Construct a Personal Asset Allocation Models with Particle Swarm Optimization – An Example for Wealth Management

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Abstract: - Given the current benefits structure of the Taiwan Employee Retirement Income Security Act (TERISA), two pension plans (defined contribution and defined benefit) are examined to discuss the asset allocation of personal accounts, which may include N targets investment strategies. The study proposes Particle Swarm Optimization (PSO) approach to be compared with Genetic Algorithms for carrying out asset allocation and the findings indicate that PSO approach outperforms GAs approach in both solution quality and computation time. The adequacy of the pension plans is examined by an actuarial model based on the hypothesis that all the variables are set as random, including simulated salary growth rate, inflation rate, interest rate, and investment return rate.

Key-Words: - Particle Swarm Optimization, Genetic Algorithm, Income-Replacement Ratio, Markowitz Portfolio Theory, Asset Allocation

1 Introduction
Along with the aging of the population becoming more and more notable, the employee retirement funding of many countries around the world has appeared a structural unbalance: the payment (pension) increases while the income (contribution) decreases. This unbalance situation also occurs in Taiwan. According to the data reported in Statistical Yearbook [4], the dependency ratio is continuously increasing; for example, the dependency ratio from 2005 was 7.35 [13], which means there is one dependant for every five working-age people. The load of the whole society, especially of those working-age people, is becoming rather heavier.

The aging of the population can be viewed as the most fundamental problem brought up to pension funding. Because of population aging, the retired employees nowadays need to make longer life plans compared to those employees in the past. At the same time, people may hope to maintain their consuming ability above certain level in several years after their retirement.

These phenomena highlights the importance of pension plans with which everyone concerns: to have a sufficient and stable pension fund for living a comfortable and respectable life.

On the other hand, population aging also brings up a severe challenge to the management of Taiwan Employee Retirement Fund System. Two systems suitable for a strong-growing population environment: Pay As You Go and Defined Benefit (DB) cannot fit in with the current society need any more. Hence, the advanced countries over the world planed to reform the old systems and Defined Contribution (DC) has emerged to become the main trend. The Central Provident Fund in Singapore and the Privatization of Public Pension Fund in Chile (the private administrative units of the pension fund are “Administradoras de Fondos De Pensiones, AFP”) are two successful examples [11]. It is believed that the adoption of Defined Benefit system will be weeded out through the evolution of population composition.

Pension fund belongs to long-term capital which can take short-term price risk to exchange for high long-term profits and have the employees’ retirement life ensured. Hence, appropriate operation strategies to respond and manage the risks brought by revaluation are indispensable. Investment strategies with high risk surely have high return rate and vice versa. Therefore, each individual labor has different
strategies to invest their pension fund according to their living environment and needs.

In this research we introduce the concept of income-replacement ration to calculate the pension amount needed. Because the risk tolerance level of each individual is different, this research suggests that each one should try to find an investment strategy most suitable for their own to reasonably distribute the investment portions and ensure their retirement life the way they set up for themselves. This research proposes the PSO approach to find out the optimal asset allocation for personal investment problem.

2 Literature Review
Defined Benefit and Defined Contribution are two of the pension plans most commonly used[11][12]The pension plan of the labor standard laws in Taiwan is similar to Defined Benefit, while the plan of the Taiwan Employee Retirement Income Security Act (TERISA) belongs to Defined Contribution. According to Defined Contribution, the pension fund is accumulated by investing the amount contributed per month. In this study, Defined Benefit and Defined Contribution are discussed, and the Optimal Investment portfolio, Genetic Algorithm and Particle Swarm Optimization approach will be introduced as well.

2.1 Defined Benefit and Defined Contribution
In Defined Benefit pension plan, labors will be granted a specific amount of pension according to their working years and salary levels. In Defined Contribution pension plan, labors will be granted certain amount of fund from both the employers and themselves transferring to their retirement pension account for investment; the accumulated amount in this account will be used as the pension fund when they get retired.

Under DB system, the pension amount is decided by the labor’s salary (usually of the last year) and his/her working period. The employers give a specific amount of pension benefits when the labors apply for retirement. Because the amount is decided depending on the final year of the working period without considering the period as a whole process contribution, the investment strategy on this plan should view the pension fund as a whole unit, without considering different individual labor accounts.

As for DC, the contribution process instead of the investment result is taken into consideration; that is, the employers allot pension funds on schedule to

individual employee’s account, and the withdraw and deposit rate are determined according to the employee’s salary and working years till the termination of the working period. Thus, the employee’s pension fund is the accumulated amount of his/her personal account. When drawing up the investment strategies, it is the employee him/herself that should be taken into consideration, instead of the system as a whole, and the strategies should fit the employee’s personal preference for returns and risks.

2.2 Investment Combination Theory
Portfolio Selection was proposed by an American economist, the Noble Award winner Markowitz in 1952. Markowitz thinks that investors can find the optimal investment combination by considering the expected return rates of target strategies, related risk coefficients, the limitations of selection tools for investment and objective environment condition. The investment policies is formulated according to the tolerance risk level of an investor, which can maximize the return under fixed risk situation or can minimize risk for the investor under a situation of maximum return. An investor can have different kinds of target investment strategies to form a variety of investment combination and decrease the risk level. This can be explained by the following equation:

\[
E(R_p) = \sum_{i=1}^{N} W_i E(R_i)
\]

where

- \(E(R_p)\) = the expected return rate of investment combination
- \(W_i\) = the weights of \(i\)th asset to the whole investment
- \(E(R_i)\) = the expected return rate of \(i\)th asset
- \(N\) = the asset number of investment combination

In the past, many researchers used Artificial Neural Networks (ANNs) and statistical methods for searching solutions in investment combination optimization problems [5][7]; However, there exist certain limitations in these methods. First, the process of ANNs is like black-box operation; it is hard to explain the internal algorithm and over-fitting and local optimization may occur in the construction of data model. Statistical methods belong to linear algorithms which are not suitable for non-linear problem such as investment combination optimization.
2.3 Genetic Algorithms

Genetic Algorithms (GAs) proposed in Holland [2], adopts the natural selection scheme for reproducing better offspring than the mother generation. Evolution and Selection are two fundamental essences. Genetic algorithms use three basic operation mechanisms to imitate the process of natural inheritance: Reproduction, Crossover and Mutation. Through the evolution of these three processes, a new offspring is created from the mother generation. The selection is processed by defining a fitness function to construct its external living environment. In each generation, the fitness of the whole population is evaluated and all species evolve based on this evaluation mechanism.

Genetic algorithms are suitable for searching the optimal solution; however, the encoding process is difficult and without the ability to memorize, the calculation usually is very time-consuming when the solution space is large. In this research we attempt to try a recently developed fast search approach PSO for the asset allocation problem.

2.4 Particle Swarm Optimization (PSO)

Particle Swarm Optimization (PSO) is also an evolutionary computation proposed by Eberhart and Kennedy [3]. Its main idea is motivated by research on social behavior of organisms such as bird flocking and fish schooling. PSO is constructed by simulating a flock of birds searching for food; suppose the following scenario: a group of birds are randomly searching food in an area. Assume there is only one piece of food in the area being searched. None of the birds knows where the food is. But they know how far the food is in each iteration. So what's the best strategy to find the food? The most effective and simplest one is to follow the bird which is nearest to the food.

In a PSO system, each bird in the search space stands for a single solution, which is called a "particle". The position of each particle is expressed as \( x_{id} \), where \( i \) is the \( i^{th} \) particle and \( d \) is the dimension degree of the search space. The velocity of each particle flying at each dimension degree is expressed as \( v_{id} \), where \( i \) is the \( i^{th} \) particle and \( d \) is the dimension degree of the search space. Each particle has its fitness value of the optimization problem and is aware of its current fittest value and best position, which is called the Particle best value (\( p_{id} \)). The information to the particles is like their own experience. At the same time each particle is aware of the current fittest value and best position in the group, which is called the Globe best value (\( p_{gd} \)) of the group, and this information is like the experience of other particles. Afterward, each particle adjusts its position according to the following information:

\[
\begin{align*}
    x_{id} &= x_{id} + v_{id} \\
    v_{id} &= v_{id} + c_1 \cdot rand_1 \cdot (p_{id} - x_{id}) + c_2 \cdot (p_{gd} - x_{id})
\end{align*}
\]

Finally, the current position of each particle is adjusted by the following principles:

\[
\begin{align*}
    v_{id} &= v_{id}^{old} + c_1 \cdot rand_1 \cdot (p_{id} - x_{id}) + c_2 \cdot (p_{gd} - x_{id}) \\
    x_{id} &= x_{id}^{old} + v_{id}^{new} \\
    \text{if } V_{id} > V_{max}, V_{id} = V_{max} \\
    \text{else if } V_{id} < V_{min}, V_{id} = V_{min}
\end{align*}
\]

where \( C_1 \) and \( C_2 \) are learning factors; \( rand_1 \) and \( rand_2 \) are random variables between 0 and 1; \( V_{max} \) is the maximum speed to be set.

