Performance Evaluation for OFDM PAPR Reduction Methods

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Abstract: - The ultimate goal of Peak to Average Power Ratio (PAPR) reduction for Orthogonal Frequency Division Multiplexing (OFDM) signals is to reduce the signal’s sensitivity to nonlinearity and to improve signal quality and transmitter power efficiency. Performance evaluation has traditionally been based mainly on the PAPR distribution curves. For clipping related methods, however, tradeoffs between PAPR and signal quality must be made, but is often made in a heuristic manner. In this paper, we propose to include amplifier characteristics and use the required backoff to evaluate the performance of PAPR reduction methods. The required backoff OBO* is defined as the minimum required transmit power backoff for the signal to meet quality requirements such as spectrum mask and error vector magnitude constraints. Effective PAPR reduction should lead to smaller OBO*, not just smaller PAPR. Simulation results for IEEE 802.11a wireless local area network signals with clipping related methods are presented.

Key-words:- OFDM, PAPR Reduction, Clipping, Performance, Intermodulation, Backoff.

1 Introduction
Many methods have been proposed in recent years to reduce the peak to average power ratios (PAPR) of OFDM signals, e.g. clipping, coding, SLM, PTS, tone reservation, e.t.c. [1]–[4]. Although the immediate goal is PAPR reduction, the ultimate goal of these methods is to reduce signal’s sensitivity to transmitter nonlinearity, to improve the signal quality and transmit power efficiency. The effectiveness of these methods may not always be clear from observing the PAPR alone. A comprehensive performance evaluation involves the PAPR CCDF, the bit error rate curves (reflecting inband intermodulations), the power spectral density (reflecting outband intermodulations), and tradeoffs between them. The tradeoffs are often made in a heuristic manner. In this paper, we suggest to consider amplifier characteristics and define the required backoff at the transmitter as a comprehensive performance measure for PAPR reduction. The required backoff can be defined as the minimum backoff for the transmit signal to satisfy signal quality requirements such as spectrum mask and error vector magnitude. Simulation results using 802.11a wireless lan OFDM signals with clipping related PAPR reduction methods are given. When the amplifier is modeled as a soft limiter, it is shown that clipping below saturation does not appear to give better performance even though the PAPR can be further reduced.

In Section 2, concepts on transmitter power efficiency, backoff and PAPR are reviewed. The required backoff OBO* is defined in Section 3. Simulation results using IEEE 802.11a signals and clipping related PAPR reduction methods are presented in Section 4. Some issues on performance evaluation for tone reservation methods are also discussed.

2 Power Efficiency, Backoff, PAPR
The communication transmitter amplifier’s power efficiency has traditionally been measured by the output backoff (OBO), i.e. the difference between the amplifier’s saturation output power and the average transmit signal power after amplification, 
\[ \text{OBO} = P_{\text{sat}}(\text{dB}) - P_{\text{av}}(\text{dB}) \]

The power consumption of an amplifier is mainly determined by the saturation power \( P_{\text{sat}} \). With the same
average transmit signal power $P_{av}$, a signal that requires smaller backoff allows the amplifier to operate with a smaller saturation power $P_{sat}$ and thus reduces power consumption.

The complex baseband time sequence of an OFDM symbol in a symbol duration can be expressed as the sum of $N$ orthogonal subcarriers as follows:

$$x_k = \sum_{n=1}^{N} X_n e^{j2\pi(n-1)\Delta f k/T} , k = 0,1,...,LN - 1$$

(1)

$N$ is the number of orthogonal subcarriers, $L$ is the oversampling ratio, $NT$ is the symbol duration, $\Delta f = 1/NT$ is the subcarrier frequency spacing, $x_k$ is the $kth$ time sample, and $X_n$ is the frequency domain symbol for the $nth$ subcarrier. The peak to average power ratio (PAPR) of a symbol is defined as follows.

$$\text{PAPR} = \max_k \frac{\|x_k\|^2}{E[\|x_k\|^2]} \quad k = 0,1,...,LN - 1$$

(2)

The maximum PAPR for $N$-subcarrier OFDM symbols is equal to the number of subcarriers, PAPR=$N$. This occurs when all subcarriers add constructively at some point in the symbol duration. As the number of subcarriers increases, more OFDM symbols can have high PAPR. The existence of high PAPR OFDM symbols is the main reason leading to low transmitter amplifier power efficiency. To prevent quality degradation caused by nonlinear distortions, signal power needs to stay mostly within the amplifier’s linear region. For high PAPR signals, the average signal power must be backed away significantly from amplifier’s saturation power to reduce the chance of high power signal samples entering the nonlinear region. The required OBO for OFDM signals is much greater compare to OBO for low PAPR signals, e.g. single carrier GSM signals.

Notice, however, that high PAPR becomes a problem only when nonlinearity is present. When predistortion is performed, the amplifier’s input/output characteristics can be almost completely linearized up to saturation point. Signals entering this linearized amplifier will be distorted only if their magnitudes are above saturation. It is unnecessary to reduce the magnitude further if it is already below saturation. For example, if signal samples are “clipped” to a threshold magnitude lower than the amplifier’s input saturation magnitude, the PAPR is reduced but more inband and outband intermodulations are introduced. This concept is illustrated numerically in Section 4 through our simulation results using IEEE 802.11a OFDM signals.

3 Required Backoff OBO*

Consider the IEEE 802.11a OFDM signals [5]. The transmit signal must satisfy two quality requirements: spectrum mask and error vector magnitude (EVM) [5]. Both requirements can be satisfied with a sufficient amount of backoff. The required backoff OBO* can be defined as:

$$\text{OBO*} = \max(\text{OBO}_{\text{mask}}, \text{OBO}_{\text{EVM}})$$

(3)

where $\text{OBO}_{\text{mask}}$ is the smallest output backoff required to satisfy the spectrum mask (see Fig. 1), and $\text{OBO}_{\text{EVM}}$ is the smallest output backoff required to satisfy the error vector magnitude constraint (see Fig. 2)[6]. $\text{OBO}_{\text{mask}}$ can be viewed as a measure of outband intermodulations, and $\text{OBO}_{\text{EVM}}$ is a measure of inband intermodulations.

![Fig. 1 Transmit Spectrum Mask for IEEE 802.11a][6]

![Fig. 2 Error Vector Magnitude for IEEE 802.11a][6]

There exists a tradeoff between power efficiency loss in terms of OBO, and signal quality loss in terms of inband/outband intermodulations. An effective PAPR reduction method can improve the tradeoff between power efficiency and inband/outband signal quality.
This means that after the PAPR reduction method is applied, one can either obtain better signal quality given the same power efficiency, or obtain better power efficiency given the same signal quality. Better signal quality means fewer errors for the inband received signals and lower outband components in the signal’s power spectral density. Better power efficiency means less required backoff (OBO). Signal quality improvement can also be translated into improved receiver sensitivity. Note however that the amount of outband intermodulations (i.e. interference to users in adjacent frequency bands), which cannot be reflected by the receiver sensitivity, should also be considered.

4 Performance Evaluation
In this section, we observe the required backoff OBO* for 802.11a signals (16 QAM subcarrier modulations) with clipping related PAPR reduction methods [1]~[5]. Clipping combined with low pass filtering or peak windowing was considered to be a favorable PAPR reduction method due to its simplicity. The required backoff reveals that the optimal clipping threshold is at the amplifier’s saturation point but not lower.

Table I contains the results of clipping related methods [6][7] with several clipping thresholds where the amplifier model is taken as the soft limiter (i.e. AM/AM linearized up to saturation, AM/PM is identically zero). The clipping threshold is defined as the ratio of the clipping threshold to r sat. The parameter Th is defined as the ratio of the clipping threshold to r sat. Th=0.9 means the clipping threshold is 0.9r sat. For each OFDM signal time sample xk, the clipped sample yk is:

\[ y_k = \begin{cases} x_k & \text{if } |x_k| < Th(r_{sat}) \\ Th(r_{sat})e^{j\angle x_k} & \text{otherwise} \end{cases} \]

The clipped sample yk has magnitude \( \min(Th(r_{sat}), |x_k|) \) and phase \( \angle x_k \).

Note that with a soft limiter amplifier model, clipping at Th=1 equals no clipping since the amplifier will perform the same “clipping” action. Clipping at Th>1 also equals clipping at Th=1. We used a modified peak windowing algorithm that dynamically adjusts the hamming window width to be two samples wider than the width of the peak above threshold as in [7]. For clipping plus filtering [2] the low pass filtering impulse response length is equal to the OFDM symbol duration. Compare to peakwindowing which only “round off the corners” for samples in clipped peaks, low pass filtering affects more OFDM time samples. It can be seen in our simulation results that low pass filtering performs better in suppressing outband components compare to peakwindowing at the cost of increased inband errors (larger EVM and OBOEVM).

Table I shows the OBO mask, OBOEVM and OBO* as clipping threshold (and thus the resultant PAPR) reduces for each method. Low pass filtering and peak windowing reduces the required backoff compare to clipping alone, but their required backoff OBO* still increases as the clipping threshold (Th) decreases. For clipping only, OBO*≈OBO mask. For clipping plus filtering or peakwindowing, OBO*≈OBOEVM. Low pass filtering and peakwindowing suppress outband components at the cost of inband signal quality, leading to larger EVM and OBOEVM compare to clipping alone. Peakwindowing achieves the smallest OBO* among all three methods at Th=1.

In [1]~[4] the clipping thresholds (clipping levels) are set with respect to the signal’s average power in terms of a clipping ratio (or crest factor) without reference to the amplifier’s characteristics. Tradeoffs are made between PAPR reduction and signal degradation (bit errors, outband components) in a somewhat heuristic manner. More clipping always lead to lower PAPR and higher errors/outband components, and there is no obvious way to find the clipping threshold that achieves the optimum tradeoff. We believe it is necessary to consider amplifier characteristics when evaluating clipping related PAPR reduction methods. Table I reveals that it is not worthwhile to sacrifice signal degradation from clipping below amplifier’s saturation point in exchange for PAPR reduction. The optimum clipping threshold is at Th=1 (i.e. do not clip below saturation). This can be observed only when the clipping threshold is set with respect to the amplifier saturation magnitude, not when it is set with respect to the signal’s average power as in [1]~[4].

