Advances on Calculating Effective Dither for Audio Signals

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Abstract: - Dithering represents a well-established, attractive technique for minimizing the audibility of the error produced during requantization of digital audio signals. However, it is also known that, as dither is applied as controlled, additive random noise prior requantisation, it also increases the overall quantization noise level and audibility. In this work, a novel dither generator is presented, based on a novel stochastic approach that derives a statistically equivalent signal to the original input. Using a sequence of tests as well as objective and psychoacoustic measurements, it is shown that the addition of the proposed dither generator output to the initial input signal and the application of a low-order noise shaper significantly suppresses the quantization error perception.

Key-Words: - Digital Audio, Signal Quantization, Dither, Noise-shaping

1 Introduction
The well-known advantages of digital audio technology are nowadays available to any audiophile user. Starting from the widespread employment of high-quality digital audio sources (such as DVD, DVD-Audio and SACD) as well as the digital audio distribution over packet-based networks and the Internet, the audio distribution and playback chain is now becoming fully digital [1]. Traditional analog audio subsystems (such as power amplifiers and loudspeakers) are replaced by digital ones, such as all-digital PWM amplifiers [2] - [3] and digital transducer arrays [4] - [5], allowing the direct playback of any digital audio waveform, without the employment of digital-to-analog converters.

Hence, although digital-to-analog conversion presence is not necessary, sampling and quantization is still required for converting the recorded analog audio signals to digital for further storing, processing and playback. Moreover, the processing / filtering of digital audio signals in software or hardware DSP platforms frequently requires truncation of the output signal to the original bit resolution defined prior processing.

Dither represents a well-established technique for minimizing the distortions induced by the signal quantization / requantization process. Briefly, dither renders quantization error objectively and subjectively optimized [6], while it reduces the harmonic distortion and noise modulation raised during quantization of very-low signal levels [7], at the expense of increasing the overall quantization level. As it will be described in the next paragraph, this is performed by adding a (random) noise with specific statistical properties to the signal under quantization, combined with appropriate noise-shaping techniques for minimizing the audible effect of the increased noise presence. A number of studies have extended the theory and applications of dither to Σ-Δ modulation in order to eliminate harmonics artifacts raised due to the presence of periodic bit patterns [8], [9], as well as to Pulse Width Modulation (PWM) conversion for distortion minimization / elimination [10].

In this work, a novel additive dither generation algorithm is proposed termed as Stochastic Dither (STD), based on a stochastic approach that produces a statistically equivalent signal to the original input. The performance of the proposed algorithm is assessed using both objective and perceptual criteria, allowing the accurate effect estimation and optimization of the algorithm parameters for audio signals.

The rest of the paper is organized as follows: In Section 2, an overview of the conventional dither theory and application for audio signals is presented, followed by the analytical derivation and description of the proposed dither generation algorithm included in Section 3. In Section 4, the testing parameters, procedures and criteria for evaluating the perceptual performance of the proposed dither algorithm are described, followed by the results obtained during the tests and the conclusions summarized in Section 5.
2 Conventional dither overview

As it was previously mentioned, dither represents a random noise with controlled amplitude, which is added to the audio signal prior quantization, as it is shown in Fig. 1. In this Figure, the input signal is analog. However, dither can also be applied to digital signals during their requantization to a lower bit resolution. In general, the effect of additive dither can be summarized as following: a) it directly affects the quantization error by rendering it independent from the input signal b) it minimizes the harmonic distortion raised during the quantization of signals with amplitude near the value that corresponds to the least significant bit and c) it subjectively increases the quantized signal dynamic range.

\[ X_i = (aX_{i-1} + c) \mod B \]  
(1)

where \( a, c \) and \( B \) are integer numbers with \( a>0, c<B \). The output values \( X_i \) are uniformly distributed within the range \( 0<X_i<B \). For \( i=0 \), the initial (seed) value of the number generator must be defined within the above range. It should be also noted that eq. (1) produces a periodic random output, that is the sequence of the random values is repeated after the generation of a finite number \( L \) of dither samples. The period \( L \) can be maximized by appropriately selecting the values of \( a, c \) and \( B \) parameters appeared in eq. (1) [11].

As dither has a statistic nature, it is characterized by a specific Probability Density Function (PDF), which defines the “generation” probability of a specific dither value within the allowed dither range (usually measured as a portion of the quantization least significant bit amplitude \( \Delta \)). It is known that, for a digital signal with amplitude values limited within the range \([-S_m, S_m] \), the least significant bit value \( \Delta \) is calculated using the following equation:

\[ \Delta = \frac{2S_m}{2^N} \]  
(2)

where \( N \) denotes the digital signal bit resolution. Obviously, large maximum dither values result into increased noise floor elevation and vice versa.

