Modelling and Monitoring of Hybrid Systems by Hybrid Petri Nets and (Max, +) Algebra

ABDELAZIZ ZAIDI, NADIA ZANZOURI and MONCEF TAGINA
Laboratory ACS, Department of Electrical Engineering
National School of Engineers of Tunis (ENIT)
BP37, 1002 Tunis-Belvedere
TUNISIA

Abstract: - An hybrid dynamic system (HDS) is the result of interaction between continuous dynamics and discrete events. The modelling of this class of physical systems is possible when using tools supporting a combined modelling approach. We propose to model a system composed of three communicating tanks by Hybrid Petri Nets (HPN). To monitor the behaviour of the system, we propose an approach based on the HPN formalism for the monitoring of the liquid levels. For the monitoring of the opened and closed events of an output valve, we propose an original approach based on the representation in the (Max, +) algebra of the discrete part. We show in this paper the interest of the use of only one tool for the modelling and the monitoring of the HDS.

Key words: - Hybrid Dynamic Systems, Monitoring, HPN, (Max, +) Algebra, Application.

1 Introduction
The study of problems related to the control and monitoring of hybrid systems is relatively new. The majority of the systems classified in this category were already studied with the old approaches (non-linear systems).

The Petri Nets (PN) takes a broad place in the literature of modelling of the HDS. The first most significant stage was the extension of traditional (discrete) PN by continuous PN (CPN). The principal idea was in fact to have a real marking instead of integer one. CPN are approximations of discrete event systems (DES) with faster simulation without influencing the precision of the results. The calculation of the firing speed of a transition in timed CPN was defined in [1]. HPN were proposed for the first time by Bail and al. [2], they combine ordinary (discrete) PN and CPN.

In the second section we present the modelling and the monitoring of hybrid physical systems by only one approach: Petri Nets. The Section 3 will be the subject of an application of this approach and the proposal method to monitor three communicating tanks system. Finally, in section 4, we conclude on the advantages and limits of this approach as well as future work.

2 Modelling and Monitoring of HDS by only one tool: PN
The approaches used for the study of the hybrid systems are so varied: we can mention the hybrid automata [3], the coupling of the continuous models using the bond graph tool and the discrete automata [4] and also the coupling of bond graph model and discrete PN [5], we find finally models based only on the PN tool. The traditional representation in this formalism was extended by more elements so that the tool becomes more flexible in the representation of any continuous state or discrete event. We present, then, an approach based on HPN and the (Max, +) algebra for the modelling and the monitoring of the HDS. This approach will be applied thereafter to a system of three communicating tanks.

2.1 HPN
HPN are combination of two parts; discrete PN and continuous PN (CPN). HPN contain discrete places and transitions (D-place and D-transition) and continuous places and transitions (C-place and C-transition) (Fig.1). The input and output places of a D-transition can be continuous or discrete. The input and output places of the C-transitions are continuous, they can be discrete only if the place is looped through itself [6] [7] [8].
To solve some problems of modelling of the HDS, an extension of ordinary PN was proposed [9]. The arcs represented on figure 2 are added to the traditional representation.

\[ X(n) = A \otimes X(n-1) \]  

This solution is given in the form of a recurrent equation of order 1 [11]:

\[ X(n) = A_0 \otimes A_1 X(n-1) \]  

with \( A = A_0 \otimes A_1 \);

\[ A_0 = E \oplus A_0 \otimes A_0, \quad A_0 \geq 0 \quad \text{and} \quad E = \begin{bmatrix} e & e \\ e & e \end{bmatrix} \]

so we have:

\[ A_0 = \begin{bmatrix} e & e \\ e & e \end{bmatrix} \Rightarrow A = \begin{bmatrix} e & \tau_{p1} \\ \tau_{p2} & e \end{bmatrix} \]

### 3 Applications

#### 3.1 Description of the physical system

The physical system is composed of three communicating tanks (Fig. 4). The first tank, largest, is supplied by a permanent source of liquid. The second is an intermediate tank between the first and the third. This tank outputs outside.

