in the exhaust system. Combustion gases also flow through the same passages and volumes simultaneously, resulting in a complex two-phase flow phenomenon. The flow velocities approach sonic speeds in the ring end gaps, where engine oil is entrained in the high speed gases. The pressure gradient, and the wall velocity may have opposing effects on the flow direction during the compression stroke. Understanding the physics of this complex problem would enhance the understanding and controlling the entrainment of engine oil that is later released to the atmosphere as a source of unburned hydrocarbons. It is the aim of this study to understand the parameters leading to flow instability in reverse Couette – Poiseuille flows, where the effect of the pressure gradient and velocity at the wall are opposite, and the gas velocity approaches sonic speeds.

1.2 Review of Previous Work

The stability of Couette and Poiseuille flows has been under investigation for a long time. Analysis of the incompressible viscous and/or inviscid stability problem based on the Orr-Sommerfeld equation has been widely reported in the literature. Yih considered the stability of superposed fluids of different viscosity in plane Couette and Poiseuille flow. The variation of viscosity in a fluid can cause instability. He concluded that both plane Couette and Poiseuille flows can become unstable even for small Reynolds numbers [6]. Malik compared various numerical methods for the solution of stability equations for compressible boundary layers. He discussed both the global and the local eigenvalue methods for temporal stability analysis [5]. Hu and Zhong studied the viscous linear stability of supersonic Couette flow using two global methods to solve the linear stability equation. They used a fourth order finite difference method and a spectral collocation method. They found that two wave modes are unstable at finite Reynolds number. These modes are acoustic modes created by sustained acoustic reflections between a wall and a relative sonic line when the mean flow in the local region is supersonic with respect to the wave velocities. Effects of compressibility, three dimensionality and wall cooling on the two wave families are also studied [4]. Orszag solved the Orr-Sommerfeld equation numerically using expansions in Chebyshev polynomials and the QR matrix eigenvalue algorithm. The method was applied to the stability of plane Poiseuille flow and it was shown that results of great accuracy could be obtained very economically. He found the critical Reynolds number as 5772.22, reducing all lengths by the half-width of the channel and velocities by the undisturbed stream velocity at the center of the channel [8].

2 Problem Formulation

Under brake conditions of an engine, the pressure in the intake manifold becomes lower than the crankcase. When this occurs, lubricating oil is sucked from the crankcase through the piston clearances and ring gaps into the combustion chamber. To understand the entrainment of oil into the high-speed gas, it is necessary to investigate the instabilities at the oil and gas interface. The thickness and the velocity of the oil film moving on the stationary plate are small compared to the high-speed gas flow above the oil film and regarding the high viscosity of oil compared to that of air, the oil layer behaves like a solid wall as far as the stability of the gas phase is concerned. Therefore, it is possible to study the stability of the high-speed gas flow only, and interpret the findings to apply to the gas-oil system.

Although this approach simplifies the problem, the formulation does not compromise the physics and the omission of the oil layer in the analysis is not expected to have an effect on the magnitude of critical Reynolds number for the combined plane Couette-Poiseuille flow. The flow geometry is given in Fig.1. This approach is justified by Özgen [3] who studied the characteristics of the instability of Newtonian and non-Newtonian liquid-air system for low speed flows and concluded that for the case of air flowing over a thin layer of liquid, there is negligible effect of thin liquid layer on the stability of the two-phase flow provided that the liquid viscosity is much higher than the gas viscoisty. The fact that, the existence of a thin liquid layer has little contribution to the two-phase flow instability simplifies and allows the formulation of the problem for a single layer gas flow.

3 Problem Solution

3.1 Flow Description and Objectives

The aim of the present study is to understand the effects of viscosity, temperature, compressibility and density on the stability of high speed parallel shear flows. The upper wall is moving in reverse streamwise direction while the gas stream is flowing in streamwise direction and the lower plate stays stationary. The high speed gas flow is viscous with compressibility effects, is parallel and fully developed. Velocity and temperature profiles are functions of the normal distance to the wall only (y). The linearized disturbances are in the form of traveling sine waves whose amplification is in time.

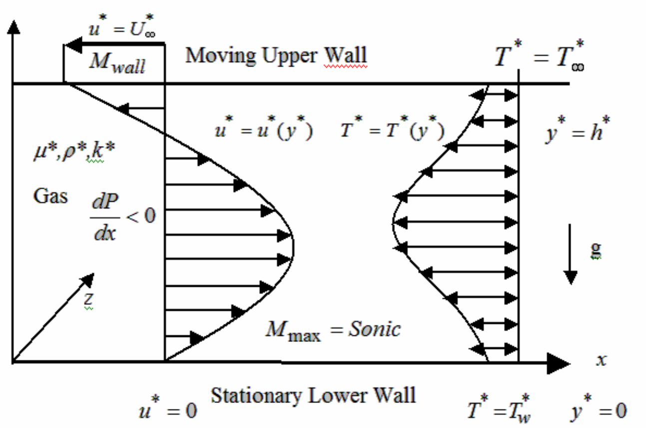

Fig.1. 2D representation of flow geometry and flow parameters

3.2 Governing Equations of Perturbation Flow and Linear Stability Analysis

Although considering two-dimensional disturbances for the lowest limit of stability is sufficient and approved by Squire's theorem, three-dimensional form of the compressible viscous equations of motion is considered in this study, resulting in a more general formulation, thus allowing conversion to the three dimensional disturbance case if needed. The linear stability equations are based on a normal mode analysis of the linearized perturbation equations of the three-dimensional Navier-Stokes equations. In the normal mode analysis, small disturbances are resolved into modes, which may be treated separately because each satisfies the linear system. The linear stability theory formulas presented in this study are valid for general compressible flows with parallel steady flow fields. The linear stability is considered for high speed viscous combined plane Couette-Poiseuille flow confined between finite parallel walls located at $y^* = 0$ (lower wall) and $y^* = h^*$ (upper wall). Each flow variable is assumed to consist of a mean part and infinitesimally small perturbations. Utilizing normal mode analysis, the perturbations are expressed in a Fourier series. The resulting disturbance equations are linear partial differential equations in the variables *x, y, z* and *t*. The disturbance equations are linear and the coefficients are functions of *y* only. Then the separation of variables using normal modes (i.e., exponential solutions in terms of the independent variables) resulting in the ordinary differential equations can be used. One possible normal mode is the single wave and is excited harmonically as in Equation (1).

$$q(x,y,z,t) = \ddot{Q}(x,y,z,t) + \tilde{q}(x,y,z,t)$$
$$\tilde{q}(x,y,z,t) = \hat{Q}(y)e^{i(\alpha x + \beta z - \omega t)} \quad (1)$$

The complex frequency is $\varpi = \alpha c$, while the real part of c is the dimensionless phase speed, c_r, and the imaginary part is the temporal amplification factor, c_i. Disturbances are classified according to normal mode analysis in which are either amplified ($c_i > 0$), neutral ($c_i > 0$), or damped ($c_i < 0$). The wave number α. is real and positive and represented as $\alpha = 2\pi / \lambda$, where λ is the wavelength.

The Squire theorem states that when the mean flow velocities in y – and z – directions are zero, the lowest value of the critical Reynolds number occurs when β = 0. The equation governing a three-dimensional oscillation is the same as that of a two-dimensional oscillation except the transverse wave number, β, and other terms for the z – momentum equation. If α and β are real, the presence of β acts in a way to effectively increase the viscosity. In the stability calculations, two-dimensional disturbances are considered and the wave number, β, in z – direction taken to be zero. The set of equations of motion, continuity, energy, equation of state and Sutherland's rule of viscosity for viscous compressible ideal gases in dimensional form are used for the linear stability analysis. Equation (2) gives the Sutherland's law for the viscosity [7].

$$\frac{\mu^*}{\mu_\infty^*} = \left(\frac{T^*}{T_\infty^*}\right)^{3/2} \frac{T_\infty^* + S_1^*}{T^* + S_1^*} \quad (2)$$

In case of parallel flows, the flow parameters are functions of y^* only, i.e., $u^* = u^*(y^*)$, $w^* = w^*(y^*)$, $T^* = T^*(y^*)$, $\mu^* = \mu^*(y^*)$, $k^* = k^*(y^*)$ and normal component of the mean velocity is zero, $v^* = 0$. Cartesian coordinate system and the following scaling factors are used in non-dimensionalization of the conservation equations. The length scale is the channel height, h^*, velocity scale is the velocity at the upper moving wall, U_∞^*. Density ρ_∞^*, viscosity μ_∞^* and thermal conductivity k_∞^* are all at the reference temperature of 288 K of upper wall, pressure is nondimensionalized by $\rho_\infty^* {U_\infty^*}^2$ and the time by h^*/U_∞^*. All other variables are nondimensionalized by their corresponding values on the upper wall. The dimensionless variables are represented by the same symbol as those used for the dimensional variables but without the asterisk, *. When compared with the mean flow, the perturbations are small, therefore, quadratic fluctuating terms such as $\tilde{u}\frac{\partial \tilde{u}}{\partial x} << U\frac{\partial \tilde{u}}{\partial x}$ can be neglected. In all the above equations, there are also fluctuating components of viscosity and thermal conductivity which are also functions of temperature as $\tilde{\mu} = \frac{d\mu}{dT}\tilde{T}$, $\tilde{\lambda} = \frac{d\lambda}{dT}\tilde{T}$, $\tilde{k} = \frac{dk}{dT}\tilde{T}$.

3.3. Method of Normal Modes and Generalized Eigenvalue Problem

The linear stability analysis is based on normal mode analysis of the linearized perturbation equations of the three-dimensional Navier-Stokes equations. In the normal mode analysis for the linear disturbances, the fluctuations of flow quantities are assumed to be represented by harmonic waves of the following form in three dimensions as: $\tilde{u},\tilde{v},\tilde{w},\tilde{p},\tilde{T} = \left[\hat{u}(y),\hat{v}(y),\hat{w}(y),\hat{p}(y),\hat{T}(y)\right]e^{i(\alpha x + \beta z - \varpi t)}$. The real part of ϖ, represents the frequency of the disturbance modes while the imaginary part of ϖ represents their temporal amplification rate.

Introducing the perturbation terms into the set of equations and differentiating with respect to y constitutes the set of generalized eigenvalue problem.

Linear disturbances satisfying all of the equations results in the generalized eigenvalue problem shown as in Malik [6].

$$(\mathbf{A}D^2 + \mathbf{B}D + \mathbf{C})\Psi_i = 0\,, \quad (3)$$

where Ψ_i is the five element vector defined by $(\hat{u},\hat{v},\hat{P},\hat{T},\hat{w})^{tr}$ and $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ which are $(5N+1)x(5N+1)$ matrices of functions of $\alpha, \beta, \omega, \mathrm{Re}$ and M_w. The disturbance waves are three-dimensional in general, while two-dimensional disturbance modes correspond to a special case of $\beta = 0$. We are interested in two-dimensional basic flow, then the velocity component $w(y)$ may be set to zero. The boundary conditions for Equation (3) are imposing the isothermal wall temperature at the upper wall. The lower wall assumes either isothermal or adiabatic wall boundary conditions.

$$y^* = 0 \quad \Psi_1 = \Psi_2 = \Psi_4 = \Psi_5 = 0 \text{ or } d\Psi_4/dy = 0\,,$$
$$y^* = h^* \quad \Psi_1 = \Psi_2 = \Psi_4 = \Psi_5 = 0 \quad (4)$$

Equations (3) and (4) constitute the homogeneous boundary value problem and the main scope is to determine the relation between the α, β, Re, M_w and ϖ that satisfies the system which constitutes a generalized eigenvalue problem, in the form of $\varpi = \varpi(\alpha, \beta, M_w, \mathrm{Re})$.

4. Numerical Approach

In order to implement a numerical solution, the computational domain, η, is divided into grids with equal spacing and the physical properties of the fluid are evaluated at the grid points in y – direction. The 2nd order differential equations are discretized using $O(h^2)$ finite difference formulae. Due to the staggered mesh generation there is no need to have an artificial pressure boundary condition on the walls. The three momentum and energy equations are written at full nodes. The pressure information for those equations at full nodes is obtained from the neighboring half nodes at which continuity equation is written. The total number of full nodes is N and the total number of half nodes is N+1. For each full node there are 4 conservation equations and 1 equation, the continuity, is written at the half nodes. The total number of equations written at all the nodes is 5N+1. Equation (3) with the boundary conditions in Equation (4) represents the 5N+1 equations and 5N+1 unknowns to be solved simultaneously. The discretization of the governing equations reduces the system to a generalized eigenvalue problem as $\mathbf{A}\Psi = \varpi\mathbf{B}\Psi$, where ϖ is the eigenvalue in the form of $\varpi = \alpha(c_r + ic_i)$. Real part of ω, $\mathrm{Re}(\varpi)$, represents the frequency of the disturbance modes, while the imaginary part, $\mathrm{Im}(\varpi)$, represents the temporal amplification rate of disturbances. The term Ψ is the discrete representation of the eigenfunction. **A** and **B** are the square coefficient matrices of the stability equations. The matrices are complex and therefore are composed of real and imaginary submatrices shown as $(\mathbf{A_r} + i\mathbf{A_i})\Psi = \alpha(c_r + ic_i)(\mathbf{B_r} + i\mathbf{B_i})\Psi$. IMSL and Eispack QZ algorithms were used for the solution of the generalized eigenvalue. The double precision complex subroutines used as solver are DGVLCG and DWRCRN for IMSL libraries.

5 Results

This section reports on the analysis of high speed Couette and combined Couette – Poiseuille flows. The effects of variable viscosity, temperature and density on the stability of Modes I and II for Couette flow and Mode 0 for Couette – Poiseuille flow are studied comparing the viscous results at finite Reynolds numbers with the results reported by Hu and Zhong for plane Couette flow [4].

5.1 Critical Reynolds and Wave Numbers

The critical Reynolds number, Re_{cr} is defined as the smallest value of Reynolds number for which an unstable eigenmode exists. The investigation of critical Reynolds numbers and determination of the stable-unstable regions on the Uwall/Umax-Re map were generated working on a range of upper moving wall Mach numbers between 0.0001 to 1.0.

In Fig.2, the critical Re numbers increase steadily as the velocity ratio increases until the U_{wall}/U_{max} ratio of roughly 0.1 for all the Mach numbers depicted. Curves corresponding to M_{max} = 0.1 and 0.3 exibit an almost flat behavior for velocity ratios greater than 0.1 and the critical Reynolds numbers remain almost constant for U_{wall}/U_{max} ratios between 0.15 and 0.45. On the other hand, curves of 0.5, 0.7 and 1.0 are showing a strong dependence of the critical Reynolds number on the U_{wall}/U_{max} ratio for the entire range of this parameter.

After the velocity ratio of 0.55, the difference between the channel M_{max} = 0.1 and 0.3 curves increases and the critical Reynolds numbers increase very rapidly with small increments in U_{wall}/U_{max} ratio. While beyond U_{wall}/U_{max}=0.5, the flow becomes unconditionally stable for the direct flow [2], same condition for the reverse flow occurs at 1.0. In the same trend, as the channel maximum Mach number increases, one sees that this strong stabilization occurs at progressively lower U_{wall}/U_{max} ratios.

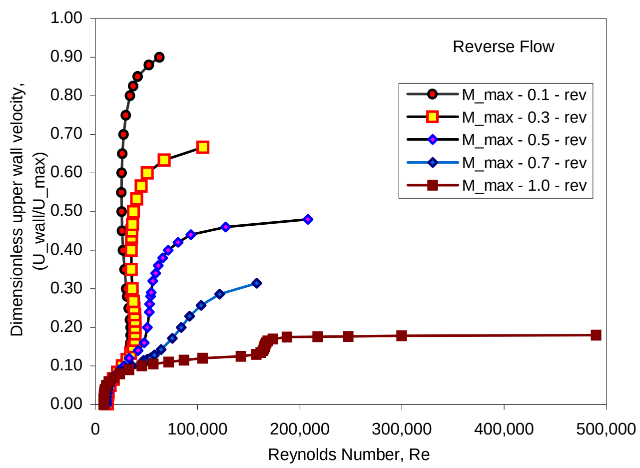

Fig. 2. Variation of critical Reynolds number with upper wall velocity.

It is seen that, increasing the channel maximum Mach number from incompressible limit (low speed) to high speed, the stability of the flow is enhanced. It means that, although increasing the channel maximum Mach numbers result in a decrease in the upper wall Mach numbers, they cause an increase in the critical Reynolds numbers.

Fig. 3 concentrates on the lower values of Uwall/Umax ratio for the same cases as in Fig. 2. In this range, increasing the maximum Mach number in the channel destabilizes the flow. The most interesting thing is the intersection of all curves at the velocity ratio of 0.080. Critical Reynolds number at the intersection point is around 22000 for all cases investigated. Similar condition occurs at the velocity ratio of 0.085 and the critical Reynolds numbers is at 26000 for the direct flow [2].

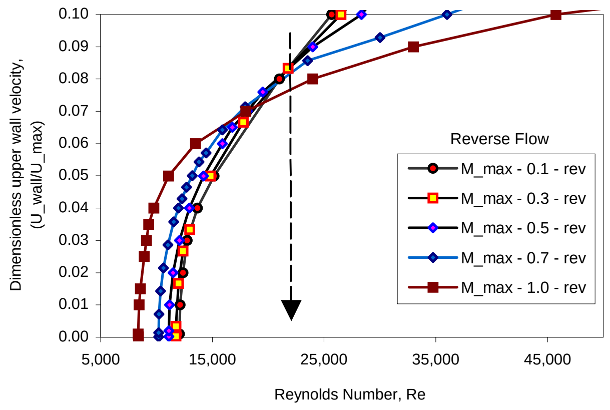

Fig. 3. Variation of critical Reynolds number with upper wall velocity between U_wall/U_max= 0 - 0.1.

In Fig. 4, the variation of critical wave numbers with respect to the U_{wall}/U_{max} ratio are illustrated for all the flow cases. Generally, at $U_{wall}/U_{max}=0$ limit, for all cases the critical wave number is around 2.0.

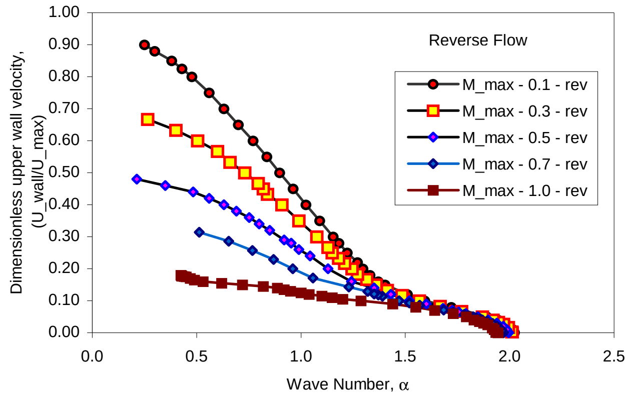

Fig. 4. Variation of critical wave number with upper wall velocity.

Compared to the curves of related critical Reynolds number for each case, the curves for the wave numbers show an important difference: unlike the critical Reynolds number curves, the wave number curves do not intersect but being closer in the range of velocity ratios 0 and 0.1.

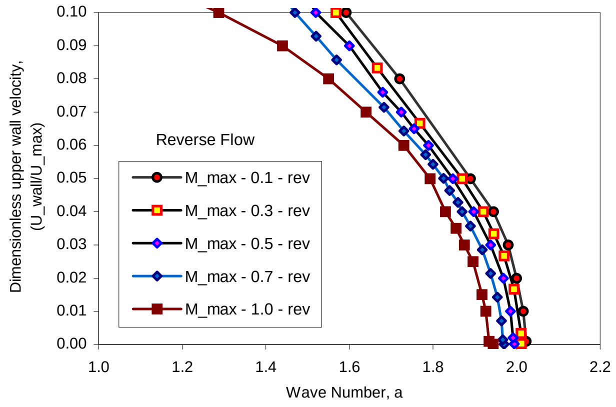

Fig. 5. Variation of critical wave number with upper wall velocity between U_wall/U_max = 0-0.1.

Fig. 5 concentrates on the results for velocity ratios between 0.0 and 0.1, for the same cases treated in Fig. 4. For high speed flow, critical Reynolds and wave numbers are both smaller than the other cases along the curves and also at $U_{wall}/U_{max}=0$. Other possible minor differences on the overall results between the incompressible (low speed) results are due to solution method used and the differential equations solved. Currently, the set of governing equations for high speed flow are solved simultaneously instead of incompressible Orr–Sommerfeld equation and $M_w=0.0001$ is employed as an incompressible limit for high speed flow for the validation.

6 Conclusions

In this study, the linear viscous stability characteristics of high speed Couette and combined Couette – Poiseuille flows have been investigated numerically. The upper wall moves in reverse direction while the flow between the walls in streamwise direction. The aim of the study is to see the effect of the reverse flow on the stability compared to the direct flow. In the combined plane Couette – Poiseuille flow, the new mode, Mode 0, which seems to be a member of even modes such as Mode II is the most unstable mode. High speed stability of channel flow with temperature effect is less commonly investigated compared to the incompressible isothermal flow. The most significant consequence of high speed flow with temperature effect is the variation of the temperature and temperature related viscosity/conductivity profiles in the channel and the inclusion of the energy equation in the stability analysis. In this study, the effect of compressibility on the flow is minimized by setting the channel maximum Mach number to 0.1, which corresponds to incompressible flow conditions, rendering the temperature, viscosity and thermal conductivity constant in the channel and results are compared to the findings of Orszag [8] and Özgen et al [1].

Regarding the cases with compressibility effects, it is observed that the effect of compressibility is to stabilize the flow. Another noteworthy results is that, the velocity ratio beyond which unconditional stabilization occurs increases significantly with increasing wall Mach number (from ≈0.5 in incompressible limit to ≈1 for M_{wall}=1.0).

As far as the wave numbers are concerned, the unstable wave number range migrates to smaller wave number values as the compressibility effect is increased. This means that longer waves become prone to instabilities, while for incompressible flow, short waves (high wave numbers) could also become unstable.

Nomenclature

Superscripts

*	Dimensional quantities
~	Dimensionless perturbation quantities
^	Amplitude of dimensionless perturbation quant
‾	Mean flow quantities

Subscripts

∞	Dimensional quantities evaluated at upper moving wall
w	Dimensional quantities evaluated at lower stationary wall
o	Dimensional mean flow terms
cr	Critical values
r	Real part of complex quantities
i	Imaginary part of complex quantities

Greek Symbols

α	Wave number in streamwise, ($\alpha = \alpha_r + i\alpha_i$)
β	Wave number in spanwise, ($\beta = \beta_r + i\beta_i$)
Ψ	Generalized eigenvalue problem eigenvector
ρ	Density, (kg/m3)
μ	Dynamic viscosity, (kg/ms)
ν	Kinematic viscosity, (m^2/s)
k	Thermal conductivity, (W/mK)
λ	Second coefficient of viscosity, $\lambda = -2/3\mu$
c	Complex propagation wave velocity for temporal stability, ($c = c_r + ic_i$)
c_r	Dimensionless phase speed
c_i	Dimensionless temporal amplification factor
ϖ	Temporal amplification rate, $\varpi = \alpha(c_r + ic_i)$
Φ	Viscous heat dissipation

Dimensionless Groups

Re	Reynolds number, $\mathrm{Re} = \rho_\infty^* U_\infty^* h^* / \mu_\infty^*$
M_{max}	maximum channel Mach number based on local velocity and temperature,
M_w	Upper wall Mach number , $M_w = U_\infty^* / \sqrt{\gamma R T_\infty^*}$
Pr	Prandtl number, $\mathrm{Pr} = \mu_\infty^* c_p^* / k_\infty^*$

Alphanumeric Symbols

A,B,C	Coefficient matrices of eigenvalue problem
$d_{1,2}$	Gas & oil layer thicknesses, (μm)
c_p	Specific heat constant at const pressure, (J/kgK)
g	Gravitational acceleration, ($g = 9.81$ m/s^2)
h	Dimensional channel height, (m)
N	Node number in computational domain
p	Pressure, (Pa)
q , Q	Velocity, temperature and pressure in linear stability analysis
R	Universal gas constant (R = 287 J/kgK for air)
S_1	Dimensional constant Sutherland's law
T	Temperature , (K)
T_{w_up}	upper (moving) wall temperature , T_∞^*
t	Time scale, (s)
U ,u	Streamwise velocity component, (m/s)
U_w	upper (moving) wall velocity , U_∞^*
U_{max}	maximum velocity in channel,
v	Velocity component normal to flow, (m/s)
w	Spanwise velocity component, (m/s)

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