Common conventional dither implementations employ the Rectangular PDF (RPDF) and the Triangular PDF (TPDF) with typical amplitude ranges shown in Fig. 2. The RPDF dither can be implemented by directly employing eq. (1), which produces random values with equal generation probability. In order to obtain TPDF dither values, as it is shown in Fig. 3, the output of two random number generators based on eq. (1) must be added. According to [7], TPDF dither perceptually achieves significantly improved performance compared to the RPDF dither; hence it is used in the following Sections in order to evaluate the performance of the proposed STD dither.

Fig. 2: Typical dither PDFs

It should be also noted that a variation of the TPDF dither was previously proposed [11], defined as the subtraction of an one sample delayed random generator output from the original output of the same generator (see also Fig. 3). This dither type is termed as high-pass TPDF dither, due to the high-pass characteristics of its spectrum. However, this dither type is not considered in this work, as the same high-pass effect can be efficiently and more accurately achieved using well-established noise-shaping techniques [12].
3 Stochastic dither implementation

The proposed “STochastic Dither” (STD) generation scheme is based on a statistical treatment of an arbitrary input signal by means of a Taylor expansion around the mean. As a result, the expansion of the input signal in terms of an arbitrary white noise process can be derived, where the appropriate “signal information” is included into the corresponding expansion coefficients [13]. The produced arbitrary white noise samples represent the dither values that are added on the original audio signal prior quantization / requantization.

More specifically, let \( S \) be the original audio signal, the statistical properties of which must be calculated. Quite general, we may assume that \( S = S(W) \), where \( W \) is a white noise stochastic process with the standard statistical properties, i.e.

\[
\langle W \rangle = 0
\]

and

\[
\langle W_i W_j \rangle = \sigma^2 \delta_{ij}
\]

with \( \sigma^2 \) being the noise amplitude. In a first approximation we may approximate \( S(W) \) locally with a parabola performing a Taylor expansion around the mean \( \langle W \rangle \) as described in [14] using the equation:

\[
S(W) = S(\langle W \rangle) + (S'_{\langle W \rangle})W + \frac{1}{2}S''_{\langle W \rangle}W^2 \quad (5)
\]

From the above equation, following reference [14] it can be shown that

\[
S'_{\langle W \rangle} = \frac{2}{3}\sigma_3^{1/3} \sigma^2
\]

where \( \sigma_3^2 \) is the variance of the original input signal.

Taking the time averages of both sides of eq. (5) it is important to note that, as usually assumed in physical systems, we can distinguish two distinct time scales: a) time averages that are calculated for time periods long enough for the rapid fluctuating noise \( W \) and b) time averages calculated over sort enough time intervals in order the expansion above has meaning in the vicinity of \( S(\langle w \rangle) \), that is

\[
\langle S(W) \rangle = S(\langle W \rangle) + \frac{1}{2}S'_{\langle W \rangle} \sigma^2
\]

where we have used that \( \langle W \rangle = 0 \) and \( \langle W^2 \rangle = \sigma^2 \). To estimate \( S'_{\langle W \rangle} \) we apply eq. (7) for the function \( S^3(\langle W \rangle) \). Since its second derivative at \( \langle W \rangle \) equals to

\[
6S(W)S'_{\langle W \rangle}^2 + 3S(W)^2S''_{\langle W \rangle},
\]

calculations and neglecting higher orders we conclude that,

\[
S'_{\langle W \rangle} = \left( \frac{1}{\sigma^2} - \frac{1}{3} \right)S - \frac{2}{3} \frac{\mu_3}{\sigma^2} \quad (8)
\]

where the abbreviation \( S = S(\langle W \rangle) \) was used and \( \mu_3 \) is the third central moment of the experimentally observed signal. In order to estimate \( S \) we return to eq. (4) and inverting,

\[
S = \langle S(W) \rangle - \frac{1}{2}S'_{\langle W \rangle} \sigma^2 \quad (9)
\]

For low noise amplitude, using eq. (8) and substituting finally in eq. (5) we end up with

\[
S_{\text{est}}(W) = \frac{2}{3}\langle S(W) \rangle + \frac{2}{9} \frac{\mu_3}{\sigma^3} \sigma^2 + \left( \frac{\sigma_3^{1/3}}{3} \right)W + \frac{1}{2} \left[ \frac{1}{\sigma^2} \left( \frac{2}{3}\langle S(W) \rangle + \frac{2}{9} \frac{\mu_3}{\sigma^3} \sigma^2 \right) - \frac{2}{3} \frac{\mu_3}{\sigma^2} \right] W^2 \quad (10)
\]

From the above analysis it is clear that the original signal \( S(W) \) may be written as a polynomial of an arbitrary white noise process where the corresponding coefficients are explicitly expressed as functions of the statistical moments (of any order depending the desired accuracy) of the original input signal.