#### 3.2 Modelling of the system by HPN

**3.2.1 Theoretical relations**

We consider \( P \) as the pressure in one tank and \( Q \) its volumic flow, we have the relation:

\[ \frac{dP}{dt} = (1/C)*Q \]  

with \( C \): the capacity of the tank. The CPN of figure 5 contains two places \( P_1 \) and \( P_2 \). Each tank is modelled by a C-place. \( P_1 \) and \( P_2 \) have continuous markings, which are respectively the heights \( h_1 \) and \( h_2 \) of the liquid on the level of the two communicating tanks.

The transition \( T_1 \) has a continuous speed \( V \) which we will determine its expression.
The height of the liquid in the second tank is:

\[ h_2(t) = \int V \, dt \Rightarrow \frac{dh_2}{dt} = V \]  

(7)

The relation (6) implies that:

\[ Q = C \frac{dP}{dt} \]  

(8)

or we have:

\[ Q = \text{sign} (h_1 - h_2) \times H_2 \times K_2 \times \sqrt{\rho g |h_1 - h_2|} \]  

(9)

with \( H_2 \in \{0,1\} \), which represents the state of the valve (opened or closed), \( K_2 \) is a linear gain, \( \rho \) the density of the liquid and \( g \) the constant of gravity. This flow is determined by the Bernoulli’s equation of energy for an incompressible liquid in the assumption of a turbulent flow. We can simplify the equation (9):

\[ Q = K \times \text{sign} (h_1 - h_2) \times \sqrt{|h_1 - h_2|} \]  

(10)

with \( K \) a linear constant which characterizes the connection between the two tanks.

(8) and (10) =>

\[ \frac{dP}{dt} = \frac{K}{C} \times \text{sign} (h_1 - h_2) \times \sqrt{|h_1 - h_2|} \]  

(11)

One can easily deduce from the relation \( P = \rho gh \) the relation:

\[ \frac{dh_1}{dt} = (1/\rho g) \frac{dP}{dt} \]  

(12)

The relations (7), (11) and (12) imply that:

\[ V = \frac{K}{\rho g C} \times \text{sign} (h_1 - h_2) \times \sqrt{|h_1 - h_2|} \]  

(13)

To simplify the equation for simulation, one considers that:

\[ V = \text{sign} (h_1 - h_2) \times \sqrt{|h_1 - h_2|} \]  

(14)

### 3.2.2 HPN model of the system

The figure 6 shows the HPN model of the physical system with inhibiting arcs.

- A C-transition \( T_1 \) is used to model the continuous source of liquid. The C-transition \( T_4 \) represents the controlled valve \( V_4 \). Its control (ON/OFF) is ensured by the presence of one token in the D-place \( P_4 \). The continuous places \( P_1, P_2 \) and \( P_3 \) are connected by traditional arcs to indicate the transfer of quantity of marking from a tank towards the following one. The D-place \( P_4 \) must initially have a token to order the opening of valve 4, because the system is supposed with initial conditions (for \( t=0 \), \( h_1 = h_{10}, h_2 = h_{20} \) and \( h_3 = h_{30} \)). The two D-transitions \( T_5 \) and \( T_6 \) are fireable under the following respective conditions:

\[ h_3 < h_{3\text{min}}; h_1 > h_{1\text{max}} \]

The presence of an token in the place \( P_4 \) inhibits the firing of \( T_6 \). The simulation results in normal operation of the continuous places \( P_1, P_3 \) and the discrete place \( P_4 \) by the software Visual Object Net++ [12], is presented at figure 7.

![Simulation of normal operation](image)

The simulation shows that when the token is present at the place \( P_4 \), \( T_4 \) fires and valve 4 will be opened. The level of the liquid decreases until the firing of \( T_5 \) which causes the closing of valve 4 (\( P_4 = 0 \)). The level of the liquid increases and there will be again the firing of \( T_6 \) from where \( P_4 = 1 \) and valve 4 opens and so on.

### 3.3 Monitoring of the system

#### 3.3.1 Monitoring of the continuous part

In this paragraph we propose an approach for monitoring measurable outputs of the system by extended HPN.
We propose to monitor the heights of the liquid $h_1$ and $h_3$ which must, in a correct operation of the system, respect the conditions: $(h_1 \leq h_{1\text{max}})$ and $(h_3 \geq h_{3\text{min}})$ for $t > t_0$ with $t_0$ the moment of beginning of operation cycle. The model of the monitored system is presented at figure 8.

We introduce two comparators because of the presence of two conditions for commutation. The model of a comparator was proposed in [13] for the control of a robot. This model is used to compare a continuous variable with a reference one.

The figure 8 shows the existence of three various states which must be distinguished. These states are represented by the places $\text{lo}(\text{lower})$, $\text{gr}(\text{greater})$ and $\text{eq}(\text{equal})$ indicating respectively the results of comparison: $h_i < h_j$, $h_i > h_j$ and $h_i = h_j$ (the equal term represents a confidence interval indicating that $h_i$ is close to $h_j$). The PN must be initially marked by a token put in an arbitrary place (for example eq). This token will have then only two possibilities; it goes towards the place $\text{lo}$ if $T_4$ (or $T_9$) is fireable ($h_i < h_j - \delta h$), or towards the place $\text{gr}$ if $T_1$ (or $T_7$) is fireable ($h_i > h_j + \delta h$).

Consequently, the token would circulate between these three places to give, in real time, the state of the comparison of $h_i$ and $h_j$.

We propose a tolerance of error of the level of liquid $\delta h = 0.2 \text{ l}$ for the variables $h_1$ and $h_3$. The PN ($P_5$, $T_{11}$, $P_6$) was introduced to model a temporization to avoid false alarm at the beginning operation.

### 3.3.2 Monitoring of the discrete part

The proposed approach is based on the (Max, +) algebra, we transform the PN with inhibiting arc into traditional PN (Fig.9), knowing that this transformation is possible if the PN is bounded [6].

![Diagram](image-url)

**Fig.9:** Transformation of PN with inhibiting arcs into ordinary PN

The choice of $X(1)$ as initial condition can be raised from the simulation of normal operation system (Fig. 10).

![Diagram](image-url)

**Fig. 10:** Marking of $P_1$ in normal operation

We consider:

$$X(1) = \begin{bmatrix} x_1(1) \\ x_2(1) \end{bmatrix} = \begin{bmatrix} t_0 + \tau_{p_1} \\ t_0 + \tau_{p_1} + \tau_{p_2} \end{bmatrix}$$  \hspace{1cm} (15)

with $t_0$ moment of beginning of the periodic cycle. By using of the recurrent relation (5), one can deduce:

$$X(2) = \begin{bmatrix} x_1(2) \\ x_2(2) \end{bmatrix} = \begin{bmatrix} x_1(1) + \tau_{p_1} + \tau_{p_2} + t_0 \\ x_2(1) + \tau_{p_1} + \tau_{p_2} + t_0 \end{bmatrix},$$

$$X(n) = \begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} = \begin{bmatrix} n \tau_{p_1} + (n-1) \tau_{p_2} + t_0 \\ n \tau_{p_1} + n \tau_{p_2} + t_0 \end{bmatrix}$$  \hspace{1cm} (16)

According to the preceding representation, we can define: $T(n)$ as the difference between two successive firing time of $T_2$ (or $T_1$) and $K(n)$ the waiting time at place $P_1$

$$T(n) = x_2(n) - x_2(n-1) = x_1(n) - x_1(n-1) = \tau_{p_1} + \tau_{p_2} \text{ with } n \geq 2; \text{ and } K(n) = x_1(n) - x_1(n-1) = \tau_{p_1}.$$  

We can with each firing of $T_1$ and $T_2$ memorize the corresponding moments, and determine the durations $T(n)$ and $K(n)$. These values compared with the normal operation values (references), with the adequate tolerance, permit to detect a failure in the system.

### 3.3.3 Models of disturbance

To verify the combined monitoring approach, we introduce two types of disturbances:

![Diagram](image-url)
Disturbance 1: a significant leakage in the third tank (Fig. 11). This disturbance consists in introducing a leakage flow equal to $0.8 \times \sqrt{h_3}$ which lasts 10s after 100s since the starting of the cycle.

Disturbance 2: a disturbance in the control of valve 4 (blocking of V4) (Fig. 12). It consists in blocking V4 during 5s. This blocking is applied after 100s since the starting.

### 3.4 Simulation and interpretations

**Parameters of simulation:**

- Initial conditions: $h_{10} = 40$ l, $h_{20} = 35$ l, $h_{30} = 30$ l
- Conditions of commutation: $h_{1\text{max}} = 35$ l, $h_{3\text{min}} = 10$ l
- Firing speeds of the transitions:
  - $V_{T1} = 0.2$ l/s
  - $V_{T2} = \text{sign}(h_1-h_3) \times \sqrt{\text{abs}(h_1-h_2)}$ l/s
  - $V_{T3} = \text{sign}(h_2-h_3) \times \sqrt{\text{abs}(h_2-h_3)}$ l/s
  - $V_{T4} = \sqrt{h_3}$ l/s

From the simulation of the discrete part monitoring (Fig. 13(c)) we can determine $T(2) \approx 25$s, which is largely lower than 50s; the reference duration of $T(n)$ for a normal operation. Figure 14(c) shows a value of $K(2)$ equal to 30s, which exceeds the value 25s for normal operation. The simulation results of the continuous part show that:

**For disturbance 1:** It was not detected by the PN of the continuous part monitoring (there is no overtaking of the thresholds $h_{1\text{max}}$ and $h_{3\text{min}}$; Fig. 13(a) and 13(b)), whereas the PN of the discrete part monitoring has detected it (Fig. 13(d)).

**For disturbance 2:** It caused a delayed detection by the discrete part (Fig. 14(d)). The detection of the continuous part is faster (Fig. 14(e)).
4 Conclusion

In this paper, an HPN model is proposed for the monitoring of hybrid dynamic systems. The simulation of the normal operation shows clearly the commutation between the two continuous states system.

The introduction of the \((\text{Max}, +)\) algebra to improve the quality of failure diagnosis is proposed. It’s possible with this tool to process the temporal data over the moments of commutation. We also showed the complementarity of the presented approaches in monitoring of the continuous part and the discrete part of HDS.

In conclusion, we find that the use of only one tool for simulation and monitoring of hybrid systems is interesting because of simplicity in handling and treatment, which is enough attracting for the analysis of the systems having combined continuous and event dynamics.

The HPN model provides a significant quantity of information which can be exploited to deduce a methodology for isolation of the faulty parameters system. This could be the subject of later studies.

Annex: \((\text{Max}, +)\) algebra theory

A dioïd \(D\) is a set provided with two laws of intern composition \(\oplus\) (addition) and \(\otimes\) (multiplication) with following equivalences with the traditional algebra:

\[
\begin{align*}
\text{Max} & \leftrightarrow \oplus \\
+ & \leftrightarrow \otimes \\
-\infty & \leftrightarrow e \\
0 & \leftrightarrow e
\end{align*}
\]

\(e\) and \(e\) are the neutral elements for these two laws which verify the following properties [10]:

- The law \(\oplus\) is associative, commutative and idempotent \((a \oplus a = a)\)
- The law \(\otimes\) is associative, distributive compared to \(\oplus\).
- \(e\) is absorbent for the law \(\otimes\):
  \[
  \forall a \in D, a \otimes e = e \otimes a = e
  \]
- \(e\) is the neutral element for the law \(\otimes\):
  \[
  e \otimes a = a
  \]
- The relation \(\geq\) is a relation of order in the dioïd \(D\):
  \[
  \forall a, b \in D, a \geq b \iff a \oplus b = a
  \]
- A dioïd is known as complete if it is closed for any infinite sum of these elements and if the law \(\otimes\) is distributive compared to these infinite sums.
- If \(A\) and \(B\) are matrices with coefficients in \(D\); the equation \(Ax \oplus B = x\) has as a solution
  \[
  x = A^{*} \otimes B
  \]
  with \(A^{*} = E \oplus A \oplus A^{2} \oplus \ldots \oplus A^{n} \oplus \ldots\)
  and \(E\) is the identity matrix in dioïd \(D\).

References: