The Simplex Method Applied to Wavelet Decomposition

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Abstract - Wavelet decomposition problems have been modeled as linear programs – but only as extremely dense problems. Both revised simplex and interior point methods have difficulty with dense linear programs. The question then is how to get around that issue. In our experiments the standard method outperforms a revised implementation for these problems. Moreover, the standard method can be easily and scalably distributed. Hence the standard simplex method should be useful in solving wavelet decomposition problems.

Key-words: Wavelet decomposition, linear programming; Standard simplex method; Dense matrices; Distributed computing

1 Introduction

Wavelet Decomposition has been modeled many ways. One method has focused on Linear Programs utilizing interior point methods [1]. Unfortunately the Linear Programs produced for Wavelet Decomposition models have extremely dense matrices and interior point methods have difficulty dealing with dense problems. Chen et al finessed this issue by restricting the wavelet decomposition problems to those with dictionaries having a special structure and by tailoring an implementation of an interior point method to take advantage of that special structure.

To overcome these problems we propose utilizing the simplex method and in particular the standard simplex method to solve both these wavelet decomposition problems as well as other problems that produce dense matrices. There are two major variants of the simplex method, the revised method and the standard method. The revised method is commonly used due to its advantage for the majority of problems which are sparse. Nevertheless although dense problems are uncommon in general they do occur in a number of applications within Linear programming [2].

Another advantage of using the standard method is that it can be easily and effectively extended to parallel and coarse grained distributed algorithms. (There are no scalable distributed versions of the revised simplex method.) When the standard method is distributed, aspect ratio becomes less of an issue. We simply divide the extra columns among more processors. If done properly, parallelization of the standard method pays off even on small problems [6].

We have written a standard implementation of the simplex method (retroLP) and compared it to the commonly used revised method as implemented by the well-known *MINOS* optimization package [3]. In this paper we solve a number of wavelet decomposition problems utilizing both the revised and the standard simplex methods. We empirically show that although the revised method is superior for the average sparse problem, when we are solving these wavelet decomposition problems which are dense the standard method is actually better suited and should be used.

2 The Revised and Standard Simplex Methods

The following is the general form of a linear program:

$$Max\ z = cx$$

$$b^l \le Ax \le b^u$$

$$l_j \le x_j \le u_j \quad for\ \ j = 1,\dots,n$$

or with $y = Ax$ we have:

$$\underset{x}{\text{Maximize}}\ z = \sum_{j=1}^{n} c_j x_j$$

$$\text{Subject to } y_i = \sum_{j=1}^{n} a_{ij} x_j \ \ (i = 1, 2, ..., m)$$

$$l_j \le x_j \le u_j \text{ for } j = 1, ..., n; \quad b_i^l \le y_i \le b_i^u \text{ for } i = 1, ..., m$$

$A = \{a_{ij}\}$ is a given m x n matrix, x is an n-vector of decision variables x_j, each with given lower bound l_j and upper bound u_j. The m-vectors b^l and b^u are given data that define constraints. The lower bound, l_j, may take on the value $-\infty$ and the upper bound, u_j, may take on the value $+\infty$. Similarly, some or all of the components of b^l may be $-\infty$, and some or all of b^u may be $+\infty$.

Table 1 summarizes the main qualitative differences between the standard and revised simplex method that affect wavelet decomposition linear programming problems.

Revised Simplex Method	**Standard Simplex Method**
Takes better advantage of sparsity in problems	Is more effective for dense problems
Is more efficient for problems with large aspect ratio (n/m)	Is more efficient for problems with low aspect ratio.
Is difficult to perform efficiently in parallel, especially, in loosely coupled systems.	Very easy to convert to a distributed version with a loosely coupled system.

Table 1: Comparison of Revised and Standard Forms of the Simplex Method

3 Previous Research

3.1 Wavelet Decomposition

Chen, Donoho and Saunders[1] have modeled Wavelet Decomposition as Linear Programs using a method called "Atomic Decomposition by Basis Pursuit." This method translates into large linear programs. For example, a typical wave signal of length 8192 results in an equivalent Linear Program of size 8192 by 212,992. Unfortunately the Linear Programs produced are not only extremely large but are dense. To quote the authors [1, p. 57] "However, the optimization problems we are interested in have a key difference from [other] successful large-scale applications…. The matrix A we deal with is not at all sparse; it is generally completely dense…"

In order to deal with this the authors implemented a specialized interior point method to derive a unique wavelet dictionary from an over complete dictionary. Among other things they took advantage of fast implicit algorithms for representations in the dictionaries they considered. They used this to develop a substitute approach for efficiently solving the systems of equations and restricted the class of wavelet dictionaries used.

3.2 Scalable Parallel Algorithms for the Standard Simplex Method

Recently there has been much research on methods to parallelize the simplex method. The standard method has proven to be more amenable to distributed and parallel algorithms than the revised method. A number of parallel algorithms have been produced both for massively parallel machines and for distributed networks of workstations [5,2,4,6].

Dense applications, such as wavelet decomposition, for which the standard method yields lower iteration times, have particular potential for increased efficiency through the use of these parallel algorithms. We therefore propose that it is both possible and advantageous to use the general purpose standard method to solve these wavelet decomposition problems without having to be limited to wavelet dictionaries with fast representations.

4 Experimental Results

Table 2 lists seven wavelet decomposition problems with varying problem sizes. Data for these problems came from the Wavelet and Atomizer packages provided by Chen et al [1]. Note that these packages limited us in that they only produced wavelet dictionaries that are powers of 2 in width and height. In addition we did not have the ability to specify how dense the resulting problems should be.

Columns two, three and four list the number of rows, number of columns and problem density respectively.

The fifth column shows the time per iteration it took MINOS, a well known revised simplex implementation, to solve these problems. Column six shows the time per iteration for retroLP, our implementation of the standard simplex method. We can see that as the densities listed in column four increase, retroLP becomes more and more efficient vis a vis MINOS. Problem 2 is the same size and aspect ratio as problems 5 through 7. The time per iteration for retroLP remains basically the same for all of them. MINOS, on the other hand, becomes between one hundred times and two hundred times slower when the problem has a density of 87.52% vs 0.39%.

Problem	M	N	Density	Minos time/iter	retroLP time/iter
1	1024	4096	0.20%	0.0002637	0.0950439
2	512	2048	0.39%	0.0003320	0.0464786
3	256	1536	33.85%	0.0055506	0.0163869
4	128	3246	34.37%	0.0015652	0.0034923
5	512	2048	87.52%	0.0532897	0.0452119
6	512	2048	87.52%	0.0553762	0.0450486
7	512	2048	87.52%	0.0527936	0.0451510

Table 2: Comparison of *retroLP* and *MINOS* for Wavelet Decomposition

5 Summary and Conclusions

In this paper we discussed Wavelet Decomposition problems modeled as linear programs. We focused in particular on the model offered by Chen et al [1] and provided empirical data and experiments that compare the standard algorithm with the revised algorithm. Our experiments comparing *MINOS* and *retroLP* indicate that for moderate values of density the standard method is competitive, and that Wavelet Decomposition can take advantage of the standard method.

An implementation of the standard method makes possible a natural Single Program Multiple Data (SPMD) approach for a distributed simplex method. Partition the columns among a number of workstations. Each iteration, each workstation prices out its columns, and makes a "bid" to all the workstations. The winning bid defines a pivot column, then all the workstations pivot on their columns in parallel, and so on. This is important for such problems as Wavelet Decomposition that are suited to the standard method.

References:

[1] Chen S.S., D.L. Donoho, and M.A. Saunders, “Atomic Decomposition by Basis Pursuit,” *SIAM J. on Scientific Computing*, 20, 1, pp. 33-61, 1998.http://www-stat.stanford.edu/~atomizer/

[2] Eckstein, J., I. Boduroglu, L. Polymenakos, and D. Goldfarb, "Data-Parallel Implementations of Dense Simplex Methods on the Connection Machine CM-2," *ORSA Journal on Computing*, v. 7, n. 4, pp. 402-416, Fall 1995.

[3] Murtagh, Bruce A., and Michael Saunders, "MINOS 5.5 Users Guide," *Technical Report* SOL 83-20R, Revised July, 1998.

[4] Stunkel, Craig B., Linear Optimization via Message-Based Parallel Processing, *ICPP: International* Conference *on Parallel Processing*, 1988, vol. III, pp. 264-271.

[5] Thomadakis, Michael E., and Jyh-Charn Liu, “An Efficient Steepest-Edge Simplex Algorithm for SIMD Computers,” *Proc. Of the International Conference on Super-Computing, ICS ’96*, pp. 286-293, May 1996.

[6] Yarmish, Gavriel, “A Distributed Implementation of the Simplex Method,” *Ph.D. Dissertation*, Polytechnic University, Brooklyn, NY, March, 2001.