Table I shows that overall, the optimum clipping is at Th=1 (clip at amplifier saturation, equivalent to no clipping) rather than Th<1 (some clipping below saturation). This somewhat surprising result can be understood since in our assumption the amplifier distorts signal samples only when the sample’s magnitude is above saturation. Even though the PAPR can be further reduced, clipping below saturation leads to unnecessary inband and outband signal quality
egradations. Even when the amplifier model is replaced by more realistic nonlinear AM/AM, AM/PM curves, similar behavior can still be observed since the nonlinearity introduced by clipping is more severe compare to amplifier nonlinearity.

**TABLE I**
**REQUIRED BACKOFFS FOR 802.11A SIGNALS**
(16 QAM-OFDM)

<table>
<thead>
<tr>
<th>Method</th>
<th>OBOmask</th>
<th>OBOEVM</th>
<th>OBO*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Th = 1</td>
<td>Th = 0.9</td>
<td>Th = 0.8</td>
</tr>
<tr>
<td>Clipping</td>
<td>5.01</td>
<td>5.92</td>
<td>6.94</td>
</tr>
<tr>
<td>CF</td>
<td>4.30</td>
<td>3.37</td>
<td>2.47</td>
</tr>
<tr>
<td>PW</td>
<td>3.83</td>
<td>4.64</td>
<td>5.67</td>
</tr>
<tr>
<td></td>
<td>OBOEVM</td>
<td>OBO*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Th = 1</td>
<td>Th = 0.9</td>
<td>Th = 0.8</td>
</tr>
<tr>
<td>Clipping</td>
<td>3.88</td>
<td>4.81</td>
<td>5.83</td>
</tr>
<tr>
<td>CF</td>
<td>4.50</td>
<td>4.98</td>
<td>5.87</td>
</tr>
<tr>
<td>PW</td>
<td>4.05</td>
<td>4.76</td>
<td>5.75</td>
</tr>
</tbody>
</table>

CF: Clipping and Filtering. PW: Peak Windowing

Another PAPR reduction method where performance evaluation using PAPR alone may be misleading is tone reservation [8]. In tone reservation, some subcarriers are reserved as “peak reduction tones” that transmit signals to reduce peak power. Reserved tone signals can be computed using gradient based or other methods [8]. Reserved tone signals are not recovered at the receiver and do not need to follow the format of signals on the data subcarriers. To see why PAPR may not faithfully reflect the performance, consider the example where one subcarrier is selected as the reserved tone. A trivial way to reduce PAPR is to increase the signal power on the reserved tone until the OFDM signal resembles a “single carrier” signal. The seemingly significant PAPR reduction will not be effective since all signal power is taken by the reserved tone that does not transmit data. This means that one cannot tell, by observing the PAPR CCDF curves alone, whether the PAPR reduction in tone reservation methods is effective.

One possible remedy is to express the signal power as the sum of reserved tone power and data tone power,

\[ P_{av} = P_{av\_res} + P_{av\_data} \]

The “peak to average data tone power ratio” (PADPR) can replace the PAPR as a performance measure. Tone reservation is effective if the maximum instantaneous power of the OFDM signal is reduced after the addition of the reserved tones. Unlike clipping, tone reservation does not lead to signal quality degradation (assuming the orthogonality between subcarriers is maintained). The goal of PAPR reduction is achieved as long as the peak power of the overall signal (data plus reserved tones) is reduced given the same data signal power, i.e. when the PADPR is reduced. One can still use the required backoff as an alternative performance measure. In this case the required backoff can be modified by replacing \( P_{av} \) with \( P_{av\_data} \) in calculating both OBOmask and OBOEVM. The overall required backoff is referred to as the OBO* data. For the tone reservation method to be effective, the PADPR or \( OBO*\_data \) should be lower comparing to not only the original signal but also to the trivial case of sending no signals on the reserved tones.

5 Conclusion

Performance evaluation for PAPR reduction methods requires tradeoffs between PAPR, error rates, and outband intermodulations. The effectiveness of a method depends on whether or not it can improve transmitter power efficiency and/or signal quality, which may not be obvious from observing PAPR CCDF curves alone. We propose to use the required backoff—defined as the smallest backoff required to satisfy transmitter signal quality requirements—as a comprehensive performance measure that summarizes inband/outband intermodulation information and directly reflect power efficiency. Simulation results with 802.11a signals when the amplifier is modeled as a soft limiter reveal that clipping (plus filtering or peak windowing) is most effective when the threshold is set at amplifier’s saturation amplitude. Peakwindowing with adjustable window width performs better compare to clipping plus filtering. The required backoff can be used to evaluate the performance of other PAPR reduction methods as well. The proposed performance measure is in not necessarily the only reasonable choice. What’s important is to realize that when evaluating PAPR reduction methods, it may be necessary to go...
beyond PAPR and to consider both the signal quality and the power efficiency through OBO* or other similar more comprehensive performance measure.

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References