In-Band Quantization Noise Power Estimation in High Order Single Bit ΣΔ Modulators using Taylor Series

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Abstract: - An analytic method for the calculation of the quantization noise power within the bandwidth of interest in stabilized high order Sigma-Delta modulators is presented. In contrast with previous approaches, the use of an expansion in Taylor series of the squared magnitude of the noise transfer function is proposed. The predicted results are compared with simulated data and good agreement is reported.

Key-Words: Analog-digital Conversion, Estimation, Quantization, Sigma-delta modulation.

1 Introduction

Oversampled A/D converters with noise shaping have become very popular in the last years. They are preferred for high resolution A/D conversion in digital audio, instrumentation and communications. Its circuit implementation is very robust; therefore, they are good candidates to be implemented in deep sub-micrometric CMOS technologies.

The design of such converters comprises both architectural and circuit aspects. The selection of the optimal architecture for a system is a critical issue, which requires to consider several factors as stability, maximum achievable signal-to-quantization noise ratio (SQNR) and dynamic range (DR). Classically, many designers obtain those values with simulations. For example, in [1] a family of high order stabilized ΣΔ modulators (ΣΔM’s) was presented, where the SQNR was estimated from the output signal spectra. This method requires long simulations, and gives a limited insight in the properties of the architectures before running the simulations, since it does not use the equations defining the system behaviour for doing

![Fig. 1. BPΣΔM under analysis.](image)

This Paper proposes the use of Taylor series expansion (T.s.e.) of the noise transfer function (NTF) of an already stabilized ΣΔM in order to find an approximated value of the module of the NTF under question. This facilitates the solution of the bounded integrals used to calculate the in-band quantization noise power (P_e). If a mathematical symbolic processor is used, the method can be programmed and its execution is very fast.
2 In-Band Quantization Noise Estimation

In general, $P_e$ is calculated by solving the following bounded integral:

$$P_e = \int_{-f_b}^{f_b} |NTF(z)|^2 N_e df$$

(1)

Where: $f_b$ is the bandwidth of interest, $N_e$ is the quantization noise power spectral density and $z$ is a complex variable defined as $z=\cos(2\pi f/f_S)+j\sin(2\pi f/f_S)$ being $f_S$ the sampling frequency. It is well known, that an $n$ order stabilized single bit $\Sigma\Delta M$ posses a NTF, which strongly deviates in form from that of a pure differentiation of the $N_e$ of the same order, rather we have a rational function in $z$ of order $n$:

$$NTF(z) = \frac{(zero_1+z)(zero_2+z)\cdots(zero_n+z)}{(pole_1+z)(pole_2+z)\cdots(pole_n+z)}$$

(2)

The same situation arrives when a bandpass $\Sigma\Delta M$ (BP$\Sigma\Delta M$) whose resonators were not derived by using the $z^1\rightarrow z^2$ transformation but by means of the continuous to discrete time bilinear transformation is analyzed [2]. Under these conditions, the evaluation of (1) by hand using the substitution $\sin(x) \approx x$ [3] to find out a value for $|NTF(z)|^2$ could become very difficult. In such cases, the use of programs for symbolic computation is a powerful tool, which significantly simplifies the work.

3 Taylor Series Expansion

For this analysis the following assumptions were taken: first: $f_b$ is a very small fraction of $f_S$, this means $f_b << f_S$. Secondly: we are interested in knowing the behavior of the quantization noise only in a small region within the normalized frequency axis, since the out of band noise is assumed to be rejected by a high order digital filter that follows the modulator. Observe that these two assumptions are realistic in well designed $\Sigma\Delta M$’s.

For a well behaved function, the one-dimensional T.s.e. of a real function $f(x)$ about a point $x = a$ is given by [4]:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

(3)

Observing (3) and thanks to the assumptions already mentioned, it is possible to expand using Taylor series the squared module of the rational function of $z$ representing the $NTF$ of the modulator under analysis, around the normalized frequency of interest. Using a symbolic processor, it is a standard and straight forward task, which requires just a few lines of code. After that, by taking the first term of the series, we have an estimation of the value of $|NTF(z)|^2$. This term can be used to substitute $|NTF(z)|^2$ and to evaluate the integral within the bandwidth of interest.

4 Validation

In order to proof the proposed method two stable $\Sigma\Delta M$’s were used. Fig. 1 shows a BP$\Sigma\Delta M$ whose resonators were derived from continuous time prototypes [5] while Fig. 2 depicts a $4^{th}$ order $\Sigma\Delta M$.
designed in [6], which has the coefficient set given in table I. We have chosen those two architectures because their NTF’s are rational functions in z of high order, as it will be seen.

The analysis shows that the NTF(z) of each architecture is

$$NTF_1(z) = \frac{8z^2 + 16z^4 + 8z^6}{1 + 6z^2 - 4z^3 + 9z^4 - 4z^5 + 8z^6}$$

and:

$$NTF_2(z) = \frac{1 - 4z^{-1} + 6z^{-2} - 4z^{-3} + z^{-4}}{1 - 3.1806z^{-1} + 3.8612z^{-2} - 2.1122z^{-3} + 0.4383z^{-4}}$$

Using the symbolic processor Mupad [7], eqs. (4) and (5) were analyzed. First, the substitution $z = \exp(\pm 2\pi j f N)$ was carried out, where $f_N = f_s / s$. The resulting functions were stored in auxiliary variables, which were multiplied with each other to find out the squared magnitude, since $z^* z = |z|^2$. Second, the T.s.e. was executed. Eq. (4) was expanded around $f_s = 1/4$ while (5) around $f_s = 0$ because it is the NTF of a lowpass modulator. The first term of the T.s.e is for (4) equal to $1024\pi^4 f_N^4$ and for (5) approximately equal to $6.17\pi^2 f_N^8$. Using these values to solve (1) it is found:

$$P_{s1} = \frac{\Delta^2 \pi^4}{15 \times M^2}$$

$$P_{s2} \approx \frac{\Delta^2 \pi^{13}}{1.5 \times M^9}$$

Where $\Delta$ is the quantization step and $M$ the oversampling ratio defined as $f_s / f_b$. If a signal with power $S=A^2 / 2$ is applied to the input of the $\Sigma\Delta$M’s, from (6) and (7) the SQNR is given by:

$$SQNR_1 = \frac{15M^5 A^2}{2\Delta^2 \pi^4}$$

$$SQNR_2 = \frac{1.5M^9 A^2}{2\Delta^2 \pi^{13}}$$

According to the last expressions, the $\Sigma\Delta$M of fig. 1 would reach a SQNR of 73 dB (12 b) if an $M=64$ is used. A high resolution of 16 b (SQNR=96 dB) would be reached by the structure of fig. 2 if an $M=73$ is used assuming that the performance of the systems is limited only by quantization noise. Both architectures were simulated with MIDAS [8] for $f_s=168$ and 26 MHz respectively. The output spectra are shown in figures 3 and 4. Figures 5 and 6 display the SQNR obtained by simulation. The peak values have discrepancies of approximately 3 dB and 1 dB from the ones predicted by (8) and (9) respectively; this corresponds to differences of less than one bit. These results prove that the proposed analytical method to estimate the SQNR agrees well with the empirical method based on simulation used previously.

5 Conclusion

An analytical method for the estimation of the in-band quantization noise power in high order $\Sigma\Delta$M’s was presented. The approach relies on the use of a mathematical symbolic processor to expand, using Taylor series, the squared magnitude of the NTF of the modulator under question. The validity of the
The proposed method was compared with results obtained by simulations showing discrepancies of less than one bit. This enables the designer to have a very good approximated value of the performance of a chosen architecture in an early design phase without using any simulation program, leaving this to a later task, where the requirements of the basic building blocks should be obtained by means of behavioral simulations as starting point of the circuit design stage.

References: