Evaluating the performance of control schemes for hybrid systems
A Practical Approach

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Abstract: - Hybrid systems are now one of the most important research topics in the field of system and control theory. They undoubtedly pose many interesting challenges from the purely theoretical point of view, however, the ultimate motivation for their study is practical. There many industrial plants whose mathematical models are hybrid, i.e. exhibit nontrivial interactions of continuous valued and discrete valued variables and elements. Despite this practical motivation, most papers on the control of hybrid systems are limited to theoretical aspects of control design and they are concluded with simulational experiments at best. This paper proposes a more practical alternative. It describes a laboratory scale plant that is designed in such a way that it exhibits most of the hybrid phenomena typical of process control applications. The paper includes a detailed description of the plant structure together with its mathematical model. Further, it focuses on hybrid control experiments that can be performed with this plant in order to test and evaluate various hybrid control schemes.

Keywords: - Hybrid systems, Laboratory scale models, Hybrid control

1 Introduction

Many industrial plants are modelled as systems that have neither purely continuous nor purely discrete (or in a special case logical) dynamics but include both continuous valued and discrete valued variables and elements (on/off switches or valves, speed selectors, etc.). Moreover, many plants (e.g. boilers) exhibit considerably different dynamic behaviours in different operation modes or in different working points. Such plants can be modelled as a combination of several continuous models and discrete valued variables that determine which of these models is valid in the current operation mode or range. In addition, discrete features are often introduced by the controller even if the system to be controlled is continuous (gain scheduling, controller switching, etc.).

These facts have been the motivation behind growing research interest in the theory of hybrid systems (see e.g. [4] for a short survey), which includes both continuous and discrete valued dynamics and variables in one general framework. Many important results concerning hybrid systems have been achieved recently (see e.g. [1] and [10]).

However, this field is still mostly the topic of highly theoretical papers and PhD level courses. Its high application potential remains almost completely unexploited. The testing of hybrid control schemes is usually done using simulational experiments at best. This paper proposes a more realistic and practical alternative. It describes a laboratory scale plant that can be used to test and evaluate various control strategies for hybrid systems in a setting that is much closer to real applications. A considerable deal of attention is devoted to the description of possible plant configuration variants and control experiments that can be performed with the plant. Besides control experiments and other research purposes, this plant can also be used for educational purposes. A detailed treatment of possible educational applications of this plant was given by the authors elsewhere [4].

It is quite a sad sign of the preoccupation of the current control literature with purely theoretical problems that a laboratory plant similar to the one described in this paper can hardly be found in the literature. There are several technical reports describing complex laboratory plants designed mainly for the purposes of European research...
projects (in particular the plants used as case studies in the Esprit project Verification of Hybrid Systems www-verimag.imag.fr/VHS/). Other papers just sketch simple model plants that are used to illustrate some ideas described in the paper but were not intended to be built and used for real testing (e.g., [7]). Still other papers include hybrid control or modelling experiments with laboratory models. However, although these models exhibit certain hybrid phenomena, they were primarily designed for continuous control experiments (see e.g., [6]). No laboratory plant designed primarily for experiments with hybrid control is described anywhere.

2 Hybrid systems

Hybrid phenomena arise as a result of the interaction of continuous and discrete controls, outputs and states. To avoid misinterpretation, it should perhaps be emphasised that the words continuous and discrete are used to express the fact that the variables take values from a continuous set (usually an interval of real numbers) or a discrete set containing a finite number of elements. Typically, some of the controls take only a finite set of values, some of the outputs are quantized and the differential equation

\[
\dot{x}(t) = f(x(t))
\]

modelling the continuous dynamics depends on some discrete phenomena. The two most important classes of this dependence are the following. Vector field \( f(. ) \) can change discontinuously either when the state \( x(t) \) hits certain boundaries or in response to a control command change. This behaviour is called autonomous or controlled switching respectively. State changes or control commands may also result in state \( x(t) \) jumps (or equivalently in vector field impulses). If this is the case, we speak of autonomous or controlled jumps (impulses).

Many mathematical models of hybrid systems have been proposed. Most of them are included in a general hybrid system model as described e.g. in [2]. A hybrid system is given by

\[
\begin{align*}
\dot{x}(t) &= f(x(t), m(t), u(t)) \\
y(t) &= g(x(t), m(t), u(t)) \\
m(t^+) &= \varphi(x(t), m(t), u(t), \sigma(t)) \\
o(t^+) &= \varphi(x(t), m(t), u(t), \sigma(t)) \\
x(t^+) &= \psi(x(t), m(t), u(t), \sigma(t))
\end{align*}
\]

Equations (2) model the continuous part of the system \((x(t), u(t))\) and \(y(t)\) are continuous state, input and output respectively). Unlike state equations of a purely continuous system they include discrete state \(m(t)\). This state indexes the vector fields \(f(\cdot, \cdot, \cdot)\). The development of the discrete state is described by (3) where \(\sigma(t)\) is discrete input and \(o(t)\) is discrete output. Equations (3) are formally similar to the standard state equations of continuous systems. However, the description of the discrete part of the system can also be in the form of a finite automaton or a discrete event system, if desirable. Equation (4) models the state jumps (if there are any).

The model described by (2)-(4) is fairly complex and it is not easily tractable in its full generality. Thus, many research results have been obtained for simpler models, in which state jumps are not present (eq. 4 omitted) and the continuous part has a simpler form. A particularly important special case is piecewise affine (PWA) systems, where (2) reduces to

\[
\begin{align*}
\dot{x}(t) &= A(m(t))x(t) + B(m(t))u(t) + f(m(t)) \\
y(t) &= C(m(t))x(t) + D(m(t))u(t) + g(m(t))
\end{align*}
\]

Despite its simplicity, (5) captures many important phenomena such as switching, relays and saturations, and it can also be used to approximate nonlinear dynamics with several locally valid linearizations. Moreover, it has recently been proved [3] that PWA systems are equivalent to several other classes of hybrid systems (mixed logical dynamical systems, linear complementarity systems, etc.). Thus, PWA systems are of special importance because most results and approaches in hybrid systems theory have been obtained either directly for PWA systems or for an equivalent class.

3 The laboratory scale plant

The plant is sketched in Fig. 1. Similarly as in most process control applications, the measured and controlled variables are water level, temperature and flow. The sensors and actuators were (if possible) chosen from standard industrial ranges. Thus, although the plant does not represent any particular industrial process exactly, both the plant itself and its instrumentation are well representative of many plants commonly used in the process industries.

The basic components of the plant are three water tanks. Water from the reservoir mounted under the plant can be drawn by pumps 1 and 3 (delivery rate 0.5-5 l/min) to the respective tanks. In both cases, the delivery rate can be continuously changed and the water flow rate is measured with a turbine flowmeter (VISION 2000, 0.5-5 l/min).

The flow from pump 3 is fed directly to tank 3. The flow from pump 1 goes through a storage water heater (2 kW, 5 l) and it is further controlled by an on/off solenoid valve S1. Due to the built-in pressure switch, the pump is automatically switched off if S1 is closed and the internal tank of the heater is full.
The temperature at the heater output is measured with a Pt1000 sensor TT4. The power consumption of the heater can be changed continuously. Depending on the selected control scenario, the inflows to tanks 1 and 3 can be used as manipulated variables or as disturbances with specified flow rates and also with a specified temperature in the case of inflow 1.

Tanks 1 and 2 are thermally insulated to make the heat losses negligible. Tank 3 and the water reservoir are not insulated, and if the air cooler motor is switched on, their temperature roughly equals the ambient temperature in the laboratory. Tanks 2 and 3 have a special shape that introduces a change of the dynamics at a certain level. Their behaviour is described by switched models. Water from tanks 1 and 3 is fed to tank 2. The flow from tank 1 is controlled by solenoid valves S3, S4 \((k_v=5 \text{ l/min each})\), and it can be changed in three steps: no valve open, one open, both valves open. Closing and opening the valves is instantaneous. Tank 1 can be by-passed by closing S1 and opening S2. The flow from tank 3 is controlled by a pump. Its delivery rate can be changed in four steps \((0, 1/3, 2/3, \text{max.})\).

The other heater (custom-made, 800 W) is mounted at the bottom of tank 2. Water from tank 2 is drawn by a pump. The turbine flowmeter and slave controller FC2 allow us to simulate the changing demand of the downstream processes. The water goes back to the reservoir through an air-water heat exchanger where it is cooled. This arrangement is used to keep the water temperature in the reservoir roughly constant during the experiments.

The water levels are measured by pressure sensors. These can be configured either as continuous sensors or as level switches (indicating three different water levels). The plant is controlled from a PC using two data acquisition boards. The real appearance of the plant is shown in Fig. 2.

**4 Mathematical model of the plant**

A reasonably accurate first principles model of the plant can be derived using mass and energy balance equations. The liquid (water) can be considered incompressible. The mass balance equation of a single tank is then

\[
\frac{dV(t)}{dt} = \frac{d}{dt} \left( \int_0^{h(t)} A(x) dx \right) = A(h) \frac{dh(t)}{dt}
\]

where \(h\) is water level, \(A(h)\) is the cross sectional area of the tank at level \(h\), and \(q_{in}\) and \(q_{out}\) are inlet and outlet volume flow rates.

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**Fig. 1 Structure of the plant** (FT, LT, TT stands for flow, level and temperature transmitter respectively, FC – flow controller, S – solenoid valve, M – motor, \(r_1=6 \text{ cm}, r_{21}=6 \text{ cm}, r_{22}=3 \text{ cm}, r_{31}=7 \text{ cm}, r_1=3 \text{ cm}\), tank height 80 cm, \(l_1=40 \text{ cm}, l_2=40 \text{ cm}\))
Assuming constant liquid heat capacity $c$, negligible heat losses and well mixed tank, the energy balance equation of a single tank can be written as

$$\frac{d}{dt}(\rho c V(t)(\vartheta(t) - \vartheta_{ref})) = q_{in}(t)\rho c(\vartheta_{in}(t) - \vartheta_{ref}) - q_{out}(t)\rho c(\vartheta(t) - \vartheta_{ref}) + P(t)$$

(7)

where $P$ is heater power output, $\vartheta_{in}$ inlet water temperature, $\vartheta$ water temperature in the tank. Due to the assumption of perfect mixing, the outlet water temperature can be equated with $\vartheta$. Eq. (7) can be modified to

$$V(h)\dot{\vartheta}(t) = q_{in}(t)(\vartheta_{in}(t) - \vartheta(t)) + P(t)/\rho c$$

(8)

The volume flow rate from a tank with water level $h$ can be expressed by

$$q = 0.1k_v\sqrt{gh}$$

(9)

Combining (6), (8) and (9), the plant model is built

$$\dot{h}_1(t) = (1/A_1)(q_1(t) - 0.1k_v\sigma_1(t)\sqrt{gh_1(t)})$$

(10)

$$\dot{\vartheta}_1(t) = q_1(t)(\vartheta_1(t) - \vartheta_1(t))/A_1 h_1(t)$$

(11)

$$\dot{h}_2(t) = \begin{cases} \frac{0.1k_v\sigma_1(t)\sqrt{gh_1(t)}}{1/A_2} + \sigma_2(t)q_0 - q_2(t) & \text{if } m_1 = 0 \\ \frac{0.1k_v\sigma_1(t)\sqrt{gh_1(t)}}{1/A_2} + \sigma_2(t)q_0 - q_2(t) & \text{if } m_1 = 1 \end{cases}$$

(12)

$$\dot{\vartheta}_2(t) = \begin{cases} \frac{A_2 h_2(t)}{\sigma_2(t)q_0(\vartheta_1(t) - \vartheta_2(t)) + \frac{P(t)}{\rho c}} & \text{if } m_1 = 0 \\ \frac{A_2 h_2(t)}{\sigma_2(t)q_0(\vartheta_1(t) - \vartheta_2(t)) + \frac{P(t)}{\rho c}} & \text{if } m_1 = 1 \end{cases}$$

(13)

$$\dot{h}_3(t) = \begin{cases} \frac{q_3(t) - \sigma_2(t)q_0}{\sigma_2(t)q_0(\vartheta_1(t) - \vartheta_2(t)) + \frac{P(t)}{\rho c}} & \text{if } m_2 = 0 \\ \frac{A_3}{q_3(t) - \sigma_2(t)q_0} & \text{if } m_2 = 1 \end{cases}$$

(14)

$$m_1(t^+) = \begin{cases} 0 & \text{if } h_2(t) \leq l_1 \\ 1 & \text{if } h_2(t) > l_1 \end{cases}$$

(15)
where \( A_j = \pi_j^2 \), \( \Delta r = r_{31} - r_{32} \), \( m_1 \) and \( m_2 \) are discrete state variables, discrete valued input \( \sigma_1 \) assumes values 0, 1, 2 (no valve open, S3 open, S3 and S4 open), \( \sigma_2 \) assumes values 0, 1, 2, 3 (pump 4 delivery rate is 0, 1/3 max., 2/3 max., maximum), \( q_0 \) is the delivery rate of pump 4 running at 1/3 of maximum.

The measurable outputs are identical with the state variables. The water levels can be used as continuous-valued or discrete-valued outputs depending on the configuration of the level sensors. The equation for \( \theta_2 \) is not included in the model. As tank 3 is neither heated nor insulated, \( \theta_3 \) is roughly equal to the ambient temperature, and the mixing dynamics of tank 3 are not important. Valves S1, S3 and S4 can be closed and S2 used as a control input. The plant model is then simplified: \( h_1 \) and \( \theta_1 \) are omitted and the square root term in (12), (13) is replaced by \( q_1(t) \sigma_2(t) \) (\( \sigma_1 = 0 \) or 1). The vector field defined by (10) to (14) involves both controlled and autonomous switching. Controlled switching due to changes of discrete valued inputs \( \sigma_1 \), \( \sigma_2 \) results in vector field discontinuity in (10), (12), (13) and (14). Autonomous switching due to changes of \( h_2 \) and \( h_3 \) results in vector field discontinuity in (12). The dynamic behaviours of (13) and (14) are also switched, but the vector field remains continuous.

5 Proposed control experiments

Although this plant is fairly simple and easy to understand, it allows us to define many control scenarios that can be used to test and evaluate various aspects of hybrid control algorithms. Some interesting scenarios will be outlined below, however many other scenarios are possible.

First, it is possible to define uncomplicated control tasks such as water level control in tank 2 or 3 or temperature control in tank 2. In this case, the controlled system features switched dynamics, and certain basic classes of hybrid systems can be demonstrated to the students. For instance, if the control objective is to control \( h_2 \) using \( q_1 \) as the manipulated variable (valve S2 constantly open, S1, S3, S4 closed) and \( q_2 \) being a disturbance (in this scenario both \( q_1 \) and \( q_2 \) can be changed in several steps) the controlled system is a simple integrator hybrid system. Alternatively, the objective can be water level control in tank 3. The dynamics of (14) are nonlinear for \( h_3 \leq l_2 \). The nonlinear range of \( h_3 \) can be divided into several sub-ranges and the whole system including switching and non-linearity can be approximated with a piecewise affine (PWA) system.

More complicated scenarios use two tanks. One suggestion (inspired in part by [11]) can be formulated as follows. Tank 1 serves as a buffer that receives water from an upstream process. Water flow rate and temperature are disturbances. Since the plant includes continuously controlled heater H1 and slave flow controller FC1 these disturbances can follow a defined function of time as required by a particular experiment. The main control objective is to deliver the water to a downstream process at a desired temperature. The flow demand of the downstream process is another disturbance. Valves S1, S2 and S3 are discrete valued manipulated variables and the power output of heater H2 is a continuous manipulated variable. The main control objective necessarily includes several auxiliary objectives. Tank levels must be kept within specified limits, and overflow as well as emptying of the tanks must be avoided. It is also necessary to avoid the necessity to close valve S1 in order to prevent tank 1 overflow. In a real control situation, closing S1 would mean that water from the upstream process cannot flow to the buffer but must be re-routed to the environment.

The controlled system is again hybrid and nonlinear, allowing approximate modeling, e.g., as a PWA system. However, unlike the previous simpler scenarios it includes nontrivial interactions of continuous and discrete controls. For example, if tank S3 is already open and S4 must also be opened to avoid tank 1 overflow, temperature \( \theta_2 \) may also be significantly affected, particularly if temperatures \( \theta_1 \) and \( \theta_3 \) are much different. The plant in this configuration can be used to test various approaches to hybrid system control. It is possible to follow the traditional approach and to design the water level control logic and continuous heater control independently. Alternatively, a hybrid approach can be followed and both the discrete and the continuous manipulated variables can be controlled with a single controller designed using hybrid controller synthesis methods (e.g., those developed in [1]). The control performances achieved with both approaches can be evaluated and compared.

Even more complicated interactions can occur in scenarios that use all three tanks. In this case, there are many possible combinations of manipulated, controlled and disturbance variables. For example, tanks 1 and 3 can be used as buffers that receive water from two upstream processes. Flow rates \( q_1 \), \( q_2 \), and \( q_3 \) as well as temperature \( \theta_4 \), \( \theta_5 \) are disturbances. The control task is to deliver the water at a desired temperature and flow rate \( q_2 \) to a downstream process using S3, S4, pump 4 and heater H2 as control inputs. The control objective must be
achieved while satisfying necessary constraints such as avoiding emptying and overflow of the tanks. Optimization sub-objectives can be included, e.g. it may be desirable to make use of the fact that $\theta_4 > \theta_3$ in order to minimize the power consumption of heater $H_2$.

6 Conclusion

The laboratory-scale plant described in this paper exhibits hybrid phenomena typical of process control applications. The variety of possible control scenarios outlined in the previous sections demonstrates the considerable flexibility of the plant. It can be configured just to demonstrate simple hybrid behaviours such as switching between two dynamic models or logic control of continuous plants. However, in its full configuration, it poses a great challenge to hybrid control synthesis because it includes several non-linearities and nontrivial interactions of continuous and logical/discrete dynamics. Sufficiently exact modelling using the most important hybrid dynamic models such as PWA or mixed logical dynamical systems is possible. Thus, it can be used to evaluate most of the recent approaches to hybrid systems control in a setting that is much closer to real application than mere simulation.

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References:


