Entropy Differences of Arithmetic Operations with Shannon Function on Triangular Fuzzy Numbers

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Abstract: - The main purposes of this paper are to probe for the entropy differences between arithmetically manipulated Triangular Fuzzy Numbers (TFNs) using Shannon’s Function; and to study the relationships between any two TFNs. Simultaneously, we expand on the articles of Wang et al. [6] and Wang et al. [5]. The entropy differences between two TFNs subjected to different arithmetic operations are classified as seven theorems. This paper finds that with the application of Shannon’s Function on triangular fuzzy numbers the grades of fuzziness are changed after arithmetic operations.

Key-Words: - Entropy, Triangular fuzzy numbers, Fuzzy sets, Measure of fuzziness, Shannon’s Function, Arithmetic operations.

1 Introduction
In the past, researchers have used myriads of methods to measure the fuzziness of fuzzy sets. Kaufmann [2] denoted the fuzziness of a fuzzy set by calculating the distance between the fuzzy set and its nearest non-fuzzy set. De Luca et al. [3] proposed using the entropy to describe the fuzziness of a fuzzy set. Indeed, there are plenty more papers which discuss the entropy of fuzzy sets [1, 2, 6, 7]. Pedrycz [4] used the fuzzy set with a triangular membership function to demonstrate the entropy change when the interval size of the universal set is changed. Wang and Chiu [6] extended the results of Pedrycz’s findings to any type of fuzzy sets. It is necessary to base the comparisons of the fuzziness of several fuzzy numbers on the same definition of entropy computation.

In the past, the study of the entropy differences of TFNs with arithmetic operations has always relied on this equation of entropy functions: \( h(x) = 4x(1-x) \). Shannon’s famous function, \( h(x) = -x \cdot \ln x - (1-x) \cdot \ln(1-x) \), is often used to measure fuzziness, yet it is difficult to find researchers who use the function to calculate entropy differences. In this paper, we use Shannon’s Function on arithmetic operations of triangular fuzzy numbers and to examine the entropy differences after the calculations. Moreover, we extend the studies of Wang et al. (2000), and Wang et al. (2005), which, using \( h(x) = 4x(1-x) \), found the entropy differences of triangular fuzzy numbers submitted to arithmetic operations.

2 Entropy and Fuzzy Sets
2.1 The Properties of Entropy
Suppose \( \tilde{A} \) is a fuzzy set and is defined in a universal set \( U \), where \( U \) is finite and real. The membership function value of \( x \) in fuzzy set \( \tilde{A} \) for \( x \in U \) is represented as \( \tilde{A}(x) \) and \( \tilde{A}(x) : x \rightarrow [0,1] \). The measurement of fuzziness of the fuzzy set \( \tilde{A} \) is denoted as \( H(\tilde{A}) \) and it holds the following properties (de Luca and Termini, 1972 [3]; Zimmermann, 1996 [8]):
1. \( H(\tilde{A})=0 \), if \( \tilde{A} \) is a crisp set in \( U \).
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274

2. If \( \tilde{A}(x) = 1/2 \), \( \forall x \in U \), then \( H(\tilde{A}) \) has a unique maximum.

3. For two fuzzy sets \( \tilde{A} \) and \( \tilde{B} \), if \( \tilde{B}(x) \leq \tilde{A}(x) \) for \( \tilde{A}(x) \leq 1/2 \) and \( \tilde{B}(x) \gtrsim \tilde{A}(x) \) for \( \tilde{A}(x) \gtrsim 1/2 \) then \( H(\tilde{A}) \gtrsim H(\tilde{B}) \).

4. \( H(\tilde{A}^c) = H(\tilde{A}) \), where \( \tilde{A}^c(x) \) is the standard complement of \( \tilde{A}(x) \), that is, \( \tilde{A}^c(x) = 1 - \tilde{A}(x) \).

The researchers use the integration of the following equation to calculate the global entropy measure of the fuzzy set \( \tilde{A} \) independent of \( x \) over the universal set \( U \) [4].

\[
H(\tilde{A}) = \int_{\infty}^{\infty} h(\tilde{A}(x))p(x)dx 
\]

where \( p(x) \) is the probability density function of the available data in \( U \), \( h(\tilde{A}(x)) \) is the entropy function, \( h(x) : [0,1] \rightarrow [0,1] \) is monotonically increasing in \( [0,1/2] \) and monotonically decreasing in \( [1/2,1] \), \( h(0) = 0 \), as \( x = 0 \) and \( x = 1 \); \( h(1) = 1 \), as \( x = 1/2 \). The following equations are the well-known entropy functions used to measure the fuzziness of \( H(\tilde{A}) \) which can be regarded as an “entropy” of a fuzzy set \( \tilde{A}(x) \):

\[
h(x) = \begin{cases} 2x & \text{if } x \in [0,1] \\ 2(1-x) & \text{if } x \in [1,2] \end{cases} \quad (1)
\]

\[
h(x) = 4x(1-x) \quad (2)
\]

\[
h(x) = -x \cdot \ln x - (1-x) \cdot \ln(1-x) \quad (3)
\]

where Eq.(4) is called Shannon’s function [8], which is applied to calculate the entropy of the TFN in this paper.

2.2 Definitions of Fuzzy Sets

Definition 1. The TFN \( \tilde{A} \) is denoted by a triplet \((a_1,a_2,a_3)\); it is usual to represent this TFN as \( \tilde{A} = (a_1,a_2,a_3) \). The membership function can be defined as follows [2, 8]:

\[
\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{if } x < a_1, a_1 < x \\ \frac{x-a_1}{a_2-a_1} & \text{if } a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & \text{if } a_2 \leq x \leq a_3 \end{cases} \quad (5)
\]

where \( \mu_{\tilde{A}}(x) \) is the degree of membership or the membership function value of \( x \) in fuzzy set \( \tilde{A} \), \( U \) and \( \mu_{\tilde{A}}(x) \) represent a universal set and membership function, respectively. In the expression below \( \mu_{\tilde{A}}(x) \) and fuzzy set \( \tilde{A} \) have the form:

\[
\mu_{\tilde{A}}(x) : U \rightarrow [0,1].
\]

Definition 2. Two TFNs \( \tilde{A} \) and \( \tilde{B} \) are defined as \( \tilde{A} = (a_1,a_2,a_3) \) and \( \tilde{B} = (b_1,b_2,b_3) \), so [2]

(1) Addition:

\[
\tilde{A} \oplus \tilde{B} = (a_1,b_1,b_2) \oplus (b_1,a_2,b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)
\]

(2) Subtraction:

\[
\tilde{A} - \tilde{B} = (a_1,a_2,a_3) - (b_1,b_2,b_3) = (a_1 - b_1, a_2 - b_2, a_3 - b_3)
\]

(3) Multiplication:

\[
(a) \ n \tilde{A} = (n a_1, n a_2, n a_3) \quad n \in N \quad N : \text{ denotes a natural number.}
\]

\[
(b) \ m \tilde{A} = (m a_1, m a_2, m a_3) \quad m \in I, m < 0 \quad I : \text{ denotes an integer.}
\]

\[
(c) \ r \tilde{A} = (r a_1, r a_2, r a_3) \quad r \in R, r \geq 0 \quad R : \text{ denotes a real number.}
\]

\[
(d) \ s \tilde{A} = (s a_1, s a_2, s a_3) \quad s \in R, s < 0 \quad n \tilde{A} , \ m \tilde{A} , \ r \tilde{A} , \ s \tilde{A} \text{ are triangular fuzzy numbers.}
\]

(4) \( \tilde{A} \) is the image of triangular fuzzy number \( \tilde{A} \), defined as: \( \tilde{A} = -(\tilde{A}) = (-a_1,-a_2,-a_3) \).

The results are still triangular fuzzy numbers through the arithmetic operations of addition to, subtraction from and multiplication by two triangular fuzzy numbers.

3 The Entropy Differences on TFNs through Arithmetic Operations

We will discuss and prove the entropy differences of TFNs through arithmetic operations in this section. According to Definition 2, the result of putting two TFNs through arithmetic operations is always a TFN.

Suppose that we have two triangular fuzzy numbers \( \tilde{A} \) and \( \tilde{B} \), briefly indicated as \( \tilde{A} = (a_1,a_2,a_3) \) and \( \tilde{B} = (b_1,b_2,b_3) \), respectively. Their membership functions are shown as below:

\[
\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{if } x < a_1, a_1 < x \\ \frac{x-a_1}{a_2-a_1} & \text{if } a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & \text{if } a_2 \leq x \leq a_3 \end{cases} \quad (6)
\]
Theorem 1. For two TFNs \( \tilde{A} \) and \( \tilde{B} \), expressed as \( \tilde{A} = (a_1, a_2, a_3) \) and \( \tilde{B} = (b_1, b_2, b_3) \), supposing \( p(x) = s \), where \( s \) is a constant, the relationship of the entropy of \( \tilde{A} \) and \( \tilde{B} \) has:

\[
H(\tilde{A}) = kH(\tilde{B})
\]

where

\[
k = \frac{2a_2 - a_3 - a_1}{2b_2 - b_1 - b_3}
\]

\[
\mu_\tilde{A}(x) = \begin{cases} 
0, & x < b_1, b_2 < x \\
\frac{x - b_1}{b_2 - b_1}, & b_1 \leq x \leq b_2 \\
\frac{b_2 - x}{b_3 - b_2}, & b_2 \leq x \leq b_3 
\end{cases}
\]

(7)

Proof. Applying Eq.(4) to Eq.(6), we obtain

\[
h(\mu_\tilde{A}(x)) = \begin{cases} 
0, & x < a_1, a_3 < x \\
\frac{x - a_1}{a_2 - a_1} \ln \left( \frac{x - a_1}{a_2 - a_1} \right) - \left( 1 - \frac{x - a_1}{a_2 - a_1} \right) \ln \left( 1 - \frac{x - a_1}{a_2 - a_1} \right), & a_1 \leq x \leq a_2 \\
\frac{a_3 - x}{a_3 - a_2} \ln \left( \frac{a_3 - x}{a_3 - a_2} \right) - \left( 1 - \frac{a_3 - x}{a_3 - a_2} \right) \ln \left( 1 - \frac{a_3 - x}{a_3 - a_2} \right), & a_2 \leq x \leq a_3
\end{cases}
\]

Applying Eq.(1) to the above equation, and computing the entropy of TFN \( \tilde{A} \), the result is shown as

\[
H(\tilde{A}) = \int_{a_1}^{a_2} \left[ -\left( \frac{x - a_1}{a_2 - a_1} \ln \left( \frac{x - a_1}{a_2 - a_1} \right) - \left( 1 - \frac{x - a_1}{a_2 - a_1} \right) \ln \left( 1 - \frac{x - a_1}{a_2 - a_1} \right) \right) \right] p(x) dx
+ \int_{a_2}^{a_3} \left[ -\left( \frac{a_3 - x}{a_3 - a_2} \ln \left( \frac{a_3 - x}{a_3 - a_2} \right) - \left( 1 - \frac{a_3 - x}{a_3 - a_2} \right) \ln \left( 1 - \frac{a_3 - x}{a_3 - a_2} \right) \right) \right] p(x) dx
\]

Let \( y = \frac{x - a_1}{a_2 - a_1} \), then \( dx = \left( a_2 - a_1 \right) dy \). Let \( z = \frac{a_3 - x}{a_3 - a_2} \), then \( dx = \left( a_3 - a_2 \right) dz \).

\[
H(\tilde{A}) = (-s)(a_2 - a_1) \int_{0}^{1} \left[ y \ln y + (1 - y) \ln(1 - y) \right] dy + (s)(a_3 - a_2) \int_{0}^{1} \left[ z \ln z + (1 - z) \ln(1 - z) \right] dz
\]

\[
= (-s)(a_2 - a_1) \left[ \frac{1}{2} y^2 \ln y - \frac{y^2}{4} + \frac{(1 - y)^2}{4} \ln(1 - y) \right] + (s)(a_3 - a_2) \left[ \frac{1}{2} z^2 \ln z - \frac{z^2}{4} + \frac{(1 - z)^2}{4} \ln(1 - z) \right]
\]

\[
= (-s)(a_2 - a_1) \left[ \ln 1 - \frac{1}{2} \right] + (s)(a_3 - a_2) \left[ \ln 1 - \frac{1}{2} \right] = s(a_1 + a_3 - 2a_2) \left( \ln 1 - \frac{1}{2} \right)
\]

\[
= \frac{s}{2} (2a_2 - a_1 - a_3)
\]

Hence, \( H(\tilde{A}) = \frac{s}{2} (2a_2 - a_1 - a_3) \).

(8)

Lemma 1. For the two TFNs \( \tilde{A} \) and \( \tilde{B} \), if the following conditions are satisfied, then \( H(\tilde{A}) = H(\tilde{B}) \).

1. When \( a_1 = a_2 \), that is \( \tilde{A}(x) \) and \( \tilde{B}(x) \) are left-facing right triangles and they have the same “base width”.
2. When \( a_3 = a_1 \), that is \( \tilde{A}(x) \) and \( \tilde{B}(x) \) are right-facing right triangles and they have the same “base width”.
3. When \( a_1 + a_3 = 2a_2 \), that is \( \tilde{A}(x) \) and \( \tilde{B}(x) \) are the isosceles triangles.
4. When \( 2a_2 - a_1 - a_3 = 2b_2 - b_1 - b_3 \).

Theorem 2. Supposing \( \tilde{C} = \tilde{A} + \tilde{B} \), \( \tilde{C} \) being a TFN, then the entropy of \( \tilde{C} \) is the summation of the
entropies of \( \tilde{A} \) and \( \tilde{B} \). That is, \( H(\tilde{C}) = H(\tilde{A} + \tilde{B}) = H(\tilde{A}) + H(\tilde{B}). \)

**Proof.** For \( \tilde{C}(x) = \tilde{A}(x) + \tilde{B}(x) \), the membership function can be expressed as

\[
\mu_{\tilde{C}}(x) = \begin{cases} 
0, & x < a_1 + b_1, a_3 + b_3 < x \\
\frac{x - (a_1 + b_1)}{(a_2 + b_2) - (a_1 + b_1)}, & a_1 + b_1 \leq x \leq a_2 + b_2 \\
\frac{(a_3 + b_3) - x}{(a_3 + b_3) - (a_2 + b_2)}, & a_2 + b_2 \leq x \leq a_3 + b_3 
\end{cases}
\]

Let \( y = \frac{x - (a_1 + b_1)}{(a_2 + b_2) - (a_1 + b_1)} \), \( z = \frac{(a_3 + b_3) - x}{(a_3 + b_3) - (a_2 + b_2)} \).

Applying Eq.(4) to the above equation, we obtain

\[
h(\mu_{\tilde{C}}(x)) = \begin{cases} 
0, & x < a_1 + b_1, a_3 + b_3 < x \\
-(y) \ln(y) - (1 - y) \ln(1 - y), & a_1 + b_1 \leq x \leq a_2 + b_2 \\
-(z) \ln(z) - (1 - z) \ln(1 - z), & a_2 + b_2 \leq x \leq a_3 + b_3 
\end{cases}
\]

Applying Eq.(1) to the above equation, and computing the entropy of TFN \( \tilde{C} \), the integration result is shown as

\[
H(\tilde{C}) = \int_{a_1 + b_1}^{a_2 + b_2} \left[-(y) \ln(y) - (1 - y) \ln(1 - y)\right]p(x)dx + \int_{a_2 + b_2}^{a_3 + b_3} \left[-(z) \ln(z) - (1 - z) \ln(1 - z)\right]p(x)dx
\]

\[
= \int_{a_1 + b_1}^{a_2 + b_2} \left[y \ln y + (1 - y) \ln(1 - y)\right]dy + \int_{a_2 + b_2}^{a_3 + b_3} \left[z \ln z + (1 - z) \ln(1 - z)\right]dz
\]

\[
= \left[\frac{y}{2} - \frac{1}{2}\right]_{a_2 + b_2}^{a_3 + b_3} + \left[\frac{z}{2} - \frac{1}{2}\right]_{a_1 + b_1}^{a_2 + b_2}
\]

\[
= \frac{s}{2}(2a_3 + b_3 - a_2 - b_2) + \frac{s}{2}(2a_2 + b_2 - a_1 - b_1)
\]

\[
= \frac{s}{2}(2a_2 - a_1 - a_3) + \frac{s}{2}(2b_2 - b_1 - b_3) = H(\tilde{A}) + H(\tilde{B})
\]

Hence,

\[
H(\tilde{C}) = \frac{s}{2}(2a_2 - a_1 - a_3) + \frac{s}{2}(2b_2 - b_1 - b_3) = H(\tilde{A}) + H(\tilde{B})
\]

**Theorem 3.** Supposing \( \tilde{C} = \tilde{A} - \tilde{B} \), \( \tilde{C} \) being a TFN, then the entropy of \( \tilde{C} \) is the difference of the entropies of \( \tilde{A} \) and \( \tilde{B} \). That is, \( H(\tilde{C}) = H(\tilde{A} - \tilde{B}) = H(\tilde{A}) - H(\tilde{B}) \).

**Proof.** For \( \tilde{C}(x) = \tilde{A}(x) - \tilde{B}(x) \), the membership function can be expressed as

\[
\mu_{\tilde{C}}(x) = \begin{cases} 
0, & x < a_1 - b_3, a_3 - b_3 < x \\
\frac{x - (a_1 - b_3)}{(a_2 - b_2) - (a_1 - b_1)}, & a_1 - b_3 \leq x \leq a_2 - b_2 \\
\frac{(a_3 - b_3) - x}{(a_3 - b_3) - (a_2 - b_2)}, & a_2 - b_2 \leq x \leq a_3 - b_3 
\end{cases}
\]

Let \( y = \frac{x - (a_1 - b_3)}{(a_2 - b_2) - (a_1 - b_1)} \), \( z = \frac{(a_3 - b_3) - x}{(a_3 - b_3) - (a_2 - b_2)} \).

Applying Eq.(4) to the above equation, we obtain

\[
h(\mu_{\tilde{C}}(x)) = \begin{cases} 
0, & x < a_1 - b_3, a_3 - b_3 < x \\
-(y) \ln(y) - (1 - y) \ln(1 - y), & a_1 - b_3 \leq x \leq a_2 - b_2 \\
-(z) \ln(z) - (1 - z) \ln(1 - z), & a_2 - b_2 \leq x \leq a_3 - b_3 
\end{cases}
\]

Applying Eq.(1) to the above equation, and computing the entropy of TFN \( \tilde{C} \), the result is shown as

\[
H(\tilde{C}) = \int_{a_1 - b_3}^{a_2 - b_2} \left[-(y) \ln(y) - (1 - y) \ln(1 - y)\right]p(x)dx + \int_{a_2 - b_2}^{a_3 - b_3} \left[-(z) \ln(z) - (1 - z) \ln(1 - z)\right]p(x)dx
\]

\[
= \int_{a_1 - b_3}^{a_2 - b_2} \left[y \ln y + (1 - y) \ln(1 - y)\right]dy + \int_{a_2 - b_2}^{a_3 - b_3} \left[z \ln z + (1 - z) \ln(1 - z)\right]dz
\]

\[
= \left[\frac{y}{2} - \frac{1}{2}\right]_{a_2 - b_2}^{a_3 - b_3} + \left[\frac{z}{2} - \frac{1}{2}\right]_{a_1 - b_3}^{a_2 - b_2}
\]

\[
= \frac{s}{2}(2a_3 - b_3 - a_2 + b_2) + \frac{s}{2}(2a_2 - b_2 - a_1 + b_1)
\]

\[
= \frac{s}{2}(2a_2 - a_1 - a_3) - \frac{s}{2}(2b_2 - b_1 - b_3) = H(\tilde{A}) - H(\tilde{B})
\]

Hence,

\[
H(\tilde{C}) = \frac{s}{2}(2a_2 - a_1 - a_3) - \frac{s}{2}(2b_2 - b_1 - b_3) = H(\tilde{A}) - H(\tilde{B})
\]
Theorem 4. Supposing $\tilde{A}^-$ is the image of TFN $\tilde{A}$, then the entropy of $\tilde{A}^-$ is equal to the negative entropy of $\tilde{A}$. That is, $H(\tilde{A}^-) = -H(\tilde{A})$.

Proof. For $\tilde{A}^-(x) = -\tilde{A}(x)$, the membership function can be expressed as

$$\mu_{\tilde{A}^-}(x) = \begin{cases} 0, & x < -a_s, -a_i < x \\ \frac{x + a_s}{a_i - a_s}, & -a_i \leq x \leq -a_s \\ \frac{-a_i - x}{a_s - a_i}, & -a_s \leq x \leq -a_i \end{cases}$$

Let $y = \frac{x + a_s}{a_i - a_s}, z = \frac{-a_s - x}{a_s - a_i}$. Applying Eq.(4) to the above equation, we obtain

$$h(\mu_{\tilde{A}^-}(x)) = \begin{cases} 0, & x < -a_s, -a_i < x \\ -(y)\ln(y)-(1-y)\ln(1-y), & -a_i \leq x \leq -a_s \\ -(z)\ln(z)-(1-z)\ln(1-z), & -a_s \leq x \leq -a_i \end{cases}$$

Applying Eq.(1) to the above equation, and computing the entropy of TFN $\tilde{C}$, the result is shown as

$$H(\tilde{A}^-) = \int_{-a_i}^{-a_s} [-(y)\ln(y)-(1-y)\ln(1-y)]p(x)dx + \int_{-a_s}^{-a_i} [-(z)\ln(z)-(1-z)\ln(1-z)]p(x)dx$$

Using mathematical induction, we can prove that when $\tilde{C} = n\tilde{A}, n \in N$, then $H(\tilde{C}) = nH(\tilde{A})$ is established.

Remark 2. If the “base width” of the TFN $\tilde{C}$ is $n$ ($n \in N$) times the “base width” of the TFN $\tilde{A}$, then the entropy of $\tilde{C}$ is $n$ times the entropy of $\tilde{A}$, using Shannon’s Function to compute their entropies.

Theorem 5. Supposing $\tilde{C} = n\tilde{A}, n \in N$, $\tilde{C}$ being a TFN, then the entropy of $\tilde{C}$ is $n$ times the entropy of $\tilde{A}$. That is, $H(\tilde{C}) = H(n\tilde{A}) = nH(\tilde{A})$.

Proof. $H(\tilde{C}) = n\times\frac{s}{2}(2a_s - a_i - a_s), n \in N$.

Let $y = \frac{x + a_s}{a_i - a_s}, z = \frac{-a_s - x}{a_s - a_i}$. Applying Eq.(4) to the above equation, we obtain

$$h(\mu_{\tilde{C}}(x)) = \begin{cases} 0, & x < -na_s, -na_i < x \\ -(y)\ln(y)-(1-y)\ln(1-y), & -na_i \leq x \leq -na_s \\ -(z)\ln(z)-(1-z)\ln(1-z), & -na_s \leq x \leq -na_i \end{cases}$$
Applying Eq.(1) to the above equation, and computing the entropy of TFN $\tilde{C}$, the result is shown as

$$
H(\tilde{C}) = \int_{-a_2}^{a_2} (-x) \ln(x) - (1-x) \ln(1-x) \, p(x) \, dx + \int_{-a_2}^{a_2} (-z) \ln(z) - (1-z) \ln(1-z) \, p(x) \, dz
$$

\[ \therefore \quad y = \frac{x + na_2}{na_3 - na_1} \quad \therefore \quad dx = (na_3 - na_2) \, dy, \]

\[ \therefore \quad z = \frac{-na_1 - x}{na_2 - na_1} \quad \therefore \quad dx = -(na_2 - na_1) \, dz \]

$$
H(\tilde{C}) = (s)(na_2 - na_1) \int_{-1}^{1} y \ln(y) + (1-y) \ln(1-y) \, dy + (s)(na_3 - na_2) \int_{-1}^{1} z \ln(z) + (1-z) \ln(1-z) \, dz
$$

\[ = \frac{s}{2} \left[ (5-s) \times 2a_2 + na_1 + na_3 \right] = (5-s) \times \frac{s}{2} (2a_2 - a_1 - a_3) = -s \cdot H(A)
$$

Hence, $H(\tilde{C}) = H(-n \tilde{A}) = (-n)H(\tilde{A})$.

**Remark 3.** Combining Theorems 5 and 6, the result is for any integer $m$, $m \in I$, $\tilde{C} = m \tilde{A}$, then the following relationship is always true: $H(\tilde{C}) = H(m \tilde{A}) = mH(\tilde{A})$.

**Theorem 7.** Suppose $\tilde{C}, \tilde{A}$, and $\tilde{B}$ are three TFNs, and they exit the linear relation as follows:

$\tilde{C} = n \tilde{A} + m \tilde{B}$, $n, m \in I$. The entropies of $\tilde{C}, \tilde{A}$, and $\tilde{B}$ exit the relation as:

$$
H(\tilde{C}) = H(n \tilde{A} + m \tilde{B}) = nH(\tilde{A}) + mH(\tilde{B}).
$$

**Proof.** We set $\tilde{C}_1 = n \tilde{A}$ and $\tilde{C}_2 = m \tilde{B}$. From Theorems 7 and 8, we know that

$$
H(\tilde{C}_1) = nH(\tilde{A}), \quad H(\tilde{C}_2) = mH(\tilde{B}) \quad \text{(A)}
$$

We set $\tilde{C} = \tilde{C}_1 + \tilde{C}_2$. From Theorem 2 we know that

$$
H(\tilde{C}) = H(\tilde{C}_1) + H(\tilde{C}_2) \quad \text{(B)}
$$

From formulas (A) and (B), we obtain $H(\tilde{C}) = nH(\tilde{A}) + mH(\tilde{B})$. Hence, the theorem is true.

**4 Conclusions**

In this paper, we used Shannon’s Function to find the entropy differences of triangular fuzzy numbers with the entropy of arithmetic operations and we summarized the above theorems. The main conclusions are as follows.

**References:**