PSO as an optimization tool has certain advantages such as:

1. Distribution Search
2. Be capable of memorization
3. Easier to fulfill because of fewer elements
4. Suitable for searching in a continuous area

Compared with the encoding difficulty of GAs, PSO can directly process real numbers. Further, GAs do not have the ability to memorize information, so the differences between chromosomes in each generation are irrelevant. However, since each particle in PSO can memorize the flight route and update every movement according to its past experience, all the particles tend to converge to the best solution more quickly. Basically, PSO has potentials to outperform GAs in both solution quality and computation time.

3 The Proposed Model

An actuarial model is established to consider the pension fund accumulated during work seniority and withdrawn after retirement. The required amount of pension fund is calculated according to the income-replacement ratio and the pension gap is calculated based on the required pension fund and accumulated amount. An expected return rate will be obtained after a simulating process and its investment combination allocation can be achieved through PSO searching.
### 3.1 An Actuarial Model of Defined Contribution

According to Taiwan Employee Retirement Income Security Act (TERISA), the pension fund is contributed to the employee's personal account per month; employers must contribute at least 6% of worker's monthly wage determined by the Monthly Contribution Wages Classification of Labor Pension Table [12] and workers voluntarily contribute maximum 6% to their pension accounts. Supposed that a worker starts working at the age of $x$ and gets retired at the age of $n$, the accumulated amount should be:

$$C_y = \sum_{i=1}^{12} (C_{e\%} + C_{c\%}) \times S_y \times \prod_{k=1}^{12} q_{y,k}$$

in which the fund is set to be contributed in the beginning of each month.

- $y$: contribution start age
- $n$: retirement age
- $C_{e\%}$: percentage contributed from employers $\leq 6\%$
- $C_{c\%}$: percentage contributed from employers $\geq 6\%$
- $S_y$: monthly wage of contribution start age
- $q_{y,k}$: monthly investment return rate
- $i_{y,k}$: monthly investment return rate

At the age of $t$, the total accumulated amount of an employee's personal account should be:

$$C_t = C_y \times (1+i_t) + \sum_{s=1}^{n} C_{s\%} \times S_s \times \prod_{k=1}^{12} q_{s,k} \times \prod_{m=s}^{t} (1+g_m)$$

where

- $g_m$: yearly salary growth rate (supposed to be adjusted every year)
- $i_t$: investment return rate of the year $t$

### 3.2 Income Replacement Ratio

Currently all over the world the required expense of elderly life is mostly expressed by income-replacement ratio, which is used as an index to measure if the living standard of life after retirement is the same as that before retirement.

Generally, the income-replacement ratio is higher in developed countries; however, the basic living demands of each individual are not exactly the same, and there is no absolute answer to the question of how much income is sufficient for guaranteeing the elderly life. Labors with more salary relatively require lower income-replacement ratio and labors with less salary require higher income-replacement ratio. The calculation formula is as follows:

Income Replacement Ration =

$$\frac{\text{Pension fund paid per month after retirement}}{\text{monthly salary before retirement}}$$

Thus, the required amount of pension fund is:

$$PV = A \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$A$: amount of pension fund paid per month after retirement
- $i$: annual interest after retirement

### 3.3 Asset Allocation Approaches

This research focuses on calculating the pension gap based on income-replacement ratio to simulate the expected return rate of invested assets with risks and establish two optimal models by PSO and GA approaches according to the simulated investment return rate. These two approaches are applied to determine the DC pension system and to compare the accumulated return rates of asset allocation of equally-weighted portfolios. When the return rate of target investments combination goes higher, the accumulated amount of the account will increase and so will the income-replacement ratio. Thus, the pension gap will become smaller. As for the asset allocation comparison, two approaches are considered in this research:

1. **1/N method**:
   
   Supposed that there are $N$ targets investments and the amount of funds invested into each target investment is $1/N$ of the total capital and the allocation portion of each contribution is the same unless the target investments number is changed. In the research the target investments number is fixed: 29 funds under 7 asset classifications recommended by $M$ company.

2. **Markowitz Mean-Variance Method**

   According to the method proposed by Markowitz, the allocation ratio of individual target investment of next period is calculated based on the return and risk rate of each target investment. The accumulated amount in the account and the contribution fund will be allocated according to this ratio.
4 Experiment
Both Table 1 and Table 2 show the Assumptions of Actuarial Model and its estimated parameters of fitness.

Table 1 Assumption of actuarial model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>30</td>
</tr>
<tr>
<td>Monthly salary</td>
<td>50,000</td>
</tr>
<tr>
<td>Investable Asset</td>
<td>500,000</td>
</tr>
<tr>
<td>Investable per each month</td>
<td>7,000</td>
</tr>
<tr>
<td>Age to retire</td>
<td>60</td>
</tr>
<tr>
<td>Age to death</td>
<td>80</td>
</tr>
<tr>
<td>Salary growth rate</td>
<td>2%</td>
</tr>
<tr>
<td>Inflation rate</td>
<td>2%</td>
</tr>
<tr>
<td>Income replacement ratio</td>
<td>70%</td>
</tr>
<tr>
<td>Percentage contributed</td>
<td>from employers 6%</td>
</tr>
</tbody>
</table>

Table 2 Estimated parameters of fitness.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required amount of pension fund</td>
<td>13,058,516</td>
</tr>
<tr>
<td>Monthly salary at retirement</td>
<td>90,568</td>
</tr>
<tr>
<td>Pension fund paid per month</td>
<td>63,398</td>
</tr>
<tr>
<td>Pension fund sources:</td>
<td></td>
</tr>
<tr>
<td>1. Social insurance</td>
<td>1,890,000</td>
</tr>
<tr>
<td>2. DC</td>
<td>1,995,830</td>
</tr>
<tr>
<td>Pension fund gap</td>
<td>9,176,004</td>
</tr>
<tr>
<td>Estimated expect return rate</td>
<td>5.6%</td>
</tr>
</tbody>
</table>

Table 3 Execution Time Comparison based on Optimization Performance

<table>
<thead>
<tr>
<th></th>
<th>PSO Exec. Time(ms)</th>
<th>GA Exec. Time(ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best Performance</td>
<td>10290</td>
<td>37058</td>
</tr>
<tr>
<td>Worst Performance</td>
<td>970</td>
<td>910</td>
</tr>
</tbody>
</table>

4.1 The Analysis of Experimental Results
29 funds under the 7 asset classifications recommended by M investment consulting company are used as the subjects for the simulation test. The experimental results are illustrated as follows:
In this section the findings can be summarized and discussed in two parts:

1. The comparison of solution quality: Fig. 1 and Fig. 2 indicate that the solution quality of PSO with swarm size of 400 after 500 iterations is much better than that of GA with population size of 400 after 5000 iterations.

2. The comparison of execution time: Fig. 3 and Fig. 4 display the execution time of PSO with swarm size of 400 after 5000 iterations is far less than that of GA population size of 40 after 500 iterations. This indicates that compared with GA, PSO has superior execution efficiency. Further, it can be seen in Table 3 that PSO takes similar time as GA in the initial stage; while reaches the best performance in a far more efficient way than GA.

3. The comparison of cumulative return: Fig. 5 indicates that both approaches outperform those of equal weight and benchmark.

5 Conclusions and suggestions

The investment performance of pension account is more and more important role in the life after retirement nowadays. Thus, how to enhance the investment performance highlights the importance of asset allocation and once an appropriate allocation has been made, it would increase a lot the accumulated value of the pension account Meanwhile an inappropriate allocation would worsen the quality of the after-retirement life.

Two asset allocation algorithms are utilized to simulate the return rates of 29 funds under 7 asset classifications in the research. Regarding the part of the actuarial model, the pension payment conditions regulated in the Labor Pension Act approved on June 11, 2004 are taken into consideration for the purpose of establishing an optimal pension model suitable for everyone.

In the experiment results, the performance of PSO has been shown to be much better than GAs both in the solution quality and execution time. As for the asset allocation, whether by PSO or GAs, the allocation performance results are all better than the accumulated performance of unallocated portfolio or of benchmark indices (as in Fig. 3). The future research can be extended to study the following aspects:

1. The life financial planning can be added to the actuarial model.

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