4 Test description and results

4.1 Test methodology

The performance of the proposed STD dither generation scheme was measured and compared to the TPDF dither described in Section 2 using requantization of a 16 bit audio signal to 8 bits. Two different audio signal types were considered: a) digital sine waves with amplitude \( M \) and frequency \( f_{\text{in}} \) shown in Table 1 and b) a 16-bit typical audio (music) track. The sampling frequency of the audio signals in all cases was equal to 44.1kHz, while 3rd order noise shaping was always applied during requantization.

Table 1: Sine wave test signals parameters

<table>
<thead>
<tr>
<th>Amplitude ( M ) (dB relative Full Scale)</th>
<th>0dB-FS</th>
<th>-20dB-FS</th>
<th>-40dB-FS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency ( f_{\text{in}} ) (kHz)</td>
<td>0.5kHz</td>
<td>1kHz</td>
<td>5kHz</td>
</tr>
</tbody>
</table>

The implementation of the conventional TPDF dither during the above requantization tests was performed using two random number generators with parameters shown in Table 2 (see also eq. (1))
for the definition of these parameters). It should be also noted that the values $X_i$ produced by eq. (1) where normalized in the range of $[-\Delta, \Delta]$, where $\Delta$ was defined previously as the signal amplitude that corresponds to the least significant bit. The same normalization process was also performed in the STD dither case, in order to produce STD dither values within the same amplitude range $[-\Delta, \Delta]$.

Table 2: Random number generator parameters

<table>
<thead>
<tr>
<th>Generator</th>
<th>$a$</th>
<th>$c$</th>
<th>$B$</th>
<th>$X_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generator 1</td>
<td>3453</td>
<td>1</td>
<td>65536</td>
<td>1531</td>
</tr>
<tr>
<td>Generator 2</td>
<td>2945</td>
<td>1</td>
<td>65536</td>
<td>18531</td>
</tr>
</tbody>
</table>

4.2 Test criteria and measurements

The performance of the STD dithering algorithm was evaluated using a) the well-known Signal-to-Noise Ratio (SNR) objective measurement and b) subjective measurements, that take into account the psychoacoustic properties of human hearing. More specifically, the SNR value (in dB) was evaluated using the equation:

$$\text{SNR} = 10 \log_{10} \left( \frac{\sum_{n=1}^{NS} S^2(n)}{\sum_{n=1}^{NS} (S(n) - S_q(n))^2} \right)$$

where $S(n)$ is the original 16-bit input signal and $S_q(n)$ is the 16-to-8 bit requantizer output. $NS$ denotes the total number of input samples.

Moreover, in order to perceptually assess the audible effect of the proposed STD dithering scheme, the well-established Noise-to-Mask Ratio (NMR) subjective criterion [15] was employed which can assess the perceived requantization effect, taking into account spectral and temporal masking effects [16]. It should be noted that lower NMR values (expressed in dB) indicate the presence of less audible distortions. For NMR value estimation, the original 16-bit input audio track (prior to requantization) was used as reference.

4.3 Test results

Fig. 4 shows the measured SNR values in the case of sinewave input signals, as described in Table 1. The SNR value in the case of typical audio material is also shown in Table 3. From these results it is clear that in all test cases, the proposed STD dithering scheme achieves a significant improvement in terms of quantization noise level, that is, under the same additive dither amplitude range (defined previously equal to $[-\Delta, \Delta]$ for both dither cases), the STD dither noise floor elevation is significantly less than in the case of TPDF dither.

Table 3: Measured SNR for typical audio material

<table>
<thead>
<tr>
<th>Dither type</th>
<th>Measured SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>TPDF</td>
<td>22.61dB</td>
</tr>
<tr>
<td>STD</td>
<td>25.17dB</td>
</tr>
</tbody>
</table>

The same trends were verified in the case of measuring the NMR perceptual criterion values. As it is shown in Table 4, the measured NMR in the case of typical audio material is lower when the STD dithering scheme is employed. Hence, it is clear that the proposed STD algorithm significantly improves the overall audibility in terms of the noise induced during the requantization process.

Table 4: Measured NMR for typical audio material

<table>
<thead>
<tr>
<th>Dither type</th>
<th>Measured NMR</th>
</tr>
</thead>
<tbody>
<tr>
<td>TPDF</td>
<td>18.99dB</td>
</tr>
<tr>
<td>STD</td>
<td>16.71dB</td>
</tr>
</tbody>
</table>

5 Conclusions

In this work, a novel additive dithering scheme is presented based on the derivation of a noise signal, which is statistically equivalent to the original input under quantization / requantization. Compared to existing, well-established dithering methods (such as TPDF white noise dither), the proposed STD dithering scheme demonstrates a significantly improved objective as well as perceptual performance. Typical applications of the STD scheme include quantization of analog signals, as well as requantization of high-resolution digital audio signals to lower bit resolution values in order to decrease their storage requirements and the bandwidth required for their transmission and distribution (e.g. over packet-based networks).
